

Synopsis Mathematica Universalis:

OR THE

Universal Mathematical **SYNOPSIS**,
Translated from the third and last
Edition of *John James Heinlin*, Prelate
of *Bebenhusan*. *J R 161*

CONTAINING

Variety of useful Practices in *Arithmetick*,
Geometry, *Trigonometry*, *Astronomy*, *Dialling*,
Chronology, *Geography*, the *Optics*, *Catoptrics*,
Dioptrics, the *Statics*; Together with *Astro-*
nomical and *Geographical* Tables, with their
Uses, and also the use of the Globes. The
whole illustrated with several Mathematical
Sculptures on Copper Plates.

Corrected and enlarged with many Additions.

By *Venterus Mandey*, Philomat. pr. 6 s.

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To the Honourable Society of Lincolns-
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the Members of the Bench of the said
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Honoured Sirs,

I Humbly beg your Pardon for bringing
this Book under your Protection. Were
it a Work of my own, or I any thing but the
Translator, I should constrain my Thoughts
to a meaner Dedication: But being a Colle-
ction of some of the greatest Masters in the
World, I mean, *Heinlin, Mestlin, Kepler, &c.*
And never made *English* till now, I conceiv-
ed I might the sooner procure their Welcom,
to Persons so eminent for noble Candor, as
well as for all those intellectual Excellencies,
wherewith your rich Souls are known to be
furnished: I resolved to be as kind to this
Book as I could, and seriously considering

Epistle Dedicatory.

which way to effect it, I at last concluded (with Submission) to prefix your Names to it, who having a higher Sense, and Judgments of excellent Parts, are best able to defend my Lapses and Imperfections.

The *Mathematical Sciences*, although in their Speculations most noble and pleasing to a *Philosophick* Mind, and in their Practice very useful and profitable to Mankind, yet have they not been equally approved by all; For which cause, such Writings as promulge and offer at the advancing of those Sciences, stand in need of such Patrons as are able to defend them against the Cavils of Pride, Envy and Ignorance.

And for as much as the Study of Knowledge is incumbent on all great Men, such as Judges, Rulers, Councillers, &c. And that they who delight and have Knowledge in *Mathematical Sciences*, are greatly assisted in the right governing in a Commonwealth, as well in times of War as of Peace, to the discerning of all things aright.

Whence also many Emperors, Kings, and Princes, &c. Have been so ravish'd with the *Mathematical Study*, that they have not only diligently studied it themselves, but have also been at great Charge in employing the most excellent Artists they could get to propagate and increase it.

It is commonly said that all Learning is of a Piece, and that *Nescit generosa mens ignorantiam pati*

Epistle Dedicatory.

pai; An Instance of the Truth of which saying, appeared in that late great and learned Person the Lord chief Justice *Hale*, lately a Member of your Honourable Society, who besides his great Knowledge in the Law, was a great *Divine* and *Mathematician*, as appears by his Books and Manuscripts in the Library of your Society, some of which I have seen.

Now having had the Honour (for so I esteem it) of serving your Honourable Society for about Thirty three Years in my Trade, and having received some Advantage thereby, I thought it just Gratitude to make this small Present, as the Poor Man doesto his rich and great Neighbour, by whom the Nosegay of Flowers, or Basket of Fruit is not ill taken, altho' he hath more Plenty, and far better of his own Growth. Small Things become valuable when they are made the Offerings of Respect, Esteem and Gratitude, for *Seneca* saith, there neither is, nor can be any Law against Ingratitude.

I know, you Gentlemen, have so much and great Employment for his Majesty and his good Subjects, that I shall not rob you of another Minutes Space, besides my Prayers to God to bless and prosper your Honourable Society, and the Liberty of Subscribing my self

Your most Humble and
most Obedient Servant.

Venterus Mandey.

THE
OLD PREFACE
TO THE
FIRST EDITION.
TO THE
Pious and Candid Reader
The Author wishes Happiness.

HOW much the pleasure and advantages are, of the Mathematical Sciences, as they are usually called, *viz.* which are exercis'd about the Numbers and Magnitudes of things, which serve not only to the better understanding of all other Arts and Sciences inferior and superior: But also are necessary to all Men, to the more convenient living a Civil life, we think it not convenient in this place to enlarge upon; for concerning this of Mathematical things, no Ingenious Men doubt: but the unskilful, unless they were entred into this study, and perceived some sweetness of it, can never be drawn to it as they believe, by any Elogy of words.

Therefore leaving those; we shall declare a few things, by what Authority, or to what end and purpose, *this Mathematical Synopsis* was compos'd; and in what order and method, we Imagine the studious in the Mathematicks, may advantageously be Employed in it.

The knowledge of the Mathematicks among other studies also, always flourish'd from the first beginning of this Academy, if not among all Learners, yet among Teachers; which have been most expert Artists in the Knowledge of the Mathematicks; and not in *Germany* only, but in all *Europe*, the most Famous, *John Staffler*, *Samuel Siderocrates*, *John Scheubelius*,
Philip

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Philip Appian, Michael Mestlin, Eberhardus Scultesius, and not only he that was very Famous and had in great honour among us, but also among Foreigners, and is now in Heaven, *VVilliam Schickard*, but also the most Ingenious and most excellent Mathematician *John Kepler*, altho he spent not his Days with us, yet he came forth from our School, and was Famous through all *Europe*: But alas! the dismal Fire of the late Wars, as it brought all other kinds of mischief along with it, so it miserably broke this School of ours at *Tubingen*, and with other Sciences and Arts. Also the most noble study of the Mathematics was almost destroyed: which ill fortune falling grievously on it, our most *Serene Prince and Governour the Lord Eberhard the III. Duke of Wirtemburge*, &c. following the footsteps of his Ancestors, among other things did also reestablish the *Mathematicks*, which was at it were destroyed; not only to be taught in the Publick University, but also, would have the first fundamentals of it, taught in the Schools of Monasteries; therefore for this end, and by the command and mandate of our most Serene Prince, *this present Synopsis* is collected, not of one, but of many Mathematical Disciplines, as well from the Lucubrations of ours, which of late flourish'd in this Academy, as of others, more noble Lovers of this Science; it being made use of by the advice of Learned Men: In which, what shall be appointed, or in what order and manner any one in our opinion may profitably be conversant in it, we shall briefly shew.

Therefore in the beginning, least any one by seeing the bigness of the Book should be offended at the Title *Synopsis*, he must know, that we deliver here not one, but as in time past five, so now six distinct Disciplines; yea if after the manner of others, we were willing to divide it into small parts, we might make it into more than Ten: If this were divided into so many parts, it might very well still retain the name of *Synopsis*, or *Compendium*. But why are we not content with one Science, but would gather together many Systems? The nature it self of the *Mathematicks* requires it: For often times the parts of it are so bound, and as it were joined with the bond of Relation, that one without the other, can neither be disposed in due order, nor learned with any benefit. For Example, some Knowledge of *Astronomy*, especially to the studious devoted to *Theology*, was always requisite in our Schools: But what Knowledge I pray, can any one have from a Treatise of *Astronomy*, without somewhat of *Arithmetic*, of *Geometry*, of *Geography*, and also of *Optics*? which if any one

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wants, he will scarce learn more in *Astronomy*, than a blind Man in the Art of Painting. That therefore these should be joyned together, Necessity it self required it.

In laying down these things, we observe this Order, viz. That we premise those things, the Foreknowledge of which is chiefly necessary in the latter: Whence *Arithmetic* and *Geometry* preceed *Astronomy*; for as much as they are, as it were Wings, with which we fly into Heaven: *Geography* follows *Astronomy* (which by Reason of its great Dignity, we have indeed handled more fully; but if you consider the Multitude of things contained in it, we have handled it compendiously enough) which is so joyned with *Astronomy*, that neither without the other, can entirely be understood. The *Optics* follow *Geography*, which although it hath hitherto for the most part been neglected, even by those who would hearken to *Philosophy*; yet the Benefit and Light it affords to almost all the other Sciences is such, that it may truly be said (considering its great Merit) that it is the *Sun*, the *Eye* and *Beauty* of them all: Therefore we were willing also only to delineate it, if perchance after a long time of *Exile*, it may recover its Dignity and Place among some or other.

Those things which we have annex'd by the way of *Appendix*, or scatter'd any where by *Digression*, although they are not properly of these Disciplines, yet by reason of the great Pleasure and Utility of them, we would not neglect them; supposing that a Lover of these Studies, if he should find any thing that's good out of the way, he will no more despise it, than if in digging his Field, he should find a hidden Treasure.

We have omitted the long Demonstrations of *Geometrical Things*, since because I study Brevity, we respect Practice more than Theory, also because all the Conclusions drawn from *Geometrical Things*, are approved of, by the Opinion of all learned Men; whence 'tis fit that those that learn should also believe them. Not to speak of some Demonstrations which by reason of their Subtily, are sometimes more obscure than the Conclusions themselves: yet if any one require them, he may satisfy himself with *Euclids Elements*, or the Commentators upon them, the excellent *Clavius*, *Ambrose*, *Rhodes*, *Orontius*, *Pisiscus* and others.

And in handling these Disciplines, we would have *Infermers* or *Instructors* observe these things; for we require the Informer to be altogether learned, diligent and prudent: For what ill Accrues, from the Studies of those Youths, who rashly take into their Hands any Books without a Teacher, and presently

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sently from them perswade themselves to be learned, Experience testifies; for oftentimes they stick a great while on some light matter, and vex and faitgue themselves on it in vain, from which they might disintangle themselves with a great deal of Ease by the help of a faithful Instructor: But at length when they perceive themselves to be exercised in such things in vain, where they scarce touch upon the Entrance of any Discipline but they conceive a Disdain of it, and they reject it again as insuperable.

For this reason we require an Informer which will explain all things faithfully; and not to propose to Youth, the learning of so much; which they usually do, who think they have performed their Duty according to their Reward, if they obtrude on their Schollars, the charging their Memory with some Prolix, not understood Lesson.

Therefore we wholly deliver the studious from this Labour of learning by heart, for the *Mathematics* are not to be learnt by heart, but to be understood, and also they are to be exercised by hand; to which Exercise, a *Ruler*, a pair of *Compasses*, a *Celestial and Terrestrial Globe*; or instead of these, *Charts* and *Geographical Maps* are necessary: In these if the Masters and Schollars are sometime conversant, it cannot but happen, that also they will bear the Words themselves in memory; from which right Understanding of things, they may also speak plainly, that by reason of it there will scarce be any need of peculiar learning by heart; especially since *Mathematical* things are not only laid before the Understanding as *Metaphysics* are, but are also contemplated by the Eyes and handled by the Hands.

Nor yet also do we think it convenient, that Learners in the Beginning, should run through the whole Body of the Sciences, as they are delivered in this *Synopsis*: But by degrees not only to ascend from one Discipline to another, but also from one of one Part, to the other Part.

And therefore we constitute three Orders, in the Study of the *Mathematics*; beginning, going forward, and making a stand; Nevertheless no Law or Punishment to be constituted, if any one should invade the Ends of another, before 'tis time; only that he insist in a right way, and observe a convenient order, not by Girds and Leaps, to be conversant, sometimes in one Part, and other times in another.

I. Therefore in the Beginning, is designed the first and second Part of *Theoretical Arithmetick*, as well in whole as in broken Numbers, which Men ought to know, by reason of the great

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great Use which they have in all Human Affairs: From whence if any one being dull of Wit, proceed no further, but would abstain from all the other Parts of the *Mathematics*, yet methinks he should not think much of the Labour to add from the third Part concerning Proportions, only *the Golden Rule of Proportion*, which is the chief Point, of the five Species or Rules of *Arithmetick*.

But if our *Beginner* would ascend higher also, to other Parts of the *Mathematics*, we may propose to him the First and Second Part of *Geometry*: In the First are contained *the Definitions of Geometrical things*, and as it were, exposes all the Furniture of *Geometry*, and subjects it to the Sight: The Knowledge of which also, although he would proceed no further, yet this Advantage would not be contemned, which so admirably brings into the Mind, not without great Pleasure, the *Ideas* of Corporeal Things, from the Divine Mind into the Human Kind of fallen Men, of which these rude Figures of ours, are as it were a Defluxion of Material Things.

To this most pleasant Contemplation, if our *Beginner* also add the Second Part of *Cyclometry*, being easie to be understood and comprehended: 'Tis no doubt but he will be then incensed, from the Love and Desire of the knowing *Mathematical Things*, to proceed with a chearful Mind, and hasten on to the Order of going forward.

II. Then after *these Proceedings*; he may first run through the two other Parts of *Theoretical Arithmetick*, in which, because he is already exercised in the Five Species or Rules of *Arithmetick* (especially in Fractions which have a great Affinity with Proportions) he may pass with an inoffensive Foot, and accomplish them in a short Space of time.

But from hence, to them that would sometime do Service in the Common Wealth, they may profitably enter upon the other Parts of *Geometry*: In which, indeed not all things are contained in these Arts, which are necessary to be known, to Politick Men, in times of Peace and of War, yet for the most part the Foundations are here laid, without the right Understanding of which things, their Knowledge will be either imperfect, or doubtful.

But because this Labour is designed chiefly for them *who think sometime to serve in Ecclesiastical Affairs*, on whom there is incumbent a greater Care of Heavenly than of Earthly Things, after the Understanding of *Theoretical Arithmetick*, and the First and Second Part of *Geometry*, they may forthwith enter upon the three Books of *Astronomy*, and after these to the First

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First Part of Geography: To which may commodiously be joyned the Astronomical and Geographical Problems (by laying aside for some time the *Trigonometrical Calculation*) which the Use of the Celestial and Terrestrial Globes shew, and they will also subject those things to the Sight, which they have heard and read in *Astronomy* and *Geography*.

He that shall have learned these things, will have more right to the Title of a Master of *Philosophy*, than if he would acquire and continue only, in some Knowledge pertaining to *Logick* and *Rhetorick*.

III. All the rest which remain, in *Arithmetick*, in *Geometry*, in *Astronomy*, in *Geography*, and in *Optics*, are designed for them that are *durable* and *consistent*; among which we propose those who are consecrated to any Superiour Faculty, who cannot grow old in the *Mathematics*, and in other Studies of *Philosophy*.

Nevertheless least we should seem to pass over with ungrateful Silence, any of them who have contributed any thing either of their own, or of others, to this Work. Of the Ancients, they are these, *Euclid*, *Ptolomey Geber*, *John de sacro Busto*, *G. Purbachius*. Of the Moderns, *Nicholas Copernicus*, *Erasmus Reinholdus*, *Tycho Brahe*, *Peter and Philip Appian*, *Gemma Frisius*, *Bartholomew Pitiscus*, *G. Lud. Frobenius*, *Tho. Winshemius*, *C. Peucerus*, *Adrian Metius*, *Christo. Clavius*, *Galilaus*, *Amb. Rhodius*, *John Fleischerus*, *Alstedius*, *Keckerman*, *Havemann* and others,, &c. And of our time the aforelaid *John Stoffler*, *Sam. Siderocrates*, *John Sceubelius*, *Mich. Mestlin*, *Joh. Kepler*, *Will. Schickhard*, whose Optical Manuscript was freely communicated to us by the excellent and generous *D. Joh. ab Hohenfeld*, a great Lover and Promoter of these Studies. From which we have collected many things in this *Synopsis* lest they should perish, as much as the Scope and Reason of our purpose would bear, and this therefore we were willing to remember, that the studious in the *Mathematics*, might know to whom Thanks is due, if any Benefit redound to them from these things.

What remains, is that we admonish the Pious and Candid Reader in Equity: If by chance some things from the more unskilful Workmanship, being handled more dexterously by others, being received more awkwardly by us, or the things themselves that are added by us, be understood to be less exquisite, that he would not make too sharp a Censure, but that he pardon somewhat of the Difficulty of the Times, and something of the Weakness of Human Nature, and that he would suppose somewhat to be left to himself, from which he
may

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may gratulate himself concerning his greater Erudition, and with us, not to subsist in the careless Contemplation of these mundane Things, but refer all Things to the great end, viz. that to God the most wise Creator of all things, acknowledging the Power and Goodness of the most wise Disposition, being made in *Number, Weight and Measure*, we may not only break forth into Praises of him, with a full Heart and Mouth, but also to mingle together our Manners, to the Pious Universal Harmony. To conclude, being allured by the sweetness of these Contemplations, we may desire more earnestly the Solution of this Body, and the Felicity of the future Life, where our Knowledge of things which is now in part, will be perfected most fully, when we shall behold, not only intimately, the things created by God, but we shall also see the Creator himself Face to Face, to him be Praise from henceforth and for ever, *Amen.*

T O T H E
Candid Reader.

The New Preface, or the Preface to this Third Edition, In which the Reason is given, of the Design of Reprinting it.

After the exact Space of Ten Years from the first Publication, is come now by chance another Mathematical Synopsis of that great Man, Johan. Jac. Heinlin Prelate of Bebenhusan, hitherto desired by the wishes of many: Nor yet without Important reasons, since that (and which of Learned Men instructed in the first place, in the knowledge of those Studies,) it hath been acknowledged to be taken from, and to consist of the whole Mathematical Learning, both pure and mixt, (also, that, they call abstract, but this Concrete) the better Disciplines being handled with more singular care than in the first Edition: So that if all the other works of this Author by which he hath deserved well of the Common wealth of Learning, and which are very many and very large, were lost; yet this little Book alone, has eternally obliged all that have a just value for knowledge themselves, or are concern'd in teaching others: For although from the manifest Love of his, to the People taking pains in learning, the collecting and publishing of this SYNOPSIS was in part, yet he was chiefly prevail'd upon, to wit, that the Youths of Wirtemberg should have it as a certain Guide, and that by the command and solicitous Desire of the most Serene Father of the Country, which founded his Mathematical Study, increased it, and order'd it to the proper Business

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of Life; Nevertheless the Fruitfulness and Dignity of this Labour appeared the greater, when also the same was approved of by very many abroad, and moreover they shew'd an earnest Regard to its Uses not less when enjoyed: Truly this little Book contains nothing superfluous, as is usual sometimes, nor remote from Practice, but is a most useful, goodly and most profitable Treatise, being looked into diligently, and in the First Place they praise it, because the Definitions, Axioms, Postulata, Theorems and Problems, are distinguished most skilfully, and nevertheless compact together in a well order'd Series, whence they afford a greater Light in the Mathematical Studies, as appears most plainly. It happens that this same indeed, seems providing enough for the Apprehension of Scholars, neither yet is it necessary that being made Proficients, they should lay aside the Book, in which also those things being studied (according to the Advice given in the Old Preface) may be plainly understood.

When therefore Heinlin himself, the Author of this SYNOPSIS, while he was yet alive, perceived that this Method of treating] of the MATHEMATICKS, took so mightily with Foreigners beyond his Expectation, (for he was a modest Man) and when he was likewise informed so, by him who had been at the Charge of the first Edition. He presently designed and prepared a New one, least the Copies being everywhere dispersed, it should be done by some other Hand, before he had reviewed and mended it. Also he increased either the Chapters, or Numbers of Books of Discipline, but however for the most part, this Philosopher and great Divine having so sufficiently finished all things as he designed, he departed this Life devoutly and pleasingly in the Year 1660.

Nevertheless somethings of which he was admonished by his Friends, especially concerning Geography and Statics, of which, both Places seem'd to require, that some enlarging, this a new Addition through the whole, whereby

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whereby this Book may more truly maintain, the renowned name of the universal SYNOPSIS of the MATHEMATICS, was not quite finished, but by his last Will he committed that gleanings to his Friends, to whom also he committed the Care of his Manuscripts, and in the first Place those belonging to Philosophy.

Therefore by these, and the strict Bond of the holy Friendship, and by the Publisher, being oftentimes called upon by the Printer, at length they applied the Hand to the Work, and that residue which the Author designed, is annex'd to fulfil the Mind of him.

And indeed concerning GEOGRAPHY, some Chapters are increased, in which the Connexion of the Countries is better, they being reduced according to the natural Order, of which sort there is usually a Want, almost generally in all the Writers of this Discipline. 'Tis not convenient to detain the Reader with a longer Preface, but concerning STATICS, which is now the first Time added, it seems not foreign to the Business, to add somewhat more.

Forasmuch as it hath been confessed, that enough appears of the Causes, why some (which Vossius also testifies concerning himself) that he can scarcely unfold the meaning of them, who consider STATICS separately, and so, not to be a Part of mixt or concrete MATHEMATICS; as the Ancients would; but they presume to imagine them a part of the pure and meer MATHEMATICS. Indeed it cannot be denied, but that STATICS, its Object being abstracted from all matter, it may conveniently be consider'd as they do ARITHMETICK and GEOMETRY, from which notwithstanding it is altogether diverse.

Wherefore considering the Thing accurately, they seem wholly to appoint it not only two Sciences which are properly MATHEMATICAL but rather three, (according to the common saying, GOD hath made all Things in Number, Measure and Weight) to wit, ARITHMETICK, GEOMETRY and STATICS

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TICS, of which the former is conversant only about Number, the other about Magnitude, the latter about Weight, to be consider'd in an abstracted Manner.

And indeed it were to be wished, since right reason advises not otherwise, that these things had been so delivered in old time, but because this Method of Handling, is hitherto unusual, the Author would not too far recede from the Old Custom, since no Man had done it before him, also which he acknowledges ought so to be done; but here also he, as elsewhere oftentimes, especially in describing the System of the World, (in ASTRONOMY Book I of the SPHERE Chap. 3) hath esteemed to himself following nicely; if a more diligent Searcher should inquire why the STATICS in the first Edition of this SYNOPSIS was wholly omitted, it will appear to the judicious Reader not difficult, if he consider, that for the most part in vulgar Compendiums, STATICS are wont to be handled alone, being concluded to be shewn in some Chapters of Book the 2 of Practical ARITHMETICK, Part the 2, which is written concerning Politic or Civil ARITHMETICK, such also as now ought not to be moved from its Place: however in that Discourse which treats particularly of STATICS, the Intent of the adding of which, we have given an Account of before, we shall find many things belonging to this Science to be there delivered it, being assigned in order after the OPTICS. But why the Author would not cut all things after the manner of others into little pieces, and make to each Particle new Titles of Discipline, yea why he rather more willingly, scattered in some Places either an Appendix, or by Digression, (such as occurs in Book the First of Theoretical ARITHMETICK, Part the Third of Musick) the Cause he declares himself, in the most learned Ancient Preface, to which we remit the Reader. But the other lesser Disciplines scattered variously through this SYNOPSIS we desire him to learn from the Index which follows, and so bid him

FAREWELL.

T H E
T R A N S L A T O R
T O T H E
R E A D E R.

I Have carefully, though not curiously, disrobed the Author of these ensuing discourses, that great *Philosopher* and Celebrated *Mathematician* *Joh. James Heinlin, Abbot of Bebenbusan*, out of his Latin Vesture, and put him into our English Habit; that such an excellent Piece may not lye Cloystered in the Precinct of a few Principalities, but that our own Nation; and especially such of them which understand not Latin, may participate of the Hony, this Laborious Bee hath brought to his Hive: If all Relish not at the first Gust, the fault is not in the Work, but their Judgments which understand him not, nor the Jewels presented unto them. The Translation it self was not performed without Labour, and Sweat; partly because the Edition which was made use of from *Tubing* was unperfect and incorrect; partly also because the Author being more intent on Matter than Words, on Things than Expressions, doth sometimes leave his Sense to be Picked out by the Analogy of his Doctrine; If Morose and Supercilious Censurers shall object, Impoliteness, *Solacisme*, inept Cadencies, & Cohasions of Words and Sentences; Obscurities, length of Parentheses, and other Flaws, usually incident, or at least imputed to Translations. In bar to such Hypercriticks, the following particulars are offer'd, viz. That every Language hath its Idiom, which is not properly transferable to another; That a Translator is no Paraphrast, but being

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limited

limited to the Sence, and in great part confined to the words of the Author, hath no allowable liberty to make excursions, or to add any Explanatory Embellishments to another Man's Work: It is his province to write by the Copy before him (*i. e.*) to Vestigate and overtake the Author's Sence, and faithfully to render it, which I hope is here not unhappily performed.

Several in *French*, as *Perrault*, *Herrigon* and *Ozanam*; some in *Italian*, as *Bonaventura Cavalerius*; Others in *Latin* as *De chales*, *Tacquet* and *Dr. Wallis*; besides many in *English* as *Mr. Kersey* and *Mr. Leybourn* and others, have testified their love to their Native Countries, by Publishing in their own Languages such rare Treasures as these: Nor fear I Detraction from any, for these my Labours, unless it be from those, who not being Friends to the Publick good, are neither willing themselves to instruct the less Learned sort of Mankind, nor that any one else should do it: I am not Ignorant of the Course of the Mathematicks writ by *Mr. Leybourn* in Folio, which is the only Book I know, that treats of the most parts of the Mathematicks together; which although it be a much larger Volume, and consequently a much dearer Book than this, yet I am sure the diligent Reader may find here, several things which are not in his *Cursus Mathematicus*, and also the several parts so ingeniously connected and explained, as (I had almost said) is possible to be done; so that it may justly be said of it, that it instructs much in a few words. The kind reception that my little Book entituled *Mellificium Mensionis* or the *Marrow of Measuring* hath met with abroad, hath Animated me to the Publishing of this *Synopsis*, which unless my Judgment fails me, will prove as useful a Book as ever was writ of this Subject, in *English*: Which little Book, I mean *Mellificium Mensionis*, having passed the *Second Impression*, I do intend God willing to reprint a *Third*, with several useful additions, some of which I have already writ, and the Diagrams are drawn and lye by me: I shall not need to say any thing more in praise of the *Author* of this *Synopsis* or his Work, I leave that to those that Peruse the Book; It only remains that I acquaint the Reader with what I have added in this *English Synopsis* which is not in the *Latin*, to make this as useful in *England* as the original in *Germany*, viz. at Page 103 and the following, I have compared the *Hebrew* and *Grecian* Money with the *English*; At page 113 I have added a Table of Equation of *English* Liquid Measure: At page 117 A Table of Equation for *English* Dry-measure, at page 118 a Table of Equation of *English* Money, also a Table of Equation for *Troy-weight*; At page 119 a Table of Equation for *Averdupois-weight*: At page 125 a

Table of Equation for English Long-measure: I have also added the Versed Sines, to the Table of Sines, Tangents and Secants, which the Author makes use of often times, altho' the Versed Sines are not in his Table: I have also added to the Astronomical Tables, A Catalogue of most of the chiefest Cities in the World, shewing the temporary difference of their Meridians from London, with the height of the Pole in each place; Likewise a Table of the Oblique Ascensions of all the Points of the Ecliptick for the Latitude of London being $51^{\circ}:32'$. Also I have added the use of some Tables which were not Treated of by the Author; and whereas at page 89 of Practical Arithmetick, I make mention of *Bonaventura Cavalerius*, having adapted his *Logarithms* to the working of the Rule of Three in *Sexagenary numbers*, which saves the Labour of Multiplying and Dividing those numbers; I have been thinking, if his *Arithmetical Logarithmical Table* as he calls it, were continued to 100 Chiliads of numbers (whereas they end now at 10) in the same method as he hath constructed it; and a *Canon* were made of the Natural Sines, Tangents and Secants and Versed sines, to every Tenth second Minute of the Quadrant, together with the *Logarithms* (or an *Artificial Canon*) of the same, standing opposite to them; It would be the most compleat thing extant, and such as I suppose is not any where yet effected, which would save much labour to those who have occasion to use Calculation: I mention this here, that if any Ingenious publick spirited Person, be willing to be concerned in so good and great a Work, I shall be very glad to confer with him concerning it, for I think the business is too weighty for one Person to accomplish, especially one in my Circumstance, who am a Tradesman: And if to these *Natural and Logarithmical Tables*, were adjoyned the said Authors *Directorium Generale Uranometricum*, and his *Trigonometria Plana & Spherica, Linearis, & Logarithmica*, both done into English, (which I have both already done) it would be an excellent work indeed: such Tables have been often wished for by many Ingenious Persons, one whereof is in Print, viz. Mr. Collins, who at page 89 of his *Sector on a Quadrant*, the Reader (says he) may observe how necessary it is, to have such Tables as have Natural Sines and Versed Sines, &c. standing against the Logarithmical Sines, &c. But as yet there are none such made as have the *Versed Sines*, but will in due time be added to Mr. Gellibrands Tables; thus far Mr. Collins: But I suppose the Versed Sines have not as yet been added to those Tables, although his Book of the Sector was Printed in the Year 1658. Indeed such a chargeable and

great Work ought rather to be done by Subscription, than at the charge of one or two Persons; which method I suppose might be encouraged, if reasonable Proposals were made, and the thing undertaken by any. Which I should be very glad to see done, and would willingly assist in doing it.

And lastly, since 'tis an easie thing to be witty in another Man's Labour, I shall conclude with the Poet, *Carpere vel noli nostra, vel ede tua.*

An.

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ARITHMETICK.

The PREFACE.

Arithmetick is a Doctrin prescribing the Reason and Manner of handling Numbers discreetly.

And 'tis two fold.

1. *Theoretical* or *Abstract*, which acts separately concerning Number from all other things, in Book I.

2. *Practical* or *Concrete*, which considers Number applied to certain Measures, in Book II.

Theoretical may conveniently be divided into 4 Parts, of which it acts.

I. *Of Whole Numbers.*

II. *Of Broken Numbers.*

III. *Of Proportionality.*

IV. *Of Figurate Numbers.*

The First Book of Theoretical Arithmetick: And its First Part.

Of Whole Numbers.

Definitions.

1. **N**UMBER is the measure of multitude, or many, collected from Unities.

2. Multitude, is the distinction of many things bound together by some common band.

3. Unity is the beginning of Number, according to which, every one of them which are, is called One.

4. Numerical Characters, are Notes or Figures, by which Numbers are express'd in Writing.

1. Notes of Numbers are various, and at pleasure, yet some more expedite than others.

2. The *Hebrews* and *Greeks* use the Elements of Letters instead of Numbers: The *Latinists* use these, M. D. C. L. X. LXVI.

3. Characters are now chiefly used, and are most expedite, which some will have to be from *Greek* Letters corrupt, others from the *Arabians*, or *Indians*: to wit,

1. 2. 3. 4. 5. 6. 7. 8. 9. 0.

4. Of these Notes or Figures, the foremost Nine are significative; but the Tenth, or 0. is called Nothing, or a Cipher, and by it self signifies nothing, notwithstanding it fills up a place, and increases the

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the next antecedent by ten times: Whence it cannot possess the first place from the Left Hand.

The Parts of Numeration are,

Notation.

Addition.

Subtraction.

Multiplication.

Division.

5. *Notation*, is the true writing and expressing of any number proposed.

6. *Addition*, is the collecting into one, two, or more numbers taken severally, which is called the Total or Sum.

7. *Subtraction*, or *Subduction*, is the separating of two numbers, by taking the lesser from the greater, that the residue may be found; or the difference of the two given Numbers.

8. *Multiplication*, is the gathering together the number of the Multiplicand so many times as there are Unites in the Multiplier; whence is made the Fact or Product.

Its Kinds are,

Doubling, Tripling, Quadruplicating, &c.

9. *Division*, or *Partition*, is the finding of a Number which shews by its Unities, how often the Number Dividing, or the Divisor is contained in the Dividend; whence the Number found is commonly called the Quote or Quotient.

Its Kinds are,

Halving or Bipartiting, Tripartiting, Quadripartiting, &c.

10. A Number is said to measure a Number, when one so exactly divides the other, that nothing remains: But if there be some residue, the Divisor is said to divide, but not to measure.

(These are commonly called the Five Particulars of Arithmetick, the Barbarous explanation whereof is called by the half *Arabian* word *Algorithme*.)

11. An Aliquot Part, is a number of a number, the lesser of a greater, when the lesser measures the greater: So 4 is an Aliquot part of the Numbers 8. 12. 16. 20. 24. 28.

12. Aliquant Parts are Numbers, the lesser of the greater, when the lesser do not exactly divide the greater, as 3 and 4 with respect to 10.

13. Multiplex or manifold, is a number of a number, the greater of the lesser, when the greater contains the lesser a certain number of times exactly, and the lesser measures the greater: As 30 is the Multiplex of 6, which is contained 5 times in 30.

14. An Even Number, is that which may be divided in the middle, or into two Equal Parts; so 8 may be divided into 4 and 4.

15. An Odd Number, is that which cannot be divided in the middle into two Equal Parts, as 5. 7. 9. 11. &c.

This always differs by Unity from the next even number; whence by adding two odd Numbers is made an even one.

16. In like manner, that is an even number which measures an even number by an even number: such is the number 12, which 6 measures by 2.

17. Also

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17. Also an Even Number is that which an even number measures by an odd number : so is the number 30, which 2 measures by 15.

18. Moreover, an unequal Odd Number is that which an odd number measures by an odd number ; as the number 15 is unequally odd, because the odd number 5 measures it by the odd number 3.

19. A *Perfect Number*, is that, of which all the Aliquot Parts taken together are equal to the whole, and restore it : so 6 is a perfect number, because its Aliquot Parts 1. 2. 3. being added, restore the same.

And there are found but few Perfect Numbers among a great many great ones, to wit, from 1 to 40000000, only these.

6
28
496
8128
130816
2096128
33550336

All the rest are abundant or deficient ; and all the Perfect Numbers begin by turns from 6 and 8.

Requests or Petitions.

1. That to any Number we may take a greater.
2. To any numbers, how many soever, we may take their Equals or Multiplexes.

Therefore although a number cannot be divided infinitely, but it will necessarily at length be diminished to individual Unity ; yet all number may be infinitely increased by Addition.

Axioms and Principles.

1. **I**F to equal things, there are added or taken away equal things, the total, or the residue also will be equal.

2. If equal things to unequal, or unequal to equal things be added or taken away, the total and the remainder also will be unequal.

3. The whole is greater than any of its parts, and is equal to all the parts taken together.

4. Unity measures every number by the number it self: so 1 measures 7 by 7.

5. Every number measures it self by unity; so 7 measures it self by 1.

6. A number measuring other numbers, measures likewise their compounds: As, because 4 measures 8, 12, 16: it also measures 36 the Fact of 8, 12, and 16.

7. A number measuring any other number, it measures also any number which that measures; As, because 4 measures 8; and 8 measures 24, 32, 40, &c. 4 also will measure the same.

8. A number that measures the whole, and that which is taken away, will also measure the remainder: So, because 4 measures 32, whence if you take away 8 is left 24: then because 4 measures 8, it will also measure 24.

THEOREMS.

1. **U**Nity is the beginning and end of all number; for from it all numbers do arise, and in it all numbers are resolved: Therefore 'tis the Symbol of God from whom all things are, and to whom all things return.

2. Among infinite numbers no one is like another, but every one differs. So in the innumerable multitude of Creatures produced from God, no one is like to another in every respect.

3. Unity is subject to no mutation; for whatsoever is changed, is changed into somewhat divers from it self; the which Unity hath not.

4. Every number is Mutable; for Unity being added or taken away, it is not any longer the former number: So God only is Immutable, but all Creatures Mutable, being deficient when God takes away his Spirit: Like as also,

5. Unity being taken away, it necessarily follows that all numbers are taken away.

6. 'Tis necessary for Unity to precede all number, which if you be ignorant of, you know not the number.

Therefore they are not very wise who begin the generality of things from number, and take away all Science.

7. In Unity all numbers are Virtually, although they are Infinite. So all things are in God, in him they live and move.

8 Seven is a Sacred Number, chiefly used in Holy Scripture.

It seems to have its Original from the Inscrutable Unity of Divine Essence, and Sextuple in respect of the Divine Persons among themselves : Whence also the Description of a Septangled form is impossible, and cannot be known by Human Minds.

9. Ten is the Image of Unity ; for 0 or a Cypher added to it, signifies Nothing, and it denotes a returning to Unity, even as all numbers added after 10 repeat the first order of Numbers in this manner.

10	0	16	6	22	2
11	1	17	7	23	3
12	2	18	8	24	4
13	3	19	9	25	5
14	4	20	10	0	6
15	5	21	1	27	7, &c.

10. An Even Number is the weaker ; an Odd Number the stronger : for Even Numbers cannot produce Odd ; but Odd both produce Odd, if they are either multiplied in themselves, or being mixt they produce even : and they produce even numbers, if they are multiplied with even ; hence an Odd Number is always more worthy than an Even, as God also delights in an Odd Number.

11. The number 8 continually multiplied, the Figures arising, being added together among themselves, they always decrease.

Example.

Example.

The numb. 8 multiplied.	The figures arising of the figures added, as, 1 & 6
8 ————— 8	is 7 : 2 & 4 is
16 ————— 6	6 : 3 & 2 is
24 ————— 4	5, &c.
32 ————— 2	
40 ————— 0	
48 ————— 8	
56 ————— 6	
64 ————— 4	

12. The number 9 continually multiplied ; the Figures arising being added to each other, the same always remains, and produces 9.

Example.

The numb. 9 multiplied.	The figures arising of the figures added, as, 1 & 8
9 ————— 9	is 9 : 2 & 7 is
18 ————— 8	9, &c.
27 ————— 7	
36 ————— 6	
45 ————— 5	
54 ————— 4	
63 ————— 3	
72 ————— 2	
81 ————— 1	

Therefore the number 8 is the Symbol of Death and Destruction : And the number 9 is the Symbol of Firmness and Constancy : In the middle of the 9 numbers, the numbers of the right hand are changed with those of the left hand, and they go backward in order.

P R O B L E M S.

TO declare any Number express'd by Figures, and being declared, to express the Figure.

The Solution of this and of all the other Problems is learnt much easier by lively instruction, than by dead and dumb Letters. Therefore we shall only point at the chief things which are to be observed; leaving the rest to the industry of the skilful informer.

The things to be attended in this place are,

1. The order of numbers, proceeding from the right hand to the left.

2. Every three Figures are distinguished from the rest by putting a small dash between; and every fourth after the small dash is noted by a prick or point.

3. The first Figure of the three is esteemed for single, the other for decuple or 10 times, the third for Centuple or 100 times, and the fourth being noted with a point is esteemed as Millecuple or 1000 times; the foremost of which, if there be any, constitute Myriades or 10 thousands in the same order.

4. But larger numbers are by some pronounced otherwise, the most simple manner, is, if the two former points are pronounced, by thousands of thousands; to the rest the word (thousands) is always repeated.

Example I.

The number employed by Xerxes King of the Persians, which he led forth against the Greeks, was

3 | 283 | 220.

And

And may be thus read, Five thousands of thousands; two hundred eighty three thousand; two hundred and twenty heads.

Example II.

A number Thewing how often the 22 Hebrew Letters may be transposed and varied.

11124100017271777160716801000

Thus read: the word (thousand thousand times) in the first, is 5 times repeated; then four times; afterwards (passing by the Cyphers) tis twice repeated: the Figures placed between the small strokes have each one their value assigned: and the two last points is pronounced by thousands of thousands and thousand.

Example III.

A number exceeding many times the number of the Stars is this,

534142317441000; that is,

Five hundred thirty four thousand of thousands of thousands; Four hundred twenty three thousands of thousands; Seven hundred forty four thousand of Stars the Heaven doth not contain.

II. To add a number to a number, or to collect many into one Sum.

1. The order proceeds from the right hand to the left.

2. The single are placed under the single, the tens under tens, and the hundreds under hundreds.

3. The right Series, or Rank of like numbers are collected into one Sum.

4. A line being drawn underneath, the first figure of the collection is set under the first Rank; if there be another figure tis numbered with the figures of the following Rank.

A Chronological Example.

From the beginning of the World to the	}	1656
Deluge or Flood, are years		
To the calling of <i>Abraham</i>		0368
To the giving the Law		0429
To the Temple of <i>Solomon</i>		0479
To the first Destruction of it		0427
To its Restauration		0184
To the Christian <i>Epocha</i>		0420
To the year Compleat		1700
The Sum. From the beginning	}	5663 years.
of the World to this time is		

The Examination of Addition.

Of the numbers added subtract each from the Sum, if 0 remains the Sum is right.

III. To Subtract a lesser Number from a greater, and to find the difference of them, or the remainder.

1. The order proceeds from the right hand to the left, taking the lesser number from the greater; the greater number being placed above, the lesser beneath.

2. Like numbers in the Minorand are writ under their like, that are to be Subducted.

3. The lower being taken out of the upper, and a line drawn, they are writ under the same Rank.

4. If the upper be the lesser, ten must be added in the mind from the antecedent figure, (noted against that point) to be taken together.

5. And it must be lessened by unity in the following operation, because of this mutual borrowing, or giving, and if there be Cyphers, it must be accounted for 9.

Exam.

Example I.

How many years is it since the Deluge or Flood?

Ans^w. From the beginning of the World } 5663
to the year 1700 Compleat, is years — }
The Flood hapned in the year of the World—1656

This Subtracted from that leaves — 4007
And so many years it is from the Flood to this present year of 1700.

II.

How many years is it since Christ Suffered? Ans^w.
The year of the Christian Epocha compleat—1700

We number from the beginning of the World—5663
Christ Suffered in the year of the World—3997

This taken from that, there remains years } 1666
since Christ Suffered — }
— — — — —

III.

The Minorand — 62001002789

The Subducend — 1998769346

The Difference and Remainder—60002233443

The Examination of Subtraction.

The Remainder added to the Subducend, if the Sum makes the Minorand, 'tis right.

IV. Two Numbers given, to multiply one by the other, or to draw one into the other.

To the performing of this, the foreknowledge of single Multiplication is required, to wit, of single Figures with single: the Description of which may be this.

2. <	2—4	4. <	4—16	6. }	6—36			
	3—6		5—20		7—42			
	4—8		6—24		8—48			
	5—10		7—28		9—54			
	6—12		8—32		10—60			
	7—14		9—36					
	8—16		10—40					
	9—18							
	10—20							
	3. <		3—9		5. }	5—25	7. }	7—49
4—12		6—30	8—56					
5—15		7—35	9—63					
6—18		8—40	10—70					
7—21		9—45						
8—24		10—50						
9—27								
10—30								
						8. }		8—64
								9—72
				10—80				
					9. }		9—81	
			10—90					
			10. <	10—100				

The Table of *Pythagoras*, likewise shews the same Multiplication.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

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These things being foreknown.

1. The order is observed from the right hand to the left.

2. In significative Figures, the first Figure of the Multiplicator is writ under the first of the Multiplicand, the second under the second, and so on.

3. Each Figure of the Multiplicator must be multiplied by every one of the Multiplicand, and a line being drawn underneath, the first Figure of the Product is writ under the Multiplier, the other is kept in memory, and added to the following Product.

4. Lastly, all the Products are collected into one Sum by Addition.

Some Rules serving as Compendiums in Calculation.

I.

A 0 or figure Nothing, begets nothing, notwithstanding it possesses its place.

II.

Therefore Cyphers at the beginning are cut off before the Operation; and after the Operation you must adjoin so many as you cut off.

Example I.

Multiplicand ————— 5972

Multiplicator ————— 343

Products — { 17916
23898
17916

The Sum ————— 2048396

Examp. II.

Example II.

Multiplicand ——— 46057

Multiplier ——— 402

$$\begin{array}{r} \text{Products} \left\{ \begin{array}{r} 92114 \\ 1842280 \end{array} \right. \end{array}$$

The Sum ——— 18514914

Example III.

Multiplicand ——— 3674 | 0

Multiplier ——— 3 | 000

The Product ——— 110220000

Example IV.

Multiplicand ——— 478912 |

Multiplier ——— 1 | 000

The Product ——— 478912 000

V. To Divide one Number by another, or to Distribute one into another.

1. This order is contrary to the former, proceeding from the left hand to the right.

2. The first figure of the Divisor must be writ under the first figure of the Dividend, if it be lesser, but under the second if it be greater.

3. Seek how many times the first figure of the Divisor is contained in the Superior figure of the Dividend? and write it in the Quotient after the Lunula or Semicircle.

4. Multiply the Divisor by the Quotient, and subscribe the Product of the Divisor.

5. The same Product also you must subtract from its superiours of the Dividend.

6. The

6. The Remainder being subscribed under the line drawn, adjoin another figure of the Dividend.

7. The Divisor being removed one station, repeat this Process, until all the figures of the Dividend be wasted.

8. If there be a remainder after the Division (which must be always less than the Divisor) that must be adjoined to the Quotient also, yet distinctly, to which the Divisor must be again subscribed, making a dash between.

For this Remainder is a Fraction, or part of the Dividend, either an Aliquot or Aliquant part, which the Divisor denominates.

CAUTIONS.

1. Beware that the Quotient be taken neither greater nor lesser than just; which being multiplied into the Divisor, may produce a number either equal, or next lesser to the Vertical, or number over it, to be divided, that so Subtraction may be made.

2. If the Divisor, being removed, and the superior Vertical, or upper number, to be divided, be less than the Divisor, you must write a Cypher in the Quotient, after the Lunula, and assume another figure of the Dividend, and remove the Divisor forward.

COMPENDIUMS.

1. Unity, as it doth not Multiply, so neither doth it divide, but restores the Dividend it self, yet cutting off so many Figures as there are Cyphers belonging to the Unity.

2. The extream Cyphers of the Divisor under the last figures of the Dividend are rejected, and the Operation instituted by the significative figures only, until you come to those Cyphers, whose Verticals are all to be added to the Remainder.

3. If the Divisor be 2, then you institute halving; take half of the Even, but the last unity of the Odd is accounted for 10, and must be adjoined in mind to the following figure.

C

4. If

18 PROBLEMS of Part I.

4. If the Divisor consists of one or two figures only ; Multiplication and Subtraction may be performed conveniently in the mind only, and the Remainder from Subtraction to be placed over the Dividend may be left out.

Example I.

$$\begin{array}{r}
 \text{The Dividend} \text{---} 21445 \quad (893 \frac{1}{4}) \\
 \text{The Divisor} \text{---} 24 \\
 \hline
 192 \\
 \hline
 224 \\
 \hline
 24 \\
 \hline
 216 \\
 \hline
 85 \\
 24 \\
 \hline
 72
 \end{array}$$

Example II.

$$\begin{array}{r}
 \text{The Dividend} \text{---} 77445392 \quad (27041 \frac{1}{4}) \\
 \text{The Divisor} \text{---} 2864 \\
 \hline
 5728 \\
 \hline
 20166 \\
 \hline
 2864 \\
 \hline
 20048 \\
 \hline
 11839 \\
 \hline
 2864 \\
 \hline
 11456 \\
 \hline
 3882 \\
 \hline
 2864 \\
 \hline
 968
 \end{array}$$

Example III.

Example III.

Dividend — 679842 (295 $\frac{11}{23}$)

Divisor — 23 | 00

46 |

219 |

—

23 |

207 |

12842

2300

11500

—

1342

Example IV. by Halving.

Dividend — 384200904671

Divisor is 2. 192100452335 $\frac{1}{2}$ this Sum is the
(Quotient.)

Example V. Of a Divisor of one Figure.

XXXX(I)

Dividend — 879428 (226473 $\frac{1}{2}$)

Divisor — 888888

Example VI. Of a Divisor of two Figures.

XX(2)

2238(3)

Dividend — 48987 (1956 $\frac{11}{23}$)

Divisor — 24444

222

Note, If when, as sometimes it happens, a lesser number is to be divided by a greater ; for Example, 3 by 4, place the lesser above the greater in this manner $\frac{3}{4}$, and you have done, and the Quotient will be a Fraction of a Whole Number.

The Examination of Division.

Multiplication and Division examine each other :
For,

In Multiplication.

If the Product be divided by the Multiplicand, it produces the Multiplier ; if the Product be divided by the Multiplier, it produces the Multiplicand.

In Division.

If the Divisor be multiplied by the Quotient, it produces the Dividend, but you must add the remainder again, if there be any ; unless it produces the Dividend, you have committed an Error ; and if it doth, you need not doubt.

The

The First Book of Theoretical Arithmetick :

And its Second Part.

Of Broken Numbers.

Definitions.

1. **A**N whole Number is that whereby we number whole things.

2. A *Fraction* or *Broken Number*, is that which we assign for a part or parts of any whole thing. It is also wont to be called a *Mite* or *Fraction*, because these things are broke into small parts : And they are usually writ with two Terms or Numbers, with a stroke or dash between them, of which the uppermost is called *Numerator*, and the lowermost *Denominator*.

3. *Numerator*, is the superior Term of the Fraction, shewing how many Aliquot parts must be taken.

4. *Denominator* is the inferior Term of the Fraction, distinguished from the superior by a small line between them, shewing into how many parts the Integer, or whole thing is divided.

Example.

Numerator — 34691

Denominator — 458102, &c.

5. A *Fraction of a Fraction*, is that whereby not an Integer is designed, but a part of a part, or the parts of two distinct Fractions, the lesser being placed before, distinguished by a Comma from the greater.

C 3

Examp.

Example.

$\frac{3}{4}$, are three fourth parts of one Integer : But $\frac{3}{2}$ is a Fraction of a Fraction, signifying three Fourths of one Half, or of two Fourths.

6. A *Prime Number*, is that which Unity only measures ; or it divides it exactly without any thing remaining ; such are 3, 5, 7, 11, 13, 17, &c.

7. *Numbers* are *Primes* between themselves, all which Unity only measures, as 5, 7, 9 : also 3, 11, 13.

8. A *Compound Number*, is that which some other Number measures besides Unity, as 10 ; for 1 and 2 and 5 measure 10.

9. *Compound Numbers* between themselves, are those, all of which another Number also will measure, besides Unity : such are 8 and 12, for both are measured by 1, 2, and 4.

10. The *greatest Common Measure*, is the greatest Number of them, by which the given Numbers, compounded between themselves, are exactly divided : As 90 and 24 may be divided exactly by 1, by 2, by 3, and by 6 ; but 6 is the greatest Common Measure of them ; for no greater Divisor can be given, by which 90 and 24 can be exactly divided.

11. The *least Common Dividend*, is a Number compounded ; the least among those which will exactly divide the given Numbers.

Example.

There are two Numbers given, 13 and 27 ; by these two, more Numbers may be divided exactly, to wit, 1404, 1053, 702, 351. But this last is the least common Dividend which can be divided by 13 and 27, beneath which no other can be found.

Axioms and Maxims.

1. **T**HE Number 2 measures every Even Number.

Whence it follows, that all Even Numbers are compounded by and among themselves, except the Number 2.

2. The Number 3 measures every Number, whose Numeral Figures collected together may be divided exactly by three.

As the Number 5439, the figures collected constitute 21; which, because 'tis divisible by 3, the Number also 5439 may be divided by 3.

3. The same property is of the Number 9.

So because 4869 make 27, a number divisible by 9, therefore also 4869 may be divided by 9.

4. The Number 4 measures every Number, whose two first figures, numbring from the right hand, the said Number 4 measures.

Example.

69816, the two first figures 16 may be divided by 4; therefore also the whole.

5. The Number 5 measures every Number, whose first figure is 5 or 0: such are 4715, 1600, 3430.

6. The Number 6 measures every Even Number which 3 will measure.

So the Even Number 4262 is measured by 3, by the second Axiom: therefore 6 will also measure the same, since it is an Even Number.

7. If of a proposed Number, three of the first Figures numbred from the right hand, the half may be divided by 4, the Whole Number may be divided by 8.

Example.

The Number 658368, the half of the three first figures 368, to wit, 184 may exactly be divided by 4; Therefore the same Whole likewise may be measured by 8.

8. *Parts or Fractions whose Denominators increase, they decrease in quantity.*

So among the Fractions $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$, the least is $\frac{1}{2}$, because its Denominator is the greatest of the given.

9. *Parts or Fractions whose Numerators increase, they increase in Quantity.*

So among $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$, the greatest is $\frac{3}{4}$, because its Numerator is the greatest of the given.

P R O B L E M S.

I. *Of two Numbers given, to know whether or no they are Primes or Compounds between themselves.*

1. Divide always the greater by the lesser, both in the given, and in the remaining Numbers.

2. If Unity remains, they are Primes: If Nothing remains, they are Compounds.

Example I.

Given the Numbers 134 and 49, and you would know whether they are Primes or Compounds.

Then always divide the greater by the lesser.

134 (2	49 (1	36 (2	13 (1	10 (3
49	36	13	10	3
98	—	26	—	9
—	13	—	3	—
36		10		1
				Then

Then because after all the Divisions, 1 remains, it follows, that the given Numbers are Primes between themselves.

Given the Numbers 330 and 135, of which you would know, whether they are Primes or Compounds.

Always divide the greater by the lesser.

330 (2	135 (2	60 (4
135	60	15
270	120	60
—	—	—
60	15	0

Then because after all the Division, nothing is left, it argues, that the Numbers given are Compounds between themselves : The same is done, if one after another you Subtract the lesser from the greater, until you come to 1 or 0.

II. Of given Numbers, being Compounds between themselves, whether they are two or more, to find the greatest Common Measure.

1. Of the two given Numbers, always divide the greater by the lesser.

2. The last Divisor, wasting all even to 0, is the greatest Common Measure.

3. With this compare the third of the given Numbers, if you have a third Number, and proceed in the same manner : and also proceed in the same manner, if there be more than three Numbers.

4. For then the last Divisor, all being wasted, will be the greatest Common Measure of the given Numbers.

Example.

Example.

Given two Numbers 234 and 144, the common measure of which is sought : divide the greater by the lesser, in this manner.

234 (1	144 (1	90 (1	54 (1	36 (2
144	90	54	36	18
90	54	36	18	0

Then since the last Divisor (all being waisted) is 18 ; it is the greatest common measure of the Numbers 234 and 144 ; you may effect the same thing by the continual Subtraction of the lesser from the greater.

III. Given two Numbers, or more, to find the least Common Dividend of the same.

1. In Numbers that are Primes between themselves.

1. Multiply the two given Numbers in themselves, and if there are three, the Product into the third.

2. The last Product is the least Common Dividend.

In Numbers Compounded between themselves.

1. First find the greatest Common Measure.

2. By this divide the rest of the given Numbers.

3. Multiply the Quotient into the other of the given Numbers.

4. The Fact will be the least from each Dividend, with which if you compare any third of the given Numbers in the same manner, you will have the least Dividend of the three given Numbers.

Example.

Example.

Given the Numbers _____ 27 and 45

Their Common Measure is _____ 9

By which the given Numbers being } 3 and 5
divided, makes _____

Now draw either 3 in 45, or 5 in 27, } 135
and they produce _____

The least Common Dividend.

IV. To find the Quantity of Fractions.

1. A Fraction is either greater, or equal, or lesser than an Integer; even as the Numerator is greater, or equal, or less than the Denominator: 'tis thus.

To the Integer.

Greater.

Equal.

Lesser.

$$\frac{9}{3}$$

$$\frac{1}{1}$$

$$\frac{3}{5}$$

2. A Fraction that is greater than, and equal to an Integer is spurious, unless it be reduced to the Integer.

3. And to find between two proper Fractions, which is greater or lesser: Work thus.

1. Draw the Numerator of one into the Denominator of the other Cross-wise.

2. Of which, that which produces the greater Numerator, is the greater.

Example.

There are given two Fractions, $\frac{6}{7}$ & $\frac{7}{9}$.

By Multiplication they make 48. 49.

Then since the Numerator 7 produces the greater Number, I conclude that the Fraction $\frac{7}{9}$ is greater than $\frac{6}{7}$.

V. To

V. To change one Fraction into another.

1. To change a Fraction that is greater than, or equal to an Integer, into an Integer.

1. For any Terms of equal Fractions substitute 1; as for $\frac{1}{4}$ put 1.

2. If the Numerator be greater than the Denominator, divide it by the lesser: so $\frac{9}{4}$ will be $2\frac{1}{4}$.

2. To transpose or change an Integer into a Fraction, no certain Denominator being given.

Example.

To the Integer 5 subscribe 1, in this manner $\frac{1}{4}$.

3. To change an Integer into a Fraction of a certain Denomination given.

1. Multiply the Integer, for Example, 5, into the Denominator given, which suppose 4.

2. The Product will be the Numerator of the given Denomination, to wit, 20 .

4. To reduce a greater Fraction, to the least Terms that are Equivalent.

For one and the same Fraction may be expressed divers ways, among which, that is most noted which hath the least Terms: Therefore,

1. If all the Terms are even, half them as often as you can: As,

$$\frac{32}{64} \quad \frac{16}{32} \quad \frac{8}{16} \quad \frac{4}{8} \quad \frac{2}{4} \quad \frac{1}{2}$$

All these are one and the same Fraction, among which the least $\frac{1}{2}$ is the most noted.

2. If

2. If the Terms have Cyphers annexed equal in Number, cast them away : As,

$\frac{100}{100}$ is the same as $\frac{1}{1}$.

3. If the Terms of a Fraction given are Compounds between themselves : Work thus,

1. Seek their greatest Common Measure.

2. By this divide the Numerator and Denominator, the Quotients of which will shew the least Numerators and Denominators.

Example.

$\frac{24}{24}$ hath the greatest Common Measure 24, by which being divided it produces $\frac{1}{1}$.

N O T E.

But if the Terms of the Fraction given are Primes between themselves, they cannot be lessened any further, as $\frac{1}{1}$ and $\frac{1}{1}$.

5. To change one Fraction into another equal to it, of a certain given Numerator, or Denominator, if it may be done.

1. Let the given Fraction be $\frac{4}{9}$, to be changed into another whose Numerator is 9.

1. Multiply the Denominator of the given Fraction by the new Numerator given, to wit, 4 by 9, and it makes 36.

2. This Product divide by the Numerator 3, of the given Fraction.

3. The Quotient, to wit, 12, will be the Denominator of the desired Fraction $\frac{9}{12}$, which is the same with $\frac{4}{9}$.

2. Let the given Fraction $\frac{4}{9}$ be to be changed into another whose Denominator is 12.

1. Mul.

1. Multiply the Numerator of the given Fraction by the new Denominator given, to wit, 3 by 12, and it makes 36.

2. Divide this Product by the Denominator of the given Fraction 4.

3. The Quotient 9 will be the Numerator of the desired Fraction $\frac{9}{4}$, being the same as $\frac{3}{1}$.

Note. If the Product cannot be exactly divided there can be no change made.

6. To reduce a Number mixt, of an Integer and a Fraction, into one Fraction.

1. Draw the Integer into the Denominator of the adhering Fraction.

2. Add the Numerator to the Product: Thus $3\frac{2}{3}$ are $\frac{11}{3}$.

7. To convert a Fraction of a Fraction into one Fraction, with respect to some Integer.

1. Draw the Numerators one after another into themselves, and in the same manner the Denominators also.

2. The Products will give the Fraction of an Integer, comprehending them both: so from $\frac{2}{3} \mid \frac{1}{4}$ are made $\frac{6}{12}$: If you have a third Fraction, with this, and those, work in the same manner.

8. To reduce two Fractions of divers Denominations, or also more; to others of the same Denomination, equal to those.

I. Case.

If two only are given.

1. Draw the Denominators into each other, and you will have a new Denominator common to both Fractions.

2. Mul-

2. Multiply the Numerators into the Denominators Cross-wise, and place the new Numerators produced, instead of the former.

Example.

Given two Fractions $\frac{2}{3} \times \frac{4}{5}$
 The Denominators 3 and 5 drawn into each other make the Common Denominator 15

And the Numerators drawn Cross-wise into the former Denominators make 10 and 12 for new Numerators.

II. Case.

If there be given three Fractions.

1. Draw all the Denominators into each other, to wit, the First into the Second, and this Product into the Third, that you may have a Common Denominator.

2. Divide this by each of the given Denominators.

3. Draw the produced Quotients into each of the Numerators; and it will produce new Numerators to be placed over the Common Denominator.

Example.

Let the given Fractions be $\frac{2}{3} | \frac{3}{4} | \frac{4}{5}$, to be reduced to the same Denomination.

1. Draw 3 into 4, it makes 12; and 12 into 5, makes 60: which number will be the Common Denominator.

2. Divide this by the given Denominators, to wit, by 3, 4, 5, and you will produce these Quotients, 20, 15, 12.

3. Draw

3. Draw (or multiply) these into 2, 3, 4, to wit, the given Numerators, and you will produce three new Numerators, to wit, 40, 45, 48, to be placed over the Common Denominator; thus $\frac{40}{48} | \frac{45}{48} | \frac{48}{48}$, which Fractions are equivalent to the given $\frac{5}{6} | \frac{3}{4} | \frac{4}{6}$.

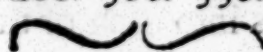
Note, First, If the given Denominators are Compounds between themselves: Then, if first you find their greatest Common Measure by Problem the second; then their least Common Dividend by Problem the third. And lastly, work according to this second manner, you will produce Fractions of the same Denomination in the least Terms.

Example.

Let there be three Fractions $\frac{1}{2} | \frac{1}{3} | \frac{7}{12}$, to be reduced to the same Denomination.

If then, without the greatest Common Measure, and the least Common Dividend, you work after the manner of the second Case, you will produce these Fractions.

288. 360. 336.

 Equivalent to the given $\frac{1}{2} | \frac{1}{3} | \frac{7}{12}$.

576.

But if first you seek the least Common Dividend of these three Denominators, which is 24, you will have it instead of a Common Denominator, and then proceeding as above (in Number 2 and 3) you will have the same Fractions in these least Terms.

12. 15. 14.


24

This reduction of the divers Denominations to the same, is necessary, if you would collect many Fractions, either to be added into one Sum, or to be Sub-

Subtracted one from another; for without this neither can be done. But the reduction being made, the thing is performed as follows.

VI. *To add Fractions of one Denomination.*

Add the Numerators, and 'tis done, as $\frac{1}{2}$ and $\frac{1}{2}$ are $\frac{2}{2}$.

VII. *To Subtract Fractions of one Denomination, one from another.*

Subtract the lesser Numerator from the greater, and you have done what's required.

Example.

From $\frac{2}{2}$ take away $\frac{1}{2}$, and there remains $\frac{1}{2}$.

VIII. *To Multiply any Fractions by others.*

That is done by drawing or multiplying the Numerators into the Numerators, and the Denominators into the Denominators, in this manner.

Multiply $\frac{2}{3}$ by $\frac{4}{5}$, and it makes $\frac{8}{15}$.

IX. *To Divide one Fraction by another.*

1. Invert the Terms of the Divisor.
2. Then multiply, and you have divided.

Example.

To divide $\frac{4}{5}$ by $\frac{6}{7}$.

Multiply $\frac{4}{5}$ by $\frac{7}{6}$, inverting the Terms of the Divisor.

And it produces $\frac{28}{30}$, or $1 \frac{14}{15}$.

The First Book of Theoretical Arithmetick : And its Third Part.

Of Proportional Numbers.

Definitions.

1. **P***roportion in Numbers*, is a certain mutual Habitude of two Numbers according to quantity. In *Greek* 'tis called *Analogy* ; in *Latin*, *Ratio* ; in *English*, *Reason* or *Proportion*.

2. *The Antecedent of Proportion*, is a Term which refers to another.

3. *The Consequent*, is that to which another is refer'd.

That is writ in the first or upper place ; this in the second, or lower place ; having no small line or dash between them, that so they may be distinguished from Fractions, with which in many things they agree.

And *Proportion* is either of Equality or Inequality.

4. *Proportion of Equality*, is between two Equal Numbers, as $1 : 1$.

5. *Of Inequality*, between two unequal, as $2 : 1$.

That is of one kind : This, of the greater or lesser inequality.

6. *The Proportion of greater Inequality*, is when a greater Antecedent is compared with a lesser Consequent, as $2 : 1$.

7. *The*

7. *The Denominator of Proportion*, (which is also called the *Quotient* or quantity of Proportion) is a Number writ like a Fraction, expressing the Habitude of one Number to another, distinctly and manifestly.

In Proportion of Equality it is always writ 2, 2, but in Proportion of Inequality it is a double Number like a Fraction, either single, or of an Integer and Fraction mixt.

The Kinds of Proportion, of greater inequality, are Five :

- | | | |
|----------------------------|--|--------------------------------------|
| 1. <i>Multiplex.</i> | | 4. <i>Multiplex Superparticular.</i> |
| 2. <i>Superparticular.</i> | | |
| 3. <i>Superpartient.</i> | | 5. <i>Multiplex Superpartient.</i> |

8. *Proportion Multiplex*, is the Habitude of a greater Number to a lesser, when the greater Number contains the lesser some times exactly, so that if Division be instituted, nothing remains, as $\frac{4}{1}$, $\frac{9}{3}$.

The Denominators of it are,

$\frac{2}{1}$ $\frac{3}{1}$ $\frac{4}{1}$ $\frac{5}{1}$ $\frac{6}{1}$, &c. That is,

Double, Triple, Quadruple, Quintuple, Sextuple, and so in *Infinitum*.

9. *Proportion Superparticular*, is the Habitude of a greater Number to a lesser, when the greater contains the lesser once, and moreover one Aliquot part of it.

The Denominators of it are,

$\frac{3}{2}$ $\frac{4}{3}$ $\frac{5}{4}$ $\frac{6}{5}$ $\frac{7}{6}$

One and an half, One one third, One one fourth, One one fifth, &c. *ad infinitum*.

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10. *Proportion Superpartient*, is the Habitude of a greater Number to a lesser, when the greater contains the lesser once, and moreover some Aliquot parts, express'd by Numbers that are Primes between themselves.

The Denominators of it are,

$1\frac{2}{3}$ $1\frac{3}{4}$ $1\frac{4}{5}$
Superbipartient, *Supertripartient*, *Superquadrupar-*
 Thirds. Fourths. Fifths.
 $1\frac{5}{8}$ $1\frac{6}{11}$
tient, *Superquintupartient*, *Supersextupartient*.
 Eighths. Elevenths. and so
ad Infinitum.

Note, That Aliquot parts ought to express Numbers, that are Primes between themselves : For if they are Compounded, and are Measured also by any other Number besides Unity, another kind of Proportion arises : Example, the Proportion between 9 and 6 is not rightly called the Proportion $1\frac{3}{2}$, or *Supertripartient* Sixths ; because 3 and 6 have also a common Measure besides Unity, to wit, 3, for 3 once taken Measures it self, and twice repeated it Measures 6 ; therefore $\frac{3}{2}$ must be reduced to its least Terms $\frac{1}{2}$; and so the Proportion between 9 and 6 makes $1\frac{1}{2}$, that is *Sesquialter*.

11. *Proportion Multiplex Superparticular*, is the Habitude of a greater Number to a lesser, when the greater contains the lesser a certain Number of times, and moreover one Aliquot part of the lesser.

The Denominators of it are,

$2\frac{1}{2}$ $3\frac{1}{3}$ $4\frac{1}{4}$ $5\frac{1}{5}$
 Double. Triple. Quadruple. Quintuple.
Sesquitertia, *Sesquiquinta*, *Sesquiseptima*, *Sesquinona*.

12. *Proportion Multiplex Superpartients*, is the Habitude of a greater Number to a lesser, when the

the greater contains the lesser a certain Number of times, and moreover some Aliquot Parts of it express'd in Numbers that are Primes between themselves.

The Denominators of it are,

2 ;	3 ;	5 ;
Double.	Triple.	Quintuple.
<i>Superbipartient,</i>	<i>Superquintupartient,</i>	<i>Supersextupartient.</i>
Thirds.	Sixths.	Sevenths.

13. *Proportion of lesser Inequality*, is when the Antecedent, being a lesser Number, is compared with the Consequent, being a greater.

Its kinds are so many, as of the Proportion of greater Inequality, which are advanced likewise in the same manner, only setting or writing before the Particle *Sub*, as, *Proportion Subduple, Subtriple, Subsesquialter, Subsuperbipartient*.

14. *Proportionality*, or as some call it, the Half of a thing, or the Middle between two, is the comparing of Proportions :

Example.

If we compare the Proportion between 12 and 4, with the Proportion between 9 and 3, where the Proportion is on both parts Triple.

The more noble kinds of this are, *Arithmetical, Geometrical, Harmonical.*

15. *Arithmetical Proportionality*, is, when Three or more Numbers have the same difference from each other, but divers Proportions : And this either continually, without interruption ; as,

2. 4. 6. 8. 10. 12. which is called *Arithmetical Progreffion.*

D 3

Or

Or separately, in an Interrupted Order; as,

2. 4. 5. 7. 8. 10. 11. 13.

16. *Geometrical Proportionality*, is, when Three or more Numbers have the same Proportion between themselves, but divers Differences; And that also, either Continual, as, 2. 4. 8. 16. 32. 64. 128.

Which is called *Geometrical Progression*.

Or separately, as, 2. 4. 8. 12. 18.

17. *Harmonical, or Musical Proportionality*, is, when three Numbers having neither the same Difference nor Proportion, are so ordered, that the Proportion is the same of the greater to the lesser, as the Difference of the Greater, and of the Mean, is to the Difference of the Mean and of the Least.

Such are the Numbers 3. 4. 6: for between 6 and 3 is double Proportion: the difference between 4 and 6, is two; between 4 and 3, the Difference is 1; between which Differences 2 and 1, the Proportion is likewise double.

THEOREMS.

1. **I**F there be Proportional Numbers whatsoever, even as one of the Antecedents is to one of the Consequents, so is all the Antecedents taken together, to all the Consequents taken together.

Example.

2 to 4; likewise 6 to 12, and 7 to 14, are in Sub-duple Proportion.

Therefore also 2. 6. 7. taken together, to wit, 15, they have the same Proportion to 4. 12. 14. taken together, to wit, 30.

2. If

2. If there are four Numbers Proportional between themselves, they will be also Proportionals, being alterned or changed; if, to wit, the third is taken instead of the second, and the second instead of the third.

As when 2. 3. 4. 6. are Proportionals.

Also — 2. 4. 3. 6. will be Proportionals.

3. If four Numbers are Proportionals, the same will be Proportionals also when they are Inverted, or in a contrary reason; if, to wit, the second is taken, or assumed instead of the first; and the fourth instead of the third.

Example.

Because — 2. 3. 4. 6. are Proportionals.

Also — 3. 2. 6. 4. will be Proportionals.

4. If four Numbers are Proportionals, they will also be Proportionals in a compounded reason; if, to wit, the two former be taken jointly alike, together with the first or second; and the two latter taken also jointly alike together with the third or fourth.

As when — 2. 3. 4. 6. are Proportionals.

Also — 5. 2. 10. 4. will be Proportionals.

Likewise — 5. 3. 10. 6. will be Proportionals.

5. If four Numbers are Proportionals, the excess of the Antecedents to their Consequents will be Proportionals, to wit, by divided reason, or being divided.

Example.

Since — 3. 2. 6. 4. are Proportionals.

Also — 1. 2. 2. 4. will be Proportionals.

Or also — 2. 1. 4. 2. will be Proportionals.

6. If four Numbers are Proportionals, they will be Proportionals also by the Conversion of Reason, or

40 **T H E O R E M S** of **Part III.**

Everting the Proportionality, if you compare the Antecedent with the Excess, whereby the Antecedent exceeds its Consequent; as,

Since———10. 4. 20. 8. are Proportionals.

Also———10. 6. 20. 12. will be Proportionals.

7. *If three Numbers with other three have such Proportion, that as the first is to the second, so is the fourth to the fifth: And as the second to the third, so the fifth to the sixth: It will be also, As the first to the third, so the fourth to the sixth.*

Example.

Let there be three Numbers———2. 4. 6.

Proportionals to these three———3. 6. 9.

So that it is as———2 to 4, So——3 to 6.

And as———4 to 6, So——6 to 9.

It will be therefore also, as 2 to 6, So——3 to 9.

8. *If four Numbers are Proportionals, the Number which is made from the first and the fourth drawn into themselves, will be equal to that which is made from the second and third drawn into themselves.*

Example.

Let there be four Proportionals——7. 8. 14. 16.

As 7 drawn into, or multiplied by 16, gives 112.

So also 8 drawn into———14, gives 112.

9. *If of four Numbers given, the first is to the fourth, as the third to the second; the fact from the first and second, will be equal to the fact from the third and fourth.*

Example.

There are given four Numbers, 6. 4. 8. 3.

Because between the first and fourth there is double Proportion, as also between the third and the second; therefore also as the fact from the first and second

second

P R O B L E M S.

I. **F**ROM any two Numbers given, to find the Proportion of the same by finding the Denominator.

1. Divide the greater Number by the lesser, if nothing remains, it is Multiplex Proportion, and of what kind, the Denominator of the Quotient will shew.

Example.

What is the Proportion of 21 and 7?

It being divided, the Quotient is 3.

Shewing the Proportion is Triple— $\frac{3}{1}$.

2. But if after Division, any thing remains, that will be a Fraction to be added to the Quotient, which being reduced to the least Terms, will give the Denominator, and the Proportion sought.

Example.

What Proportion is there between the
 Numbers $\frac{18}{5}$ } 18. 5.

It being divided, the Quotient produces— $3\frac{3}{5}$.

Therefore the Proportion is, Triple, Supertripartient Fifths.

Note, Proportions of the lesser Inequality are found in the same manner; but in expressing them, the Particle *Sub* must be added, as the Proportion between 5 and 18 is *Subtriple*, *Subsupertripartient* Fifths.

II. Any

II. *Any two Numbers being given, having Proportion between themselves; to find the Root or least Terms of the same Proportion, which may be express'd by more Numbers.*

1. The Proportion being found by the Denominator of it, by the precedent Problem.

2. If the Quotient be single, having no Fraction adjoined, put Unity over or under it, according as the Proportion is of greater or lesser Inequality, and you will have the least Terms.

3. But if a Fraction adhere, being reduced to the least Terms, convert it into one Fraction, with the Integer, and you will have the least Terms of its Proportion.

Example.

1. Of the Proportion between $\frac{4}{2} : \frac{8}{1}$
 The Denominator is $\frac{2}{1}$
 Therefore the least Terms is $\frac{2}{1}$

2. The Proportion between $\frac{3}{1} : \frac{4}{2}$
 The Denominator is $\frac{2}{1}$
 By Reduction to the least Terms $\frac{2}{1}$
 Therefore the least Terms are $\frac{1}{1}$

Note, If the Proportion given be of the lesser Inequality, the Terms found are to be inverted.

So the least Terms of the Proportion $\frac{1}{1} : \frac{2}{1}$
 Are $\frac{1}{2}$

III. *The least Terms of any Proportion being given, to increase the same Infinitely.*

Multi-

Multiply both the Antecedent and Consequent Term of the Proportion by the Number by which you would increase the same.

Example.

Let the Proportion given be *Sesqui Septima*, between 8 and 7: If you would have it in Terms Quintuple greater, multiply both by 5, and it produces $\frac{40}{35}$.

IV. Any Proportion being given, and from a certain Term, either the Antecedent or Consequent; To find the other Term to the same, in the same Proportion.

1. Multiply the given Term by the opposite of the Proportion given, viz. the Consequent by the Antecedent, and the contrary.
2. Divide the Product by the other Term of the Proportion given.
3. The Quotient will agree to the given Term in the same Proportion with the given.

Example I.

Let the Proportion given be double *Sesquitercia*—7. 3. And let there be given for the Consequent Term—24.

What is its Antecedent in the same Proportion?

Ans. Draw 7 into 24, and the fact divide by 3, it produces 56; and the Proportion will be the same between 56, 24, which is between 7, 3.

Example II.

Let the Proportion given be *Sesqui Eighths*—9, 8.

Also the Antecedent Term—36.

What is its Consequent in the same Proportion?

Ans.

Answ. Draw 8 into 36, and the Product divide by 9, it produces the Consequent 32.

Therefore there is the same Proportion between 36, 32, as between 9, 8.

Note well. 1. The Process is the same, whereby one Fraction likewise is reduced to another of a given Denominator, or Numerator; Concerning which we have shewn above in Part the Second, Problem the Fifth, Member the Fifth.

2. And that you may shun Fractions, such an Antecedent must be given, which is Multiplex of the Antecedent; and such a Consequent as is Multiplex of the Consequent, in a given Proportion, and then will be produced the like Multiplex also, of the Proportion given, to the Term sought.

V. *The Denominators of two Proportions being given, to know which is the greater.*

1. Reduce the given Denominators to their Numbers, by Problem the Second.

2. Multiply the Antecedents Cross-wise into the Consequents.

3. Whose Antecedent produces most, that Proportion is the greater; the same as you have above in Part the Second, Problem the Fourth, Member the Second, concerning Fractions.

Example.

There is given *Sesquialter* $1\frac{1}{2}$, and *Superbipartient* thirds $1\frac{2}{3}$; Which is the greater?

Answ. The Denominators reduced to their Numbers are $\frac{3}{2}$ and $\frac{3}{2}$: These drawn Cross-wise into themselves produce the Antecedent of the former 9, of the latter 10, therefore the latter is the greater.

VI. *Given an Arithmetical Progression; to find the Sum of all the Terms.*

Arithmetical

Arithmetical Progression, is, as aforesaid, when many Numbers follow one another in the same Excess: To find the Sum of the same Artificially and Compendiously.

The Practice is thus.

1. Add the first Term to the last.
2. Multiply the Sum by the Number of Terms.
3. Half the Product is the Sum of all.

Example I.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15.

The first Term added to the last makes——16

This drawn into the Number of Terms——15

Produces——240

Half of this is the Sum——120

Example II.

3. 6. 9. 12. 15. 18. 21. 24. 27.

The first added to the last makes——30

This drawn into the Number of Terms——9

Produces——270

Half whereof is the Sum, to wit,——135

VII. *An Arithmetical Progression being given in Numbers ascending, with the difference of the same; To find the Number of any Term.*

Example.

Let there be given a Progression in three Terms, ascending by 3, as, 4. 7. 10.

And the Number of the twelfth Term is sought.

Work

Work thus.

1. From the Term defined 12, take away 1, and you leave 11.
2. Multiply this Remainder by the Difference of Terms, which in this place is 3, and you produce 33.
3. To this Product add the Number of the first Term.
4. So you will have the Number of the twelfth Term, to wit, 37.

VIII. *A Geometrical Progression being given : to find the Sum of all the Terms.*

1. Lessen the Denominator of the given Proportion by Unity.
2. By it, so lessened, divide the Difference of the Extreams (found by Subtraction.)
3. Add the Quotient to the greater Extream ; and you will have the Sum sought.

Example.

Given a Progression, in Proportion Triple.

2. 6. 18. 54. 162.

The Denominator 3, unity being taken away, is 2.

The difference of the Extreams ———— 160.

Divided by 2, gives the Quotient ———— 80

Which added to the greater Extream, makes — 242.

The Sum of all.

IX. *A Number being given in Geometrical Progression, from any former, to a certain distant Interval;*

Interval, to find another, which shall be so much distant from the given, as the given is from the former, both being understood inclusive.

Example.

Let the Progression be double,

2. 4. 8. 16. 32. 64.

And a Term is sought, which is so much distant from 64, as 64 is from 8.

1. Draw 64 into it self, and it makes—4096.

2. Divide the Product by the other given, } 8.
to wit, —————

The Quotient will be ————— 512.

The Term desired in the Ninth Order.

X. Given three Numbers of Arithmetical Proportionality ; to find the Harmonical from them.

1. Draw the first of the Arithmetical Proportionality into the second, and it produces the first of Harmonical Proportionality.

2. Draw then the first of the Arithmetical into the third, and it will produce the second Harmonical.

3. Lastly, Draw the second Arithmetical into the third, it will produce the third Harmonical.

Example.

The Arithmetical Proportion—3. 7. 11.

Makes the Harmonical Proportion— } 21. 33. 77.
tion —————

(And

(And this kind of Proportionality is called Musical, or Harmonical, because the Numbers of it, for the most part have those Proportions in which Musical Harmony consists)

Example.

These three Numbers, 3. 4. 6. constitute Harmonical, or Musical Proportionality : for between 6 and 4 is Proportion *Sesquialter*, constituting the Harmony which is called a Concord of 5 Notes, or Fifth. Also between 4 and 3 is Proportion *Sesquitertia*, constituting the Harmony which is called a Concord of 4 Notes, or the Fourth. Lastly, between the Extrems 6 and 3 is discerned Proportion Double, which constitutes the Eighth : And after the same manner you may perceive in many others.

*Concerning the Proportions of Sounds used
in Musick in our Time.*

THAT the Lovers of Musick may have the Proportions in view, of all Sounds usual in our Musical time, we thought it convenient in this place to expose the Harmonicks of the Ingenious *John Kepler*.

L Of the Intervals of Musick.

| The Names. | The Signs. | Leaf
Num. | Proportions. |
|-------------------------------|--|--------------|----------------------------------|
| | | | <i>Superseptupartients</i> |
| A half Note. | G. C. f.
Ge. Ce. fe. | 135
128 | One Hundred
Twenty Eighths. |
| A quarter
Tone. | B. de.
H. e. | 25
24 | <i>Sesqui</i> Twenty
Fourths. |
| A half Tone. | Ge. A. H. Ce.
A. B. C. d.
e. fe. d.
f. g. de. | 16
15 | <i>Sesqui</i>
Fifteenths. |
| A Tone, or
Second Letter. | A. B. d.
H. C. e. | 10
9 | <i>Sesqui</i>
Ninths. |
| A Tone, or
Second greater. | G. c. f.
A. d. g. | 9
8 | <i>Sesqui</i>
Eighths. |
| A Third
Letter. | A. H. d. e.
C. d. f. g. | 6
5 | <i>Sesqui</i>
Fifths. |
| A Third
Greater. | G. c.
H. e. | 5
4 | <i>Sesqui</i>
Fourths. |
| Four Notes
or Fourths. | G. A. c. d.
C. d. f. g. &c. | 4
3 | <i>Sesqui</i>
Thirds. |

| The Names. | The Signs. | The Numbers. | Proportions. |
|--|--|--------------|----------------------------------|
| Five Notes, or Fifths. | G. A. B. C.
d. e. f. g. | 2
3 | <i>Sesquialter.</i> |
| A Sixth Lesser. | G. A.
De. f. | 8
5 | <i>Supertripartient Fifths.</i> |
| A Sixth Greater. | G. A.
e. fe. | 5
3 | <i>Superbipartient Thirds.</i> |
| A Seventh. | G.
f. | 9
7 | <i>Superbipartient Sevenths.</i> |
| Eight Notes, or Eighths. | G.
g. | 2
1 | <i>Double.</i> |
| 2. The whole Compass of one Eighth. | | | |
| The Names of the Least Intervals. | The Signs. | The Numbers. | |
| A half Note. | G. | 2160 | |
| A half Tone. | Ge. | 2048 | |
| A half Tone. | A. | 1920 | |
| A quarter Tone. | B. | 1800 | |
| A half Tone, | H. | 1728 | |
| A half Note. | C. | 1620 | |
| A half Tone. | Ce. | 1536 | |
| A half Tone. | d. | 1440 | |
| A half Tone. | de. | 1350 | |
| A quarter Tone. | e. | 1296 | |
| A half Tone. | f. | 1215 | |
| A half Note. | fe. | 1152 | |
| A half Tone. | g. | 1080 | |

And

And of these, and also of the former Numbers, the Reason is this.

Conceive in mind some Chord in an Instrument, or the String of a Lute extended, and comprehend between two Bridges.

Understand it in the former Description of particular Intervals, to be divided into so many parts, as the Antecedent Term of Proportion, or the greater Number shews.

Then the Consequent Term of Proportion, or the lesser Number, shews how many of the same parts ought to be to the Note or Sound, distant to the Interval from the next.

Example.

If in the Interval of five Notes, some Chord or String sounds G. and understand its whole length to be divided into three parts; they agree to the Note *d.* five Notes from G. distant two parts of the same: Whence if from three parts you touch one with your Finger, and, as it were, cut it off from the remaining Chord, it will Sound the remainder of the Chord or String *d.*

But in the latter description of the whole Compass of an Eighth, if the whole Chord or String in some Instrument, be understood to be divided into 2160 parts, and it will Sound the same: For

Example, G.

Hence if we seek how many of the same parts agree to the Tone or Sound *d.* which is distant from this by five Notes, I find in the Table—1440.

And it is between the Number—2160.

And the other less than this—1440.

The Difference is ————— 720.

E 2

There-

Therefore so many parts being cut off by the Finger from the whole Chord G. the remainder will Sound *d*.

And the Proportion between 2160 and 1440, is *Sesquialter* $1 \frac{1}{2}$, which is between 3 and 2.

XI. *Three Numbers being given, to find a fourth; which is to the third as the second to the first.*

This is the *Golden Rule of Proportion*, commonly called, *The Rule of Three*, and 'tis direct; of which this is,

The Practice.

1. Of three Numbers given, Multiply the third by the second.
2. The Product of this Multiplication divide by the first.
3. The Quotient arising is the fourth Number sought.

Example.

| | | | |
|-------------------------------|--|---|-------------------|
| The Sha-
dow of
10 Foot | Is cast from a
Pole, or Stick
of 6 Foot, | How many Feet high
is a Tower whose sha-
dow is 125 Feet? | Answ.
75 Feet. |
|-------------------------------|--|---|-------------------|

Some Cautions to be Observed.

1. The first and third Numbers of the given, ought to be Homogenous, and of the same thing and name: Therefore if they are Heterogenous, they must be reduced to Homogenous.
2. Oftentimes Integers, or mixt Numbers, are to be changed into Fractions, and Fractions of divers Denominations to the same Denomination.
3. Oftentimes before the Operation of the *Golden Rule*, many other Numerations, Additions, Subtractions, Multiplications and Divisions are to be instituted; all which Examples and Use will teach.

Example.

Example I.

A Gallon of Wine | Cost | What Costs a Fat ?
 $\frac{1}{10}$ | 10 s. | 1.

In this Example the Fat must be resolved into Gallons 360, that the third Number may agree with the first ; therefore the Numbers must be placed in this manner.

$\frac{1}{10}$ 10. 360.

And the third Number being drawn into the second, produces 3600 Shillings, which should be divided by the first Number 1 ; but because 1 neither Multiplies nor Divides, a Fat of Wine, containing 360 Gallons, will cost 3600 Shillings, at the Rate of 10 Shillings a Gallon ; which being reduced to Pounds, is 180 pounds.

But if we seek.

Fat of Wine | Cost | What costs Gallon ?
 $\frac{1}{10}$ | 180 l. | 1.

For this we must say,

Gallons | cost shill. | What cost Gallon ?
 360 | 3600 | 1

And the operation being made, it produces 10 s.

Examp. II. $\frac{1}{4}$ makes $\frac{1}{2}$, What makes $\frac{1}{4}$?

In this case multiply $\frac{1}{2}$ by $\frac{1}{4}$, and they make $\frac{1}{8}$, which being divided by $\frac{1}{4}$, that is by $\frac{1}{4}$, to wit, by inverting the Fraction, and multiplying, it produces $\frac{1}{2}$, or 2 $\frac{1}{2}$.

Example III. If 2 makes $\frac{1}{2}$, What makes 3 ?

Resolve the Integers into Fractions, saying, $\frac{3}{1}$ makes $\frac{1}{2}$, what makes $\frac{1}{2}$? the Operation being made, it produces $\frac{3}{4}$.

Example IV. If 2 $\frac{1}{2}$ make 5, What doth 5 $\frac{1}{4}$ make ? Here we must resolve the mixt Fractions into simple, and say : If $\frac{1}{2}$ make 5, What doth $\frac{1}{4}$ make ? And the Operation being made, it produces $\frac{5}{2}$, that is 13 $\frac{1}{2}$.

Example V.

One sells of a heap of Wheat — $\frac{1}{3}$ & $\frac{1}{4}$

And he hath remaining, Sacks — 35.

We desire to know how much Wheat he had in the beginning?

1. Reduce the Fractions to the same Denomination, and they make $\frac{8}{12}$ and $\frac{3}{12}$.

2. Since the parts taken from the whole are twenty parts, the whole must needs be $\frac{20}{1}$.

3. Hence take away $\frac{8}{12}$ and $\frac{3}{12}$ that is $\frac{11}{12}$, and there will be $\frac{1}{12}$ left; say then: If $\frac{1}{12}$ makes $\frac{1}{12}$, What makes $\frac{11}{12}$? *Ans.* $\frac{11}{12} \times 20 = 35\frac{1}{3}$, or 65.

Or without Fractions.

If 7 makes 35, what makes 13? *Ans.* 65. And so many Sacks the Man had at first.

Example VI.

Half an ounce of Silver costs Shillings $2\frac{1}{4}$.

And three Drinking Cups { A. $12\frac{3}{4}$

A. B. C. weigh — { B. $16\frac{1}{2}$

Half Ounces — { C. $23\frac{1}{2}$

Now we would know the price of these three Cups?

Work thus :

1. Gather together the Integers weight of the Cups into one Sum, which is 51.

2. Reduce to one Denomination each Fraction annex'd to the weights, to wit,

24. 18. 6.
 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{2}$ to 

3. The Sum of these by Addition, produces $36\frac{3}{4}$, or $36\frac{3}{4}$.

4. Add this to the former Sum of the Integers, and it makes $52\frac{1}{4}$: Say then,

Half

| Half ounces | Cost Shillings | What Costs |
|-------------|----------------|-----------------|
| 1 | $2\frac{1}{2}$ | $52\frac{1}{2}$ |

| | | |
|-----------------------|---------------|----------------|
| Or thus $\frac{1}{2}$ | $\frac{1}{2}$ | $4\frac{1}{2}$ |
|-----------------------|---------------|----------------|

Ans. that is, Shillings 130 $\frac{1}{2}$ or 6 l. 10 s. 10 d.

XII. From three Numbers given, to find a fourth, which shall have such Proportion to the first, as the third to the second.

This is called the Rule of Reciprocal or Inverted Proportion; its use is, As often as the values of things increase, the things themselves decrease, and the contrary.

The Practice thus,

1. Of three given Numbers, multiply the first into the second.
2. The Product divide by the third.
3. The Quotient will be the fourth sought.

Example I.

| Yoke of Oxen | Plow a Field in Days | How many days will the same be plowed in with Yokes of Oxen | <i>Answer.</i>
In days |
|--------------|----------------------|---|---------------------------|
| 6 | 4 | 8. | 3. |

Example II.

| Work-Men | Finish a certain Work in days | In how many days will the same work be finished by men | <i>Answer</i>
In |
|----------|-------------------------------|--|---------------------|
| 10 | 4. | 40. | 1. |

Note. To these Golden Rules of Proportion, Arithmeticians have added four other Rules, as,

1. The Rule of Society, or Fellowship.
2. The Rule of Double, or the Compound Rule of Three.
3. The Rule of Alligation.
4. The Rule of False.

I.

Of the Rule of Society, or Fellowship.

THIS Rule is used when many being entred into a Society, they put in divers Sums of Money to the common Negotiation: And the Gain of each Man is sought, either with, or without the Circumstance of Time.

I. Example without the Circumstance of Time,

Three Merchants A. B. C. contribute together several Sums, } A. 60 £ } they have } What ought } A.
 } B. 100 } gained } each one to } B.
 } C. 135 } 45 £ } have of the } C.
 } } } Gain }

In this Case reduce the contributed Money into one Sum : then to the part of each of the Contributors, the Rule of Proportion is to be thrice repeated, in this manner :

If $\begin{matrix} l. \\ 295 \end{matrix}$ gain $\begin{matrix} l. \\ 45 \end{matrix}$ what $\left\{ \begin{matrix} 60? \\ 100? \\ 135? \end{matrix} \right\}$ Answer, $\left\{ \begin{matrix} 9 \frac{2}{3} \\ 15 \frac{1}{3} \\ 20 \frac{2}{3} \end{matrix} \right\}$

The Sum of the three Products agrees with the second 45, therefore the Operation is just.

II. Example with Circumstance of Time.

Two have gained 60 % and the first Man A. put in 50 % for two years; the other, B. 15 % for five years:

How much ought each to have of the Gain?

In this Case,

1. Draw

1. Draw the times of each, into the Sums contributed, to wit,

2 into 50, which makes 100.

5 into 15, which makes 75.

2. Take the Sum of these, which is 175.

Say then,

175 gives 60, what $\left\{ \begin{array}{l} 100? \\ 75? \end{array} \right\}$ Answer, $\left\{ \begin{array}{l} 34 \frac{2}{3} \\ 25 \frac{1}{3} \end{array} \right\}$

II.

The Golden Rule Compound.

IS that whereby we assume from pairs of Numbers, single Numbers made from them.

Example.

If 4 Men spend 7 19 1. what will 8 Men in
in Months 3 be spent by — 9 Months?

Draw then the Pairs into themselves, saying,
If 12 makes 19, what makes 72? *Ans.* 114.

Another Example.

18 Horses eat 9 2 In days How many 2 will 24 quarters
quarters of Oats 5 14 1 days eating 5 serve 18 Horses?

The Pairs being drawn into themselves, say,
If 162 make 12, what makes 432? *Ans.* 32.

III.

The Rule of Alligation.

THIS joins several things together
at one Price.

I. Exam.

I. Example.
In which the price of the things to be mixt, and also the price after mixture is given.

A Vessel containing 160 gallons of A 12s Wine, is to be mixt of four several B 10s Shill- sorts of Wine A B C D, of which C 8s lings. the price of one gallon is, of D 6s

And a gallon after 'tis mixt ought to be worth 9 shillings : How much therefore must we take of each Wine ?

1. Seek by Subtraction, the difference between the price of the mixt, and each of those to be mixt.

Example between $\left. \begin{array}{r} 12 \\ 10 \\ 8 \\ 6 \end{array} \right\}$ which is $\left\{ \begin{array}{r} 3 \\ 1 \\ 1 \\ 3 \end{array} \right.$
9 and

2. The Sum of the Differences is 8
Say then,

If 8 makes 160, what makes $\left\{ \begin{array}{r} 3 \\ 1 \\ 1 \\ 3 \end{array} \right\}$ Answer, $\left\{ \begin{array}{r} 60 \\ 20 \\ 20 \\ 60 \end{array} \right.$

II. Example.

In which the Price of the things to be mixt are given, but not of the mixt, whose price is sought.

To be mixt $\left\{ \begin{array}{r} 60 \\ 20 \\ 20 \\ 60 \end{array} \right\}$ Gallons of Wine of $\left\{ \begin{array}{r} 12s. \\ 10s. \\ 8s. \\ 6s. \end{array} \right\}$ Which drawn in themselves, make $\left\{ \begin{array}{r} 720 \\ 200 \\ 160 \\ 360 \end{array} \right\}$ What will the price of one gallon be after the mixture.

Sum 160

1440

Say then,
If 160 makes 1440. What makes 1 ?
Answer, 9 Shillings.

IV. The

IV.

The Rule of FALSE.

Which teaches from False things put, or supposed, to produce true.

I. Case.

If the false Numbers put, be both of them Deficient, or both exceeding the True.

1. Subtract either the Excess, whose Sign is $+$ or the defect, whose Sign is $-$, the lesser from the greater, and the Remainder reserve for a Divisor.

2. Multiply the false Numbers assumed, one into the Excess or Defect of the other.

3. Subtract the lesser of the Products from the greater.

4. Divide the Remainder by the reserved Divisor; and the Quotient will be the Number sought.

Example I.

There are 3 Numbers $\left\{ \begin{array}{l} A \\ B \\ C \end{array} \right\}$ The Sum of the Numbers is — $\left\{ \begin{array}{l} AB \ 50 \\ BC \ 70 \\ AC \ 60 \end{array} \right\}$ What is the Number of each A B C?

Where observe, If one be known, to wit, A, the rest will be known; let it be then,

Position I.

Ascribe to A 16

Then will B 34

be — C 36

The Sum of A C 52

It ought to be — 60

The defect therefore 8

Position. II.

Ascribe to A 18

Then will B 32

be — C 38

The Sum of A C 56

It should be — 60

The defect then — 4

Then

Then for finding the true Number.

1. From the greater defect ————— 8
 Subtract the lesser ————— 4
 There remains the difference ————— 4
 To be reserved for a Divisor.

2. Draw the first assumed False Number into the defect of the second assumed; and the second assumed into the defect of the first; in this manner.

Draw 18 into 8, it makes — 144

Draw 16 into 4, it makes — 64

3. This Subtracted from that, leaves 80 : which divided by the reserved Divisor 4, produces in the Quotient the true Number sought, 20.

For, If A be 20 } the Sum 50
 B will be 30 }

B ——— 30 } the Sum 70
 C ——— 40 }

C ——— 40 } the Sum 60
 A ——— 20 }

The reason is altogether the same, if both the Positions, or False Assumptions, exceed the True Number.

II. Case.

If one of the posited False Numbers is deficient from the True, and the other exceed it.

1. Add the deficient to that which exceeds; the Sum of which you must reserve for a Divisor.

2. Draw

2. Draw both the assumed Numbers into the Defect and Excess of one another.

3. Join the Products.

4. Divide the aggregate by the reserved Divisor, and the Quotient will shew the true Number sought.

Example I.

Let the unknown Number be A, if we assume.

First the Quadruple of it with Unity.

Then the Triple of this Quadruple with three.

The Sum of these two Numbers makes 56.

Then we seek, what is the Number A?

| | |
|-------------------------------|--------------------------------|
| 1. Suppose, or put A to be 2. | 2. Suppose, or put A. to be 5. |
|-------------------------------|--------------------------------|

| | |
|---------------------------------|----------------------------------|
| Its Quadruple with 1 is ————— 9 | Its Quadruple with 1 is ————— 21 |
|---------------------------------|----------------------------------|

| | |
|--|---------------------------------------|
| The Triple of this with 3, is ————— 30 | The Triple of this with 3 is ————— 66 |
|--|---------------------------------------|

| | |
|---------------------|---------------------|
| The Sum is ————— 39 | The Sum is ————— 87 |
|---------------------|---------------------|

| | |
|------------------------|------------------------|
| Which ought to be — 56 | Which ought to be — 56 |
|------------------------|------------------------|

| | |
|----------------------------|----------------------|
| The defect therefore is 17 | The Excess is ——— 31 |
|----------------------------|----------------------|

1. Add the Excess to the Defect, and it makes 48 for a Divisor.

2. Draw the assumed false 2 into the Excess 31, and it makes ————— } 62.

Also draw the other false 5 into the Defect 17, and it makes ————— } 85.

3. These two Products joined, make ——— 147.

4. Which divide by 48, and the Quotient produces the Number sought $3\frac{1}{8}$.

For suppose, or put ————— $3\frac{1}{8}$.

Its Quadruple with 1, is ————— $13\frac{1}{4}$.

And its Triple with 3 is ————— $42\frac{3}{4}$.

The Sum of which two Numbers is — 56.

Example.

Example II.

1. Let any one conceive in his mind a Number. ©
2. From the double of it take away Unity.
3. Again double the Remainer, and from this double take away 2.
4. Thirdly, double the Remainer, and from this double take away 3.
5. And in the Remainer there is only 10.

And we would know what is the Number ©.

This Rule of False is very useful to the finding many unknown Numbers : And also in *Algebra*, or *Coffie Rules*, with which it nearly agrees, since many *Algebraical Questions* also may be Solved by this *Rule of False*.

But the rest, which pertains to the Amplitude, as well of the *Rule of Proportion*, as also to these annexed Rules, cannot so well be comprehended by Precepts as by Ingenuity.

The First Book of

Theoretical Arithmetick:

And its Fourth Part.

Of Figurate Numbers.

Definitions.

1. **A Figurate Number**, is a Number shewing the Unities of a Geometrical Figure from its circuit and disposition.

And it is either Plain or Solid.

2. **A Figurate Plane**, is that which resembles a Geometrical Plane, by its Circuit or Periphery.

They are called in general *Polygons*, of which there are infinite kinds, as in Plate 1. Fig. 1. is a *Triangle*. Fig. 2. a *Square*. Fig. 3. a *Pentagon*, or five sided Figure.

The sides of these are called *Factors* and *Roots*: and they arise from the natural order of Numbers; either from none or some, or from some left.

The First, }
 The Second, } Is begat } whose
 The third, } from } unities }
 The Fourth, } 1.2. } are }
 1.2.3. }
 1.2.3.4. }
 1.2.3.4.5 }

Of

Of Quadrangulars, { The First } { The Second } { The Third } is begun from { 1, 2, 3, 4 } { 1, 3, 5, 7 } { 1, 3, 5, 7 } Whole unitys { 4, 9, 16. }

| | | | | | | | | | |
|--------------------|---|------------|------------|---|-----------|---------|---|-------|-----|
| Of Penta-
gons, | { | The First | } Is begat | { | 1.4. | } Whole | { | 3. | |
| | | The Second | | | 1.4.7. | | | units | 12. |
| | | The Third | | | 1.4.7.10. | | | are | 22. |
| | | | | | | | | | |

And so onward.

Among Polygon Planes, the Quadrate, or Square is the Chief, as Fig. 4.

3. A *Quadrate* (in *Arabick*, *Zensus*) is that which hath four equal Lines, and four equal Angles, and arises from the mutual Multiplication of two equal Numbers.

Either of these Numbers is called the Side or Root of the Square, the sign of which is,

l. Re. 7. q. or fs.

4. *A Solid Number*, is that which shews a Geometrical Solid by its Circuit or Periphery, and is stretched out, or extended by its Unities into three Intervals, to wit, Length, Breadth, and Depth.

Among the infinite Kinds of these Geometrical Solids, the Cube is the chiefest: as *Fig. 5.*

5. A *Cube* is a Solid Number, made from three equal Numbers multiplied in themselves, representing a Geometrical Cube, or having equal magnitude of Length, Breadth, and Thickness.

Each of its Factors are called the side of the Cube, or the Cubick Root, the sign or mark of which is,

Lc. Bc. 7 c.

Hence arise Numbers called *Coffic* or *Algebraic*, with their *Exponents*, *Characters*, and *Names*.

6. *Coffic Numbers*, are Numbers designed by certain Names and Characters, being multiplied in any Proportion from Unity, proceeding *ad infinitum*, always in the same Proportion, as the next from Unity exceeds the Unity, of which they shew the distance from Unity, proceeding in a rank of Natural Numbers, and also how many times the Root is to be put or placed, that the *Coffic* may be produced by Multiplication.

Example in Geometrical Progression double.

| Coffic Numb. | Exponents. | Characters. | Names. |
|--------------|------------|-----------------------------|---|
| 1 | 0 | N. | Unity, or absolute Number. |
| 2 | 1 | a^1 , or γ . l. B. | Side, or Root. |
| 4 | 2 | a^2 , or 3. q. | Quadrate, Zensus. |
| 8 | 3 | a^3 , or cu. c. | Cube, Zensi Zensus. |
| 16 | 4 | a^4 , or 33. qq. bq. | Biquadrate. Squared square. |
| 32 | 5 | a^5 , or fs. | Solid. Surfsolid. |
| 64 | 6 | a^6 , or 3. cu. qu. | Quadrato Cube. Zensi cubus. |
| 128 | 7 | a^7 , or B fs. | Bisurfsolid Bisolid. |
| 256 | 8 | a^8 , or 333 qqq. | Triquadrat. Zensi zenzensus, or Squared square squared. |
| 512 | 9 | a^9 , or c. cu. cu. cc. | Bicubed. Cube cubed. |
| 1024 | 10 | a^{10} , or 3 fs. sq. | Quadrato solid. Zensus de solidus. |
| 2048 | 11 | a^{11} , or c fs. | Cube surfsolid. Ter solidus. |
| 4096 | 12 | a^{12} , or 33 cu. | Biquadrato cube. Zensi Zensi cubus. |
| 8192 | 13 | a^{13} , or D fs. | D. surfsolid. Quater solidus. |
| 16384 | 14 | a^{14} , or 3 B fs. | Quadrato Bisolid. Zensi solidus B. surfsolid. |
| 32768 | 15 | a^{15} , or cu. fs. | Cube of Solid, &c. |

The Arithmetick of these Concrete Numbers is called *Coffic*, from the Inventer *De la Cosa* an Italian; and *Algebra* from *Geber*; concerning which *Christopher Rudolph*, and *Christopher Clavius* have written accurately, but more especially our Countreyman *Mr. Kersey* in his *Elements of Algebra*.

THEOREMS.

1. **T**WO Quadrate Numbers, being mutually multiplied in themselves, produce a Quadrate.

So 8 a quadrate, drawn into the quadrate 16, produces the quadrate 144. Whence also any quadrate drawn into it self, produces nought else but a quadrate, by the definition of a quadrate.

2. A Cube multiplied in it self, and into another Cube, always produces a Cube: For Example,

The Cube 8, multiplied in it self, produces 64, which is also a Cube: Likewise the Cube 8 drawn into the Cube 64, again produces the Cube 512.

3. If from Unity, any Numbers be successively Proportionals: The third from Unity is a Square, and all of the rest intermitting one: And the fourth is a Cube, and all the rest are Cubes intermitting two: But the seventh is a Cube, and also a Quadrate or Square, and all the rest intermitting five.

Example in Proportion Treble.

| | | | | | | | | | | | |
|----|----|----|-----|-----|------|------|-------|-------|--|--|----|
| | | c. | | | | c. | | | | | q. |
| | q. | c. | q. | | | q. | | | | | q. |
| 1. | 3. | 9. | 27. | 81. | 243. | 729. | 2187. | 6561. | | | |

| | | | | | | |
|--------|--------|---------|---------|--|--|----|
| | | c. | | | | q. |
| | c. | | q. | | | q. |
| 19683. | 59049. | 177147. | 531441. | | | |

4. If

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4. If from Unity, any Numbers be successively Proportional, and if that next to Unity be a Square, all the rest in the same Proportion will be Squares: But if after Unity it be a Cube, all the rest will be Cubes.

Example in Squares or Quadrates.

1. 4. 16. 64. 256. 1024. 4096.

Example in Cubes.

1. 8. 64. 512. 4096 32768.

5. A Quadrate to a Quadrate hath double Proportion of Side to Side; but a Cube to a Cube hath Triple Proportion of Side to Side.

And Double Reason or Proportion is made from a Square, by multiplication of the Sides; but Triple Reason from the Cubic Multiplication of the Sides; by that the Proportion of the Sides is made equal to the Proportion of the Squares; by this the Proportion of the Sides is made equal to the Proportion of the Cubes.

PROBLEMS.

I. **T**O know the Squares and Cubes of Prime Roots, from Unity to Ten.

For the foreknowledge of these is wholly required, which that you may have, inspect often, and get by heart the following Table.

| | | | | | | | | | | |
|----------|---|---|----|----|-----|-----|-----|-----|-----|------|
| Roots. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Squares. | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |
| Cubes. | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 |

II. Any Number being given, To extract its Square Root.

Example.

R

978121 (989.
 81 || ||
 ———
 1681
 188
 1504 ||
 ———
 17721
 1969
 ———
 17721
 ———
 00000

The First Operation to be but once done.

1. In the place of odd posited Numbers, impone or prefix a point, beginning from the right hand.

2. Having taken from the Table the Square Root of the first point to the left hand, or (if it be not a Square) of the next lesser Square, inscribe it in the Semicircle or Quotient.

3. Subscribe the Square of it under the first point.

4. Subtract it from that above it.

5. To the residue, assume, or bring down the two other figures.

The Second Operation to be repeated according to the Number of Points.

1. Double the Root you inscribed just now in the Semicircle.

2. Subscribe this Double under the second figure to the right hand.

3. The quote of this (to wit, so many times as 'tis contain'd in the upper Number) write in the Semicircle, and also adscribe it to the Double.

4. Multiply the whole Subscription by the Quotient.

5. Subtract the Multiplied from its Superior.

6. To the Remainer, affix the new Numbers, if there be any, and repeat this second Operation ; with this proviso that you double the whole Quotient, and not only the last Figure.

III. To

III. *To find the Root very near, in Numbers not Square.*

If after all the Operation is rightly performed, there remains some residue, that shews that the number proposed is not a Quadrate or Square: Therefore in such numbers the exquisite or exact Root is not given, and therefore it is called Surd or Irrational; as also in Solids not Cubicks.

Nevertheless the near Root, in which little is wanting, is thus found.

1. To the Remainder add two, or four, or six, or eight, or yet more pairs of Cyphers.

2. Repeat the second Operation so many times as you have added pairs of Cyphers: then the Root produced hence will be a Fraction, adherent to the principal Root, and therefore to be distinguished from it by a point: And will denote Decimal Parts of the Integer, if you have added two Cyphers; Centesimal Parts, if you added four Cyphers; Thousandth Parts, if you added six Cyphers, and so on, &c.

Example.

$$\begin{array}{r}
 \overset{\cdot}{1} \overset{\cdot}{5} \overset{\cdot}{0} \quad (12. \overset{\cdot}{1} \overset{\cdot}{2} \overset{\cdot}{4} \overset{\cdot}{4} \\
 \underline{1} \\
 1. \\
 050 \\
 22 \\
 \underline{44} \\
 0600 \\
 242 \\
 \underline{484} \\
 11600 \\
 2444 \\
 \underline{9776} \\
 1824 \\
 \text{F } 3
 \end{array}$$

IV. *To*



IV. To find the Cube Root.

The first Operation to be but once done.

Example.

$$\begin{array}{r}
 967361669 \text{ (989)} \\
 729 \overline{) 111111111} \\
 238361 \\
 \underline{27} \\
 243 \\
 \underline{1944} \\
 1728 \\
 \underline{512} \\
 212192 \\
 26169669 \\
 1728 \\
 \underline{11} \\
 294 \\
 28812 \\
 \underline{259308} \\
 23814 \\
 \underline{729} \\
 26169669 \\
 \underline{00000000}
 \end{array}$$

$$\begin{array}{r}
 294 \\
 98 \\
 \underline{2352} \\
 2646 \\
 \underline{28812}
 \end{array}$$

1. Impone points, as in the example, from the right hand, always intermitting two places, making the first point over the Unites place.

2. Inscribe in the Semicircle the Cube Root, or the next lesser, of the first point.

3. Subscribe its Cube.

4. Subtract it from the superior Number.

5. To the Remainer, bring down the three new Numbers.

The Second Operation to be repeated.

1. Triple the Root.

2. Subscribe the Triple under the second Figure to the right hand.

3. And the same Triple being first drawn into the whole Root, viz. both Figures in the Quotient, subscribe under the third place.

4. Seek the Quote of this; Inscribe the Quote, and by the Quote multiply this, having first drawn an occult line underneath, subscribe it under the third Figure.

5. Multiply the Square of the Quote into the Triple of the first

Quote above, (that is, 27) and subscribe it in the second place.

6. The Cube of the Quote subscribe in the first place.

7. Add these three subscribed Numbers.

8. Subtract the Sum from the Superior, and bring down the new ones, if there be any, and add to the Remainer.

V. To

V. *To find the Root very near, in Numbers not Cubick.*

1. Add to the Remainder either 3, or 6, or 9, or more Ternaries of Cyphers.

2. Repeat the second Operation so many times, as you add Ternaries of Cyphers.

3. The Product will give the Fraction of the Root, and indeed in Decimal Parts, if you have added three Cyphers; in Hundredth Parts, if six Cyphers; in Thousandth Parts, if nine Cyphers be added.

VI. *Two Numbers being given, to find a mean Proportional.*

That is, a Number which so much exceeds the lesser, as 'tis exceeded by the greater.

And 'tis thus found.

1. Multiply the given Numbers in themselves.

2. Seek the Square Root of the Product, which will be the mean Proportional.

Example.

What is the mean Proportional between 3 & 12?

Answ. 3 drawn in 12 makes ———— 36

The Root of this is 6, the Mean between the two given Proportionals.

VII. *Two Numbers given, to find a Third Proportional.*

F 4

That

That is, a Number which is to the second, as the second is to the first.

1. Multiply the second in it self.
2. And divide the Product by the first: The Quotient is the third Proportional sought.

And this Process is no other than the Rule of Three, by repeating the second Term.

Example.

If 2 gives 6, what gives 6? *Ans.* 18.

VIII. *To two Numbers given, To find any other two, Proportional to them.*

1. Adjoin any third Number to the two given.
2. With these work according to the prescribed Rule of Three, and the Quotient with the adjoined Number will be the two Proportionals to the two given.

Example.

Given 5 and 10; add 12: then say, as 5 to 10, so 12 to 24.

IX. *To two Numbers given, To find two Mean Proportionals.*

1. Draw the lesser into it self.
2. Draw the Product into the greater.
3. The Cubick Root of this, is the first Mean Proportional.

Then,

1. The first Mean Proportional being found, draw it into it self.
2. The Product divide by the first lesser given,

3. The

3. The Quotient is the other Mean Proportional.

Example.

To find two Mean Proportionals between 4 and 32 ; proceed thus.

4 drawn (or multiplied) in it self, makes — 16.

This drawn into 32 makes — 128.

The Cubick Root of this, is — 5.

Which is the first Mean Proportional.

Then,

8 drawn into it self, makes — 64.

This divided by the lesser given — 4.

Exhibits in the Quotient — 16.

Which is the second Mean Proportional, and in this manner they make four Proportionals, to wit, 4. 8. 16. 32;

The End of the First Book.

The

SECOND BOOK OF

Practical Arithmetick.

The P R E F A C E.

1. *P*Ractical or *Concrete Arithmetick*, is that which considers, not Numbers abstracted from all things, but applied to the measures of divers things.

It Constitutes Two Parts,

I. *Astronomical.*

A N D

II. *Political or Civil.*

3. *Astronomical*, which sometimes also is called *Logistical*, is the Doctrine of handling *Astronomic Numbers*, which are usually called *Physical Fractions*.

4. *Political or Civil*, is that which compares and explains *Moneys or Coins*, the measures of *Dry things*, of *Liquids*, of *Intervals or Distances*, useful in *Commonwealths*.

The

The Second Book of

Practical Arithmetick,

And its First Part.

Of Astronomical Arithmetick.

C H A P. I.

Of Astronomical Numbers.

1. **A** Stronomick Numbers, are either of Motion, or of Time.

2. *Motion*, when 'tis Circular, is numbred by the parts of a Circle.

3. A Circle is divided into Degrees—360, which constitute the whole.

A Degree is subdivided into Minutes, or } 60'.
First Scruples ————— }

A First Scruple, into Seconds ————— 60".
Seconds, into Thirds, and so on.

4. *Degrees* are collected, either in Celestial Signs, of which 12 compleat a Circle; and each contains 30 Degrees: Or into Sexagenary Primes, Seconds, Thirds, &c.

5. One Sexagenary Prime contains 60 Degrees.
And a Sexagenary Second contains 60 Primes, or 3600 Degrees.

And

And one Sexagenary Third, hath 60 Sexagenary Seconds, or 216000 Degrees.

| Comprehends or Contains: | | |
|--------------------------|-----------------------|-------------|
| | Scruples. | |
| Degrees | 1 60 | 1 |
| | 2 3600 | 2 |
| | 3 216000 | 3 And Sexa- |
| | 4 129600000 | 4 genarys |
| | 5 777600000 | 5 contain |
| | 6 46656000000 | 6 Degrees. |
| | 7 2799360000000 | 7 |
| | 8 167961600000000 | 8 |
| | 9 10077696000000000 | 9 |
| | 10 604661760000000000 | 10 |

Also some call whole Degrees *Asses*, and divide one Degree, or its 60' into 12 parts, which they express by the Names of parts of Assis. In this manner.

As, cont. 60' | 9 parts 45' | 6 parts or $\frac{1}{2}$ 30' | 3 parts or $\frac{1}{4}$ 15'
 11 parts 55' | 8 parts 40' | 5 parts — 25' | 2 parts or $\frac{1}{5}$ 10'
 10 parts 50' | 7 parts 35' | 4 parts or $\frac{1}{3}$ 20' | 1 part — 5'

6. Time is divided, either *Vulgarly* or *Astronomically*.

Vulgarly.

A Julian Year hath Months ——— 12.
 Days in a Month — 28. 29. 30. 31.
 Hours in a Day ——— 24.
 Minutes in an Hour ——— 60.
 And so on.

An *Egyptian* Year hath Months — 12.

Their

Their Months equally contain 30 Days : Which are distinguished into Hours and Minutes, as the *Julian Day* is.

Astronomically.

1. A Day is divided as an Integer, into first Scruples 60'.

A Prime or first Scruple into 60", and so on.

According to this Computation they account 2', 30" to one common Hour.

2. And Days also are accounted in *Sexagenes*, so that 60 Days make one *Prime Sexagene* : And sixty *Prime Sexagenes*, or 3600 Days make one *Second Sexagene*, &c.

CHAP. II.

Of *Logistical Notation.*

IN this two things are considered.

1. The right placing of Numbers.
2. The Genuine, or Natural Designment of the same.

I. In Motion.

The manner of Placing and Designment is thus.

Signs. | Degrees. | Scruples.

S | ° | 16'. 9". 7''' . 8'''.

Or,

Sexagenes. | Degrees. | Scruples.

2^a. 2^a. 1^a. | ° | 29'. 6". 50''' . 44'''.

II. In TIME.

Years. | Months. | Days. | Hours. | Minutes.

Y. | M. | D. | H. | M.

6. | 7. | 26. | 4. | 56'. 34".

Or,

| Or, | Sexagenes. | Days. | Scruples. |
|-----|-------------|-------|-----------------|
| | 30. 20. 10. | D. | |
| | 4. 3. 5. | 36. | 55. 34. 48. &c. |

CHAP. III.

Of Logistical Addition.

1. **I**T places Numbers of like Kinds, or Ranks, under like; to wit, Prime Sexagenes under Prime Sexagenes; Seconds under Seconds; Degrees or Days under Degrees or Days: Prime Scruples under Primes; Seconds under Seconds, &c.

2. If any place want a Number, it must be supplied by a Cypher, or 0.

3. The collected Sum of one kind, as often as it constitutes the foregoing kind, so many Unities must be added to the former kind, and the Remainder which belongs not to the former kind, must be only subscribed.

Example. I. To be added in Motion.

| | | | | |
|---------|----|-----|-----|-------|
| S | 0 | | | |
| 9 | 27 | 56' | 48" | 32''' |
| 8 | 24 | 49 | 36 | 55 |
| <hr/> | | | | |
| The Sum | 6 | 22 | 46 | 25 27 |

Note, 1. Because every Rank or Kind of the Scruples, constitute one of the former Ranks or Kinds, therefore Unity is added to each of the foregoing or former Ranks, and under the latter Rank that only is placed which falls short of 60.

2. Because of the collected Degrees, 30 constitute or make a Sign, therefore of the Degrees, Unity is added to the Signs, and under the Degrees, that only is retained and subscribed which is less than 30 Degrees.

3. Whole Circles, or 12 Signs, for the most part are rejected, and that only retained which is under 12, and less than a Circle: For in Celestial Motions, 'tis not always sought how many times any Star hath passed over its Circle? But what part of its Circle 'tis in at any time?

Example

Example II.

| Sexagenes | | | Degrees. | Scruples. | | | |
|------------------|------------------|------------------|----------|-----------|-----|-------|-------|
| 3 ^a . | 2 ^a . | 1 ^a . | 0 | | | | |
| 47 | 36 | | 58 | 59' | 38" | 46''' | 55''' |
| 14 | 59 | | 40 | 47 | 36 | 38 | 44 |
| Sum | 02 | 36 | 39 | 47 | 35 | 25 | 39 |

Example III. To be added in Time.

| Years. | Days. | Hours. | Minutes. |
|--------|-------|--------|----------|
| Y. | D. | H. | |
| 30 | 36 | 23 | 55' 49" |
| 50 | 0 | 12 | 59 45 |
| Sum | 80 | 37 | 12 55 34 |

Note, The Hours collected are 36; and because 24 Hours constitute a Day, 24 being taken away, you retain only 12 Hours; and instead of the 24 Hours, you increase the Number of Days by Unity.

CHAP. IV.

OF SUBTRACTION.

1. **I**T is done as in Vulgar Numbers: only observing this, that like Kinds or Ranks are placed under like.

2. Then if the Number to be Subtracted be greater than the Number from which 'tis to be Subtracted, you must assume or borrow Unity from the Antecedent or foregoing Rank, which in mind is to be resolved into the Consequent Rank, and added to it.

3. And you must remember that the Antecedent Kind or Rank is lessened by Unity, which that you do not forget, you may Impone, or make a point to remember you.

Example.

Example.

| | Sign. | o | | | |
|-----------------|-------|-----|-----|-------|--|
| The Minorand—5 | 28 | 00' | 44" | 56''' | |
| The Subducend—3 | 29 | 36 | 55 | 59 | |
| The Remainder—1 | 28 | 23 | 48 | 57 | |

Note, 1. When under the Rank of Second Minutes, a First or Prime Minute should be taken together, or borrowed, and there be none, we must have recourse to the Degrees, and resolve one into 60', and we must also resolve one first Minute into 60".

2. Sometimes it happens, that not only the following Ranks or Kinds, but the first of all also of the *Minorand*, is less than the *Subducend*: Then in such case, a whole Circle is to be assumed or borrowed, and to be added to the lesser Number or *Minorand*. And then you must institute the Subtraction.

C H A P. V.

Of MULTIPLICATION.

1. **F**OR the sake of conveniency in Operation, write the greater Number, to wit, that which is compounded of most Ranks, above, or uppermost, as a Multiplicand: To which subscribe the other of fewer Ranks as a Multiplier, so that the last Rank of the Multiplier may be set under the last of the Multiplicand.

2. Neither doth it matter whether the same, or divers Ranks of this supposed Multiplier, are multiplied with the superior Multiplicand.

3. Draw each Rank of the Multiplier into each one of the multiplicand; and that which is produced, divide first apart by it self by 60" (if it exceed

a Sexa-

Example.

| | Signs. | o | | | |
|-----------------|--------|-----|-----|-------|--|
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Of MULTIPLICATION.

1. **F**OR the sake of conveniency in Operation, write the greater Number, to wit, that which is compounded of most Ranks, above, or uppermost, as a Multiplicand : To which subscribe the other of fewer Ranks as a Multiplier, so that the last Rank of the Multiplier may be set under the last of the Multiplicand.

2. Neither doth it matter whether the same, or divers Ranks of this supposed Multiplier, are multiplied with the superior Multiplicand.

3. Draw each Rank of the Multiplier into each one of the multiplicand ; and that which is produced, divide first apart by it self by 60" (if it exceed

a Sexa-

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 a Sexagenary) and the residue place under the Multiplier : But the Quotient produced, add to the Antecedent Rank.

Example.

| | | | | | | |
|---------------|-------|----|-----|------|-------|--------|
| Multiplicand— | 12 0 | | | | | |
| | 4 | 6 | 9' | 12'' | 24''' | |
| Multiplier— | | 5 | 8 | 9 | 16 | |
| | <hr/> | | | | | |
| | 1 | 1 | 2 | 3 | 6 | 24 |
| | | 4 | 36 | 24 | 12 | |
| | .. | .. | .. | .. | .. | .. |
| | 0 | 0 | 1 | 1 | 3 | 36 |
| | | 36 | 54 | 21 | 48 | |
| | .. | .. | .. | .. | .. | .. |
| | 0 | 0 | 1 | 1 | 3 | 12 |
| | | 32 | 48 | 12 | 36 | |
| | .. | .. | .. | .. | .. | .. |
| | 0 | 0 | 0 | 1 | 2 | 0 |
| | | 20 | 30 | 45 | 0 | |
| | <hr/> | | | | | |
| Sum— | 12 0 | | | | | |
| | 21 | 4 | 13' | 16'' | 40''' | 30'''' |
| | | | | | 54 | 24 |
| | <hr/> | | | | | |

The reason of this Process is thus ; 16 drawn in 24, produces 384. And because this Number contains 60 ; that is, the Antecedent Rank many times in it self, it must be divided first apart by it self by 60, and the Quotient will be 6, and the Remainder 24. So I bring 6 to the Antecedent Rank, and the Remainder 24 I place under the last Rank ; and so I act with all the rest.

4. The Multiplication being finished, you must note well the Rank and Denomination of the Numbers arising from the Multiplication ; concerning which, the following Rules must be observed.

Of the Kinds or Ranks produced by Multiplication.

A General Rule.

The first Number produced from the Multiplication (numbring from the right hand towards the left) must be so many Intervals distant from the first Multiplicand, as the first Multiplier is distant by Intervals from the Integer; to wit, from a Degree, which obtains the place of Integer.

Example.

| | | | | | |
|--|------|------|-----------------|------|--|
| Draw— | — | 2° | 3' | 4'' | |
| Into— | — | — | — | 3''' | |
| <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> | | | | | |
| They make— | 6''' | 9''' | 12 ^v | | |

For the first Multiplier 3''' is distant from the Integer 2°, by 3 Intervals: Therefore so many Intervals also ought the first of the Products, to wit 12, to be distant from the first Multiplicand 4'': And since the Multiplicand is 4'', the Product must needs be 12^v, that it may also be 3 Intervals distant from 4''.

Special Rules.

1. Integers drawn into Integers produce Integers. So if you Multiply Degrees by Degrees, they always bring forth Degrees.

2. If Scruples are drawn into Scruples, or Sexagenes into Sexagenes, they produce Scruples or Sexagenes of that Denomination as the Apexes that are added, or written over, to shew the Kind or Rank.

So

So if you draw 3^{'''} into 4^{''} they produce 12^v.

And 4^{1^e} drawn into 5^{2^e} produces 20^{3^e}

3. Scruples multiplied into Integers, or Integers drawn into Sexagenes, produce Scruples and Sexagenes of the same Denomination and Rank, as are the Apexes of the Scruples or Sexagenes.

So if you draw Second Scruples into Degrees, they make Second Scruples : If you multiply Third Sexagenes into Degrees, they produce Third Sexagenes.

4. Sexagenes drawn into Scruples, produce the same kind, (which the Apex shews that designs the kind) by subtracting the lesser from the greater : And the Products are Scruples, if the Apex of the Scruples be the greater ; Sexagenes if of Sexagenes.

In this manner, if Sexagene Thirds be drawn into First Scruples, they produce Sexagene Seconds ; for ' subtracted from ^{'''} leaves ^{''}, and they are Sexagenes, because the Apex of the Sexagenes is the greater. But if you draw Scruples ^{'''} into Sexagene Primes, they make Scruples ^{''}.

N O T E.

This manner of Multiplication is very tedious, and full of labour : The Process of the whole Multiplication may be instituted without multiplying or dividing by 60 ; to save which labour, there are invented Sexagenary Tables, in which, if the Kind or Rank to be multiplied be sought, there is shewn in the common Angle, the product already divided, and reduced to the Antecedent Rank ; which Canon is in most Mathematical Books, and the studious ought to have it.

The Use of this Canon is thus.

1. If either of the Multiplicands be greater than 30, seek the greater in the Triangular Table on the right hand, and the lesser in the upper Transverse part: But if both the Multiplicand and Multiplier be less than 30, you must seek the greater in the Quadrangular Table on the left hand, and the lesser in the Transverse Oblique.

2. With these two Numbers of the Multiplicand and Multiplier, proceed to their common Angle, in which you will find the Product under two Kinds, or Ranks, the Antecedent and the Consequent.

But if any one want this Canon, the following Table will help him much, in which the Product, or its next lesser Number being sought, the Number with a great Letter placed above it, shews how many times 60 belongs to the Antecedent Kind or Rank, that the produced Number contains.

A Table of Multiplication and Division of Sexagenaries.

| I | II | III | IV | V | VI | VII | VIII | IX | X |
|----|-----|-----|-----|-----|-----|-----|------|-----|-----|
| 60 | 120 | 180 | 240 | 300 | 360 | 420 | 480 | 540 | 600 |

CHAP. VI.

Of DIVISION.

1. **I**F any one want the Canon, he may shorten the Labour, if he resolve both the Dividend and Divisor, by continually multiplying the Sexagenaries into the last Rank, and then Institute the Division according to the Vulgar manner; and he will collect the Quotient by continual Sexagenary Division again, into the Kind or Rank.

Example.

Example.

To divide $16^{22} 51^4 9^{\circ} 12' 17'' 16'''$

By $24^{12} 23^{\circ} 35' 46''$

Make by the continual multiplying of the Dividend by 60, the last Rank of Thirds, to wit,

$12508388236'''$

Of the Divisor in like manner $915696''$

And work according to Vulgar Division, and you will have in the Quotient 1366.

Which divided again by 60, produces $22^{\circ} 46'$, and you may know what Rank they are of, by the following Rules.

2. But if you will work by the Canon, do thus:

1. Place the Dividend, with its Kinds, in the upper place, and the Divisor underneath, either in the first place, if it be less than the Dividend; or in the next place, if greater.

2. If both the Dividend and Divisor in the first Kind exceed 30, seek the Divisor in the upper side of the transverse Triangular Table, and the Dividend in the subjected Perpendicular Column, either it, or the next lesser, then in the right side over against them will appear the Quotient.

3. But if either, in the first Rank of the Dividend or Divisor, be less than 30, seek the Divisor in the left side of the Quadrangular Table; And in the same transverse line seek the Dividend, or the next less than it, and on the top of this Perpendicular Column you will find the Quotient to be written in the Semicircle.

4. Multiply this Quotient in the whole Divisor.

5. The Multiplication produced and collected into one Sum, subtract from the Dividend.

6. The Remainder set in its place, adjoin the other Rank of the Dividend; and the Divisor being removed, as is usual, repeat this Operation, until you have wasted all the Ranks of the Dividend.

G 3

7. If

7. If there be any Remainder from the Division, and you are minded to follow the Minutes, then multiply the Remainder from the right hand to the place of the last Rank by 60, once and again, as often as you please, and divide this Number further by the Divisor, so you may arrive at the smallest.

Example.

| | | | | | | | |
|---------------------|---------------|---------------|-----|-----|-----|-------|-----------|
| | ²² | ¹² | | | | | |
| 1. Dividend | 16 | 5 | 9° | 12' | 17" | 16''' | (22° 46') |
| Divisor | — 42 | | 23° | 35' | 46" | | |
| | | | | | | | |
| Numbers arising | 15 | 8 | 12 | 16 | 52 | | |
| from the Division | — 24 | | 26 | 50 | | | |
| | | | | | | | |
| Sum to be Subtract. | 15 | 32 | 39 | 6 | 52 | | |
| 2. New Dividend | — 32 | 30 | 5 | 25 | 16 | | |
| Divisor removed | — 42 | | 23 | 35 | 46 | | |
| | | | | | | | |
| Numbers arising | 32 | 17 | 26 | 35 | 16 | | |
| from the Division | — 12 | | 38 | 50 | | | |
| | | | | | | | |
| Sum to be Subtract. | 32 | 30 | 5 | 25 | 16 | | |
| ————— | | | | | | | |
| | 0 | 0 | 0 | 0 | 0 | | |

1. The Division being ended, you must mark the Ranks of the Quotient arising from the Division; of which this is,

A General Rule.

The Number of the Division, or the Quotient, must be so far distant from the Integer, as the Dividend is distant from the Divisor, and indeed, by how many Intervals the Rank of the Divisor Antecedes the Rank of the Dividend; by so many Intervals the Rank of the Quotient must follow the Integer.

Contra.

Contrariwise.

By how many Intervals the Rank of the Divisor follows the Rank of the Dividend, by so many Intervals the Rank of the Quotient must Antecedes the Integer.

Special Rules.

1. Ranks of the same kind, and alike between themselves, beget Integers by Division.

As if you divide Sexagene Primes by Primes, and first Scruples by Primes, they produce Degrees, which constitute the Integer.

2. Ranks unlike between themselves, but of the same kind, notifie the Rank arising, if you subtract the lesser Signature, or Note, from the greater: for the remaining Note shews the Rank.

Or of the same kind with both, if the Note of the Dividend be the greater.

Or of divers kinds, if the Signature of the Divisor be the greater.

Example.

Divide Scruples $12''$ by $6''$, it makes the Quotient $2'$.

Divide Sexagenes 16^{3e} by 4^{2e} , it makes the Quotient 4^{1e} .

Contrariwise.

Divide Scruples $12''$ by $6''$, they make 2^{1e} Sexagenes.

Divide Sexagenes 16^{2e} by 4^{3e} , they make $4'$ Scruples.

3. In Ranks of divers kinds, add the Notes of the Divisor and Dividend, the Sum shews the Rank of the Quotient, under which the Dividend is contained,

So if you divide Sexagenes 16^{2d} by $4'''$ Scruples; they make 4^{5e} Sexagenes.

If you divide Scruples $16''$ by 4^{3e} Sexagenes; they make 4^v in the Quotient.

4. If you divide Scruples by Degrees, they produce Scruples of the same Rank as the Dividend: But if you divide Degrees by Scruples, they produce Sexagenes of the same Rank as the Scruples of the Divisor. Therefore,

Divide $16''$ by 4° , you will have in the Quotient $4''$.

Divide 16° by $4''$, you will have in the Quotient 4^{2d} .

5. If you divide Sexagenes by Degrees, they produce Sexagenes of the same Rank as the Dividend; But if you divide Degrees by Sexagenes, they produce in the Quotient Scruples of the same Rank as the Divisor is of.

So by dividing 16^{2d} by 4° , the Quotient will be 4^{2e} .

And dividing 16° by 4^{2e} , the Quotient will be $4''$.

A Caution.

These Rules are in force, when the first Dividend is greater than the Divisor: But if it be lesser, and hath the Rank following either exprest or signed by a Cypher, it produces a Rank one place below that which the Rules shew: Which, if you would know the Value, 'tis necessary that you refer the Dividend to the following lesser Rank: So if you divide 6° by $12'$, they produce 30, not Degrees as the first Rule teaches, but First Scruples, which are one place lower than Degrees: But if you resolve $6'$ into $360''$, and divide by 12, the second Rule will be in force.

C H A P.

CHAP. VII.

Of Extracting the Square Root.

When occasion requires it in Logistical Numbers, omitting other ways more Artificial; work thus.

1. Resolve all the given Numbers by Sexagenary Multiplication into some lesser Rank, but denominated by an even Number, to wit, either Seconds, or Fourths, or Sixths, or Eighths.

2. Work according to the Method delivered above in *Book I. Part IV. of Figurate Numbers.*

3. To the Root found, give the Denomination under double of the Number of its Quadrate, that is, if the Quadrate consist of Fourth Scruples, the Root will contain Second Scruples: If it consists of Seconds, the Root will be of Primes; if of Sixths, it will be of Thirds: For the Root is drawn into it self to make the Square, and in drawing Seconds into themselves, you produce Fourths; and of Primes you produce Seconds, as is said above.

4. Reduce the Root found by Sexagenary Division, again to its greater Rank; as Second Scruples to Primes; Primes to Degrees, &c.

5. The Remainder, if there be any, will have the Denomination of the Number of which the Root is found.

Example.

Find the Root of $7^{\circ} 28' 16''$. This Number reduced to Seconds, is 26896''. The Root of which is 164'. That is $2^{\circ} 44'$.

2. So if you would extract the Square Root of $493''$, you will find it $22''$, and in place of the Remainder $9''$.

3. Seek the Root of $1^{\circ}, 10', 46'', 41''', 40''''$; which make 15288100''', whose Root is $3910''$, that is, $1^{\circ}, 5', 10''$.

Bonaventura Cavalieri hath Adapted his Logarithms to work the Rule of Three in Sexagenary Numbers, which lessens the fore-said Labour.

The

The Second Book of
Practical Arithmetick,
 And its Second Part.

Of Politick or Civil Arithmetick,

CH A P. I.
 of W E I G H T S.

I. Of Germane,

Which are,

1. **A Pound**: The word is deduced from the *Latin* word *Pondo*, which in *Latin* we also call *Libra*, and *As*.

2. From this is collected an Hundred Weight, to wit, an Hundred Pounds.

3. And the parts of our Pound are: Half, a Fourth Part, or Quarter, an Eighth, A Sixteenth Part, &c. Also an ounce, whose parts are Half an Ounce, &c. Drams.

4. And there are divers kinds of Pounds used by the *Germans*, but the chiefest are,

- I. Belonging to Physicians.
- II. Belonging to Money.
- III. Publick.

I. Of

I. Of the Physicians Pound.

It arrives from the *Ratio's* of the *Roman* Pound, or *Affis*, whence also 'tis called *Affispondium*.

And it arises,

1. From a Grain of Barley :

Hence there are ;

2. *Obolus*, or half a *Scruple*, containing Grains 10.
3. A *Scruple*, containing Grains ———— 20.
4. A *Dram*, containing *Scruples* ———— 3.
5. Half an Ounce, containing *Drams* ———— 4.
6. An Ounce, containing two Half Ounces, }
and of *Drams* ———— } 8.
7. A Quarter of a Pound, containing Ounces 3.
8. Half a Pound, containing Ounces ———— 6.
9. A Pound, containing Ounces ———— 12.

II. Of the Pound to weigh Money.

1. Of the Silver Money of Coiners.

Whose Parts are,

1. The least quantity is a Grain
2. A Penny Weight contains Grains ———— 24.
3. An Half Ounce, contains 12 Charats, or }
Penny Weight ———— } 12.
4. An Ounce, contains 24 Charats, or Pen- }
ny Weights ———— } 24.

2. Of the Gold of Coiners.

Whose Parts are,

1. A *Caract*, contains Grains ———— 4.
2. An Half *Scruple*, contains *Caracts* ———— 3.
3. A *Scruple*, contains *Caracts* ———— 6.
4. A *Dram*, contains *Scruples* ———— 3.

III. Of

III. Of the Civil or Common Pound, called Avoirdupois.

1. It commonly begins from a Dram, which is $\frac{1}{16}$ part of an Ounce.
2. Half an Ounce, contains Drams————— 4.
3. And an Ounce is 2 half Ounces and Drams 8.
4. The Eighth part of a Pound is Ounces— 2.
5. The Fourth part, is Ounces————— 4.
6. Half a Pound, contains Ounces———— 8.
7. The Pound contains Ounces————— 16.
8. Half Ounces————— 32.

II. Weights of the Hebrews.

1. *Gerah*, the least weight.
2. *Refa*, containing *Gerahs*————— 5.
Tis called also *Adorconim*, Drams 1, Parts 29.7.
3. *Machazith* or *Beka*, Half a Sicle, contains }
Refa's————— 2.
4. *Schakel*, a Sicle, contains *Refa's*———— 4.
5. *Maneh*, a Pound, contains Sicles———— 60.
6. *Kikar*, a Talent, containing Sicles———— 3000.

III. Attick Weights of the Greeks.

1. A Mite, or half a Grain.
2. *Æreolum*, $\frac{1}{32}$ of a Dram, contains Mites 7.
3. *Semiobolus* $\frac{1}{16}$ of a Dram, contains *Æreolums* 3.
4. *Obolus* contains of a Dram————— $\frac{1}{2}$.
5. A Dram, contains *Obolus's*————— 6.

6. Semi-

6. *Semimina*, contains Drams—————50.
7. *Mina*, Drams—————100.
8. The Lesser Talent, contains *Mina's*————60.
9. The Greater, contains *Mina's*————80.

The Ancient Roman Weights.

1. *Libra*, the weight of a Pound, contains Twelve Ounces, whose Names follow.

| | | |
|-----------------|--------|----|
| <i>Deunx</i> | | 11 |
| <i>Dextans</i> | | 10 |
| <i>Dodrans</i> | | 9 |
| <i>Bes</i> | | 8 |
| <i>Septunx</i> | | 7 |
| <i>Selibra</i> | Ounces | 6 |
| <i>Quincunx</i> | | 5 |
| <i>Quadrans</i> | | 4 |
| <i>Triens</i> | | 3 |
| <i>Sextans</i> | | 2 |
| <i>Uncia</i> | | 1 |

2. The division of the Ounce.

1. Into Half Ounces————— 2.
2. *Duellas*, $\frac{1}{2}$ of an Ounce————— 3.
3. *Sicillicos*, $\frac{1}{4}$ ————— 4.
4. *Sextulas*, $\frac{1}{8}$ ————— 6.
5. *Denarios*————— 8.
6. *Quinarios*—————16.
7. *Scriptula*, $\frac{1}{24}$ of an Ounce—————24.

For reducing these Weights to the *Germans*, you must Note the *Ratio's* and *Hypotheses* following.

1. The *Roman* Pound, of 12 Ounces, is the same as the *Physicians* is at this time, and used in all *Europe*.

2. The

2. The Attick Dram, and the Roman Penny are of the same Weight ; witness Pliny, Book 21. Chap. 34.

3. Eight Drams, and the same Number of Pence are equivalent to the Roman Ounce.

4. *Refa* of the Hebrews is equal to the Attick Dram, and therefore 4 Drams make the Hebrew Sicle.

5. The Common Ounce with us is near equal to the Physicians Ounce.

These Hypotheses, although some dissent from them, yet they are for the most part approved by the Learned ; whence 'tis easie to any one that is indifferently skilled in Arithmetick, to reduce the Weights of the Ancients to ours.

We shall bring some Examples from Holy Scripture, concerning reducing Hebrew Weights : Notwithstanding we will neglect the difference between the Holy and the Common Sicle, which some make, and the Holy Sicle, and the half Ounce : For they would have the Common to be lesser by an half : But this difference rests on a weak Foundation, whence 'tis rejected by many, who all make the Sicle to be half an Ounce. But the Holy, and also the Royal, they will in some place say, because they observe the Authentick Example of it in the Sanctuary, or the King's Palace, where they Examined false Weights.

Examples of Reducing Hebrew Weights to ours,
viz. The Germans.

Example I.

In Numbers chap. 7. we read, that each Prince of the Tribes of *Israel* brought to the Tabernacle of the Lord one Silver Charger, the weight whereof was,

Sicles or Shekels ————— 130.

And a Silver Bowl, weighing Shekels — 70.

The Sum of the Shekels ————— 200.

Divide

Divide these by 32.

(For so many Shekels, or half Ounces make a Pound)

And the Oblation of one Prince was 6 $l.$ $\frac{1}{4}$ of Silver.

And the 12 Princes of the 12 Tribes brought in all 75 Pounds.

II. Example.

In the second Book of *Samuel* and the fourteenth Chapter, we read, that *Absalom* cut off the Hair of his Head every Year, and that it weighed,

OF half Ounce Shekels ————— 200.

That is Pounds ————— 6 $\frac{1}{4}$.

III. Example

1 *Sam.* 17. The Head of *Goliath's* Spear weighed, Shekels ————— 600.

That is, Pounds ————— 18 $\frac{3}{4}$.

Example IV.

Numbers, Chap. 7. The Princes of each Tribe of *Israel* brought to the Tabernacle of the Lord, one Golden Spoon weighing, Shekels ————— 10.

Which was in all, there being 12 Tribes, 120.

That is, Pounds of Gold ————— 3 $\frac{1}{4}$.

Example V.

Numbers, Chap. 31. The Spoil which the *Israelites* took from the *Midianites*, was in Gold,

Shekels ————— 16750

That is, Pounds of Gold — 523 $\frac{1}{2}$

Example.

Example VI.

Exodus, Chap. 38. The Gold that was expended in building the Tabernacle of the Lord, was,

| | |
|--|----------------------|
| Talents | 29. |
| And Shekels | 730. |
| In all, of Shekels (a Talent being 3000 Shekels) | 87730. |
| Which divided by 32, produces of | 2679 $\frac{1}{2}$. |
| Gold, in Pounds | |

And of Silver there was expended,

| | |
|--------------------------|-----------------------|
| Talents | 100. |
| And moreover, of Shekels | 1775. |
| In all, of Shekels | 301775. |
| Or pounds of ours | 9430 $\frac{1}{11}$. |

Example VII.

1 Kings, Chap. 9. The Servants of Solomon brought from *Ophir*, of Gold,

| | |
|------------------|----------|
| Talents | 420. |
| That is, Shekels | 1260000. |
| Or Pounds | 39375. |

NOTE.

In *2 Chron.* Chap. 8. Verse the last, we read, that they brought 450 Talents; which the Rabbins reconcile thus: They say the 30 Talents which was not Numbred in *1 King.* Chap. 9. was spent in the Charge of the Journey.

Example VIII.

1 King. Chap. 10. The Gold that Solomon had brought yearly from *Ophir*, was,

| | |
|------------------|-----------------------|
| Talents | 666. |
| That is, Shekels | 1998000. |
| Or Pounds | 62437 $\frac{1}{2}$. |

Example

Example IX.

Dauids Charge to Build the Temple.

In Gold.

| | |
|--|-----------|
| 1 Chron. 22. David brought Talents of Gold | 100000 |
| Also 1 Chron. 29. Talents | 3000 |
| And the Princes brought | 5000 |
| The Sum of the Talents | 108000 |
| That is, of Sicles or Shekels | 324000000 |
| Add Drams 10000, that is, Sicles | 2500 |
| It makes the Sum of the Sicles | 324002500 |
| Or of Pounds | 10125078½ |

The Charge in Silver.

| | |
|---|------------|
| 1 Chron. 22. David prepared Talents of Silver | 1000000 |
| Also 1 Chron. 29. he prepared Talents | 7000 |
| The Princes | 10000 |
| The Sum of Talents | 1017000 |
| Which Multiplied by | 3000 |
| Make Sicles | 3051000000 |
| Which makes pounds | 95031312½ |

And each pound of Silver being worth 3 l. if we multiply the pounds by 3, it makes 285093937 l. 10 s. Sterling.

A large Sum to expend in Silver, besides the Gold, and Brasse, and Iron, and several other Materials.

And valuing the Gold at 36 l. a pound, and adding the value of Gold and Silver together, it makes 649596763 l. 10 s.

In the same manner all other Weights may be reduced to ours, if you resolve them into Ounces, or half Ounces, which are the Hebrew Sicles: Then dividing the Ounces by 16, or the half Ounces, or Sicles by 32, you will produce our Pounds of 16 Ounces, or of 32 half Ounces.

VI. Of the Proportion of Metals.

Since 'tis not unpleasant to know what Proportion God and Nature hath observed between Metals, we have proposed the same, as it hath been found by learned Men of late.

I. A Table of the Proportion of the Weight of Metals in parts of the same Magnitude.

| | | |
|---|------------------|----------------------|
| The Ratio of the weight of Gold to any of these Metals of the same Magnitude with it. | { Quicksilver } | { 71 $\frac{1}{2}$ } |
| | { Lead } | { 65 } |
| | { Silver } | { 56 } |
| | { Copper } | { As 100 to 50 } |
| | { Tin } | { 42 } |
| | { Iron } | { 41 $\frac{1}{2}$ } |
| | { Marble } | { 15 $\frac{1}{2}$ } |
| | { Common Stone } | { 10 $\frac{1}{2}$ } |

II. A Table of Proportion of the Magnitudes of Globes of Metal of equal Weight.

| | | |
|---|-----------------|---------------------|
| A Globe of Gold whose Diameter is assumed 100 Parts Equiponderates a Globe of ————— | { Quicksilver } | { 110 } |
| | { Lead } | { 115 } |
| | { Silver } | { Whose Dia- 121 } |
| | { Copper } | { meter is of 126 } |
| | { Tin } | { the same 133 } |
| | { Iron } | { Parts. 134 } |
| | { Marble } | { 186 } |
| | { Stone, } | { 211 } |

Notwithstanding we must know that these Proportions are deprehended by some accurate Inquilitors of pure Metals, (of which we here speak) not only among themselves mutually, but also in each kind there is some difference of weight, so that Gold to Gold, Lead to Lead, is found heavier and lighter, if they agree in Magnitude: Yea, and Forged Metal outweighs melted Metal, since its parts are pressed together by smiteing, far more than by melting or casting: Therefore we do not here seek concerning all sorts, but are content with those we have touched upon.

C H A P. II.

Of things Pertaining to Money, or of Money.

I. *Of German Money.*

I. **T**Here is great variety of Money in this Age, from which variety we shall choose two for our purpose.

1. The Gold, which is called a *Ducat*.
2. The Silver, called a *Rix Dollar*.

II. In these we must consider two things,

1. The Value. And, 2. The Weight.

For the Ligature, or mixing of more vile Metal, and the Proportion of the mixt, which in this unjust age, according to every ones Avarice, infinitely differs, in this place we do not Treat; since our intent is only to shew the Proportion of the Antient and Modern Money, to the better understanding of the Ancient Writings.

III. *The Value consists in these things.*

1. Two of the least Pieces of Money of Coiners, viz. 2 of *Strasburgh*, or in our Time, 3 of *Wirtemberg*, make a Cross Penny.

2. Four Cross Pence constitute a Bazium.

3. Fifteen Baziums, or 60 Cross Pence make a Floren.

4. 100000 Florens is a Tun of Gold, and 1000000 we usually call a Million.

IV. The greater Sums of this Money are also esteemed by Pounds, and 12 Half-pence, or 6 *Wirtemberg* Pence make a Shilling, and 20 Shillings a Pound: And between Pounds and Florens, this was the Proportion, that 7 Pounds was Equivalent to 5 Florens, viz. a Floren was 28 Shillings.

V. The weight which concerns a Dollar, by the Laws of the Empire it ought to be of an Ounce

H 2

weight,

weight, and the Ducat almost the weight of a Dram, it being somewhat lesser : For 87 Ducats make 64 Drams.

6. The Proportion of Gold and of Silver in the *Ratio* of weight, is the same in all Ages, and in its nature unchangeable, and 'tis, as aforesaid, Gold to Silver of the same Magnitude is very near, as 100 to 56, or according to others, as 100 to 60; or contracting the Terms, as 5 to 3, between which, is the Proportion of Superbipartient Thirds.

7. But the Proportion of Gold and of Silver in *Ratio* of Value, is various, and changeable according to the wills of Men.

1. In old time it was in Decuple Proportion, as appears by *Livy*.

2. Afterwards, and 'tis now in *Duodecuple* Proportion, that is to say, if a pound of Silver be worth (as 'tis now) 3 $\frac{1}{2}$ l. a Pound of Gold is worth 36 $\frac{1}{2}$ l.

3. In the Imperial Assembly in the Year 1559, it was estimated by the Governors of the Empire, that an ounce of sealed Silver should be 68 of the aforesaid Crost Pennys. But 8 Ducats almost being equal in weight to a Dollar of an Ounce, it was estimated at 832 Pence, and so it was a little more than 12 times dearer than Silver.

8. But at this day one Ducat to two Dollars, is 16 times heavier; in price *Duodecuple* to *Sedecuple*; so that one Dram of Gold is changed for 16 Drams of Silver.

9. And because the weight of a Dollar is an Ounce, and the weight of a Ducat almost a Dram, it is constant, and so 'tis usually said, that 8 Ducats are equal in weight to one Dollar, or more exactly, 67 Ducats is equal in weight to 8 Dollars, and the Sums of Gold and Silver are reduced to these Weights, from which the Value at any time may be easily estimated.

II. Of Hebrew Money.

1. The Sicle or Shekel was most in use, which word was attributed to weight, and to Silver Money of half an Ounce, of the Value of half a *Rix Dollar*.

(The Sicle of Gold is not in use : therefore where a Sicle of Gold occurs, its weight in Gold is to be understood.)

2. And the whole Sicle had its parts, to wit ;

1. The half Sicle, the Figures being the same with the whole Sicle (which on one side was a little Vessel, of Manna, *Exod. 16.* and on the other, the Rod of Aaron, *Numb. 17.*) but of half the weight of the whole, or $\frac{1}{2}$ of a Dollar.

2. A Fourth Part, or a Quadrant, of which we read in *1 Sam. 9. 8.* which was called *Refa*, being $\frac{1}{4}$ of a Dollar.

3. A twentieth part, called *Gerah*.

Of Money passing in our Time.

1. A Penny, the Money of the Romans, witness *Pliny's Natural History*, Book 21. Chap. 34. it was of a Dram, being $\frac{1}{4}$ of a Dollar : although some make it $\frac{1}{12}$ greater than a Dram.

2. *Didrachma*, of 2 Roman Pence, or 2 Drams, being $\frac{1}{2}$ of a Dollar in Germany, or of the value of Seven Pence Half-penny of English Money.

3. *Stater*, a Sicle, or 4 Drams, in value $\frac{1}{2}$ a Dollar, as *Hieronymus* witnesses, Chap. 3. in *Ezech.*

4. The Tenth Part of a Roman Penny, or $\frac{1}{10}$ of a Dollar.

5. *Minutum*, $\frac{1}{2}$ *Assarij*, or of one Cross Penny.

6. *Quadrans*, $\frac{1}{4}$ of a Cross Penny, of which we read *Mark 12. 42.*

III. Of the Money of the Greeks or Grecians.

1. Of Gold.

1. Σατὴρ χρυσός, weighing 2 Drams.

2. Ημιχρυσός, One Dram.

2. Of Silver.

1. Δραχμή, The Weight, and the Money was the same Quantity.

2. Ημιδραχμόν, Half a Dram.

3. Διδραχμόν, of 2; Τειδραχμόν, of 3; Τετραδραχμόν, of 4 Drams.

4. Οβολός, the sixth part of a Dram.

5. Ημισόβολον, the twelfth part of a Dram.

6. Διωβόλιον, 2; Τειωβόλιον, 3; Τετράβολον, 4 Halfpence.

IV. Of the Money of the Romans.

1. Their Gold was of divers Weights, especially under *Heliogabalus*, which raised it from 2 Drams, which his Predecessors used, even to 200 Drams in Weight.

2. And of Silver they had.

1. A Penny, whose Weight, according to *Pliny*, was of a Dram, and had its name thence; which, with the *Germans*, was the value of *As*; but if a Dollar be estimated 80 Crost Pence, a Penny was valued at 10 Crost Pence.2. *Quinarius*, having the Image of the Goddess *Victory* upon it, being half a Penny, with us in *England* half a Groat.3. *Sestertius*, the same as a *Quadrans*, or half a *Quinarius*.

Here we must Note, in *Roman* Writings, by *Sestertium*, or *Sestertius* in the Masculine Gender, we understand so many Silver *Sestertios*, as the present number expresses: But by *Sestertia* in the Plural Number, is signified so many Thousand *Sestertios*: Lastly, by the Adverb, and the Genitive Plural expression, they design so many Hundredth Thousands.

Some

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Some Examples of Holy Scriptures.

1. *Matthew* 18. 24. An ill Servant owed,
Talents ————— 10000

That is Sicles, or Shekels 30000000

Which is of Dollars ——— 15000000

Verse the 28. His fellow Servant }
owed him, Pence ————— } 100.

Or 16 Florens, 40 Cross Pence.

2. *John* 12. 5. *Marys* Ointment of *Spikenard*
might have been sold for, Pence—300

The Sum of which in Florens is 50.

3. The Price of the Magic Books consumed by
Fire at *Ephesus*, was estimated, Myriads—5

Where, if according to the Propriety of Holy
Scriptures, we understand Pence,

It makes the Sum of Pence—50000

That is of Roman Cross Pence 500000

Or Florens ————— 8333 $\frac{2}{3}$.

V. *The Value of Hebrew Silver Money in our English Coin.*

1. The Sicle or Shekel of the Sanctua- } s. d. q
ry, being double of the Royal Sicle, is— } 2 6 0

2. The Royal Sicle ————— 1 3 0

3. *Gherah*, *Agorah*, *Ceshitah*, Money }
having on it the Image of a Lamb ——— } 0 1 2

4. *Mina*, in *Latin* signifying a pound of 16 Ounces, but *Libra*, a pound, hath but 12 Ounces, that being *Haverdupois* weight, but this *Troy* weight : A *Mina*, or pound of Gold in old Time was worth 75 *l.* they esteeming Gold to be *Decuple* of the Value of Silver ; but 'tis now esteemed *Duodecuple* : But it seems they valued their Silver at a higher Rate than we do, for they esteemed a *Mina*, or pound of Silver of 16 Ounces, to be worth 7 *l.* 10 *s.* that is 9 *s.* 3 *d.* an Ounce, whereas we reckon it at 5 *s.* per Ounce *Troy* weight, and so a *Troy* pound of Silver is estimated at 3 *l.* and a pound *Troy* weight of Gold is worth 36 *l.*

5. The Talent which weighed 3000 Sicles, they esteemed in Gold to be worth 4500 *l.* And a Talent of Silver they valued at 357 *l.* as *Mr. Rider* saith in his Dictionary, but according to *Decuple* Proportion, a Talent of Gold being worth 4500 *l.* A Talent of Silver is worth 412 *l.* 10 *s.*

Grecian Money of Silver compared with English.

| The Attick Dram, Money of Athens, | | s. | d. | q. |
|-----------------------------------|--------------|----|----|-----------------|
| in value of our English Money— | | 0 | 7 | 2 |
| Stater, | { Attick | 2 | 6 | 0 |
| | { Corinthian | 1 | 8 | 3 |
| | { Macedonian | 2 | 9 | 1 $\frac{2}{3}$ |
| Didrachmum, or 2 Drams— | | 1 | 3 | 0 |
| Tridrachmum, or 3 Drams— | | 1 | 10 | 2 |
| Obolus, Attick— | | 0 | 1 | 1 |
| Obolus, Ægina— | | 0 | 2 | 3 $\frac{1}{3}$ |
| Semiobolus— | | 0 | 0 | 2 $\frac{1}{4}$ |
| Diobolus— | | 0 | 2 | 2 |
| Triobolus, | { Attick— | 0 | 3 | 3 |
| | { Ægina— | 0 | 6 | 1 |

Grecian Money of Brass, compared with English.

| | | | |
|---|---|---|-----------------|
| Assarius, in value of English Money— | 0 | 0 | 1 $\frac{1}{8}$ |
| Quadrans, or $\frac{1}{4}$ part of Oboli— | 0 | 0 | 1 $\frac{1}{2}$ |
| Minutum— | 0 | 0 | 0 $\frac{1}{4}$ |

Of Gold Coin of Grece, compared with English.

An Attick Stater contains 2 Drams, this, according to the Grecians Proportion, to wit, Decuple; is in value but 12 s. 6 d. but according to our Proportion, which is Duodecuple, 'tis worth —

Stater Macedonicus weighs 2 Attick Drams, two Obolis's, two Siliquas, of their Proportion 15 s. 5 d. and of ours —

Stater Doricus is equal to the Attick Stater; therefore in our Value —

Stater Cizyco, according to their Proportion 17 s. 6 d. according to ours —

Of

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Of Grecian Money compared with English.

The Attick Pound contains 100 } l. s. d. q.
 Dracms, 'tis worth of ours — } 608 2 6 0

The Greater Attick Talent is in }
 Value ————— } 250 0 0 0

The Lesser is ————— } 187 10 0 0

{ Of Egypt. 250 0 0 0

{ Of Syria. 46 17 6 0

{ Of Euboea. 125 0 0 0

The Greater Talent. { Of Rhodes. 140 12 6 0

{ Of Babylon 218 15 0 0

{ Of Aegina 312 10 0 0

{ Of Alexandria 375 0 0 0

The Lesser (from the exorbitant notion of the name) is thus called. { The old Sicutum — 0 3 9 0
 { The Neapolitan the same — 0 3 9 0
 { The new Sicutum — 0 1 10 2
 { Rheginum — 0 0 3 2

As or Assis the lesser, is $\frac{1}{2}$ of Denarium, in our value — } 0 0 0 3

As or Assis the greater, is 10 Asses, is in value Denarij — } 0 0 7 2

The old Denarium, which is in our Value — } 0 0 8 2 $\frac{1}{2}$

The new Denarium, with respect to which all our Computations are made, is in our Value. — } 0 0 7 2

The Value of Ancient Roman Money of Silver.

Victoriatas, also called Quinarius — 0 0 3 3

Sestertius, which contains Asses 2 $\frac{1}{2}$ — 0 0 1 3 $\frac{1}{2}$

Obolus — 0 0 1 1

Sembella — 0 0 0 1 $\frac{1}{2}$

Of Gold, Anciently.

The old Aureus, or Noble, weight }
 Dracms 2 $\frac{3}{4}$ ————— } 0 17 1 3

The new Aureus, or Noble, weight }
 2 Dracms ————— } 0 15 0 0

The

The value of Modern Roman Money compared with English.

Of Brass.

Follis, so called because beaten thin } *l.* *s.* *d.* *q.*
like a leaf, $\frac{1}{12}$ of a *Siliqua* ——— } 0 0 0 1 $\frac{1}{12}$

Of Silver.

Miliarisium, $\frac{1}{4}$ of a Solid Noble }
in Value, therefore since a Noble } 0 1 3 0
weighed 2 Drams, 'tis in Value ——— }

Siliqua, or *Caract*, $\frac{1}{17}$ of a Dram }
the greater is in Value ——— } 0 0 7 2

Siliqua simplex, in Value ——— 0 0 5

Of Gold.

Constantines Noble, in our Value 0 8 6 3 $\frac{6}{7}$

Valentines Noble ——— 0 10 0 0

Scrupulum ——— 0 2 6 0

Sums of Money in Gold.

Sestertium in the Neuter Gender, which contains 1000 *Sestertios*, in our Value } 7 16 3 0

Libra, or *Pondo*, a Pound ——— 3 0 0 0

Talentum, or Talent, which contains 24 *Sestertia* 187 10 0 0

Sportula, which contains 10 *Sestertios* ——— 0 1 6 3

Of English Money.

1. A *Farthing*, is the least Coin, being made of Copper.

2. A *Halfpenny*, being 2 *Farthings*, is also made of Copper; formerly there were *Halfpence* made of Silver, but since the Renovation of the Coin, Anno 1697, I have not seen any.

3. A *Penny*, containing 2 *Halfpence*, (Silver being at 5 *s.* an Ounce) weighs, Grains } 9 $\frac{1}{2}$

4. Two Pence, weighing, Grains ——— 19 $\frac{1}{2}$

5. Three Pence, weighing 1 Scruple, and Grains 4 $\frac{1}{2}$, }
that is ——— } 28 $\frac{1}{2}$

6. Four Pence, called a Groat, weighing one Scruple and $\frac{1}{2}$, and 2 Grains and $\frac{1}{2}$, that is ——— } 38 $\frac{1}{2}$

7. Six Pence, weighing 2 Scruples, 9 Grains, and $\frac{1}{2}$, }
that is ——— } 57 $\frac{1}{2}$

8. A *Shilling*, containing 2 Six pences, weighing Dram 1, Scrup. 1 $\frac{1}{2}$, Gr. 7 $\frac{1}{2}$, or Gr. ——— } 115 $\frac{1}{2}$

9. An *Half Crown*, containing 2 *s.* 6 *d.* weighing half an Ounce, or 4 Drams, that is, Grains ——— } 288.

10. A *Crown*, containing 5 *s.* weighing an Ounce, or 8 Drams, that is Grains ——— } 576

11. One Pound, is Estimated 20 *s.*

A Table

A Table of Weights, shewing the Proportion of Gold to Silver, as 12 to 1.

| | | | | <i>l.</i> | <i>s.</i> | <i>d.</i> | <i>q.</i> | |
|-----------|--------------|---------|---|-----------|-----------|-----------|------------------|---|
| A Pound | { of Silver, | { value | { | 3 | 0 | 0 | 0 | } Pound cont. 12 oun.
weight in Gr. 6912. |
| | { of Gold, | { | { | 36 | 0 | 0 | 0 | |
| An Ounce | { of Silver, | { value | { | 0 | 5 | 0 | 0 | } Ounce cont. 8 Drams
weight in Gra. 576. |
| | { of Gold, | { | { | 3 | 0 | 0 | 0 | |
| A Dram | { of Silver, | { value | { | 0 | 0 | 7 | 2 | } Dram cont. 3. Scrup.
weight in Grains 72. |
| | { of Gold, | { | { | 0 | 7 | 6 | 0 | |
| A Scruple | { of Silver, | { value | { | 0 | 0 | 1 | 2 | } Scrup. cont. 2 Obolos.
weight in Grains 24. |
| | { of Gold, | { | { | 0 | 2 | 6 | 0 | |
| An Obolus | { of Silver, | { value | { | 0 | 0 | 1 | 1 | } Obol. cont. 3. Caracts.
weight in Grains 12. |
| | { of Gold, | { | { | 0 | 1 | 3 | 0 | |
| A Caract | { of Silver, | { value | { | 0 | 0 | 0 | 1 $\frac{1}{2}$ | } It contains and
weighs Grains 4. |
| | { of Gold, | { | { | 0 | 0 | 5 | 0 | |
| A Grain | { of Silver, | { value | { | 0 | 0 | 0 | 0 $\frac{1}{12}$ | } |
| | { of Gold, | { | { | 0 | 0 | 1 | 1 | |

CHAP. III.

Of Liquid Measures.

Of German or Wirtemberg Measures.

1. **A Measure**, the general word is used for the particular, which hath its parts, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$.

This $\frac{1}{4}$ part is equal almost to the capacity of six Hen Eggs, whence also the capacity of a Measure ought to be estimated.

2. And this Measure is twofold.

1. Usual.

2. Ducal, or belonging to a Duke.

Between these, this is the Proportion, that 10 Ducats make 11 Usuals, *viz.* in Proportion *Sesquidécima*, whereby the Ducal exceeds the Usual by $\frac{1}{10}$ part.

3. Ten

3. Ten Measures added together is an *Imi*, which we usually call *Amphora*.

4. An *Urn* contains 16 *Amphoras*, or 160 Measures.

5. A *Plaustrum* contains 6 *Urns*, that is 96 *Amphoras*, or 960 Measures.

The Hebrew Measures are,

1. *Log*, the capacity of 6 Eggs, or half a Pint, whence 'tis called by the word *Sextarij*.

2. *Hin*, containing 12 Logs, or 3 Quarts.

3. *Bathus*, containing 6 Hins, or 72 Logs, or 4 Gallons and an half.

4. *Corus*, or *Chomer*, the measure of 10 Bathus, of 60 Hins, and 720 Logs.

5. *Cadus*, a common kind of Vessel, of no definite Measure, but of such a Quantity, that it may conveniently be carried on the Shoulders, or in their Arms, it holds about 4 or 5 Gallons.

To Compare the German and the Hebrew Measures, is easie.

For $\frac{1}{3}$ of a Ducal Measure, and an Hebrew Log, are of equal Measure, to wit, both of the capacity of six Hen Eggs.

First it appears to be so by Experience, also the Rabbins testify the same, as may be seen in *Kimchi*, in *lib. Rad.* And *Buxtorfium* in his Lexicon, on the Word *Log*.

A Table of Hebrew Measures reduced to German.

| <i>Imi</i> , or 10 Measures. | | <i>Masq.</i> or 1 Meas. | | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
|------------------------------|---|-------------------------|---|---------------|---------------|---------------|
| A Log is— | o | : | o | : | o | 1 |
| An Hin is— | o | : | o | : | o | 1 |
| Half an Hin is | o | : | o | : | o | 3 |
| $\frac{1}{3}$ of an Hin is | o | : | o | : | o | 2 |
| $\frac{1}{4}$ of an Hin is | o | : | o | : | o | 1 |
| $\frac{1}{8}$ of an Hin is | o | : | o | : | o | 1 |
| A Bathus is— | o | : | o | : | o | 9 |

Corus

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Corns, or *Ephemer*, contains 9 *Imi* or *Amphoras*, that is 90 Measures Ducal, but 99 usual Measures, that is, 45 Gallons of English Measure.

Some Examples of Holy Scripture.

I. In 1 Kings 7. 26. The Sea of Solomon is said to contain, Baths ~~18000~~ ~~2~~ ~~18000~~ ~~18000~~ 18000

Which multiplied by 9 (for a Bath contains of Ducal Measures) ~~18000~~ 9

Produces of Ducal Wirtemberg Measures—18000

That is, 18 *Luder* or *Plaustrums*, 4 *Eimer* or *Urns*, and 8 *Imi*, or 80 Measures.

If we would know how much Solomons Sea contains of English Measure; supposing $\frac{1}{2}$ part of a German Measure to be half a pint, for we find Log, to contain $\frac{1}{2}$ a pint, and 8 Hen Eggs contain about $\frac{1}{2}$ a Pint; then a German Measure is equal to two Quarts, wherefore multiplying 18000 Measures by 2, the Product is 36000 Quarts, that is, 9000 Gallons, which is in Wine Measure 35 Tuns, 4 Terse, and 12 Gallons.

II. 1 Kings 7. 38. Solomon made 10 Lavers upon 10 turning Bases, each containing Baths 40; that is, 2 *Eimer* or *Urns*, and 4 *Imi* or *Amphoras* in one Laver. That is in our Measure 180 Gallons.

III. 2 Chron. 2. 10. Solomon sent to Tyre, to the Hewers of Timber, of Wine and Oil, Baths 2000

That is, Measures—180000

Of *Eimer* or *Urns*—1125

Of *Luder* or *Plaustra*—187 $\frac{1}{2}$
3 Urns.

III. The Liquid Measures of the Romans are these.

1. *Ligula* or *Cochleare*, being 3 Drains and a Scruple of our English Weights.

2. *Cyathus*, contains 4 *Ligulas*, that is one Ounce and a half, and a Drain, and a Scruple of our English.

3. *Aceta-*

3. *Acetabulum*, that is 6 *Ligulas*, or *Cyathus* $1\frac{1}{2}$, that is 2 Ounces and $\frac{1}{2}$ of our *English*.

4. *Quartarius*, that is 2 *Acetabulas*, and $\frac{1}{4}$ of *Sextarij*; that is 5 Ounces of ours.

5. *Hemina*, half a *Sextarij*, or 10 Ounces of ours.

6. *Sextarius*, $\frac{1}{2}$ of a *Congij*, 2 *Hemina*, that is 20 Ounces of ours; that is, 1 Pint $\frac{1}{2}$.

7. *Congius*, or *Congiarum*, 6 *Sextarij*, 12 *Hemina*, that is 120 Ounces of ours, or 4 Quarts.

8. *Urna*, that is, 4 *Congius's*, or 40 Pounds, or 4 Gallons of ours.

9. *Amphora*, that is 2 *Urns*, or 8 *Congius's*, that is 80 Pounds, or 8 Gallons of ours.

10. *Culeus*, that is 20 *Amphoras*, that is 1600 Pounds, or 160 Gallons of ours.

Note, 1. *Amphora* is likewise called *Quadrantal*, from a Square Figure, for 'tis a Cubic Figure of 1 Foot.

2. *Sextarius*, is twofold, belonging to the City, as aforesaid, and belonging to the Camp, which is doubly greater.

3. *Sextarius*, because it contains 12 *Cyathos*, it hath the same divisions as a Pound.

The Reduction of the Roman Measure to the Germans, is thus.

1. Under *Vespasian Cæsar*, a *Congius* of Fountain Water was found to weigh 10 pounds; in which *Ratio* 80 Pounds fill the *Roman Amphora*, being 8 *Congius's*, which also *Pliny* and *Galen* testify.

2. And the *Roman Pound* was of 12 Ounces, the same as the *German Physicians* use, as aforesaid.

3. Since a Fourth Part of the *German Ducal Measure* weighs a Medical or *Troy Pound*; a *Roman Congius* will receive 10 pounds, or $\frac{10}{4}$ of Ducal Measure, that is Ducal Measures $2\frac{1}{2}$: So then, 10 Measures or *Imi* equal to 40 l.

| | <i>Imi, Measures</i> $\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1000}$ $\frac{1}{10000}$ | | | | | | |
|-------------------------|--|---|---|---|---|---|---|
| A <i>Culeus</i> will be | 40 | : | 0 | : | 0 | : | 0 |
| An <i>Amphora</i> is | 02 | : | 0 | : | 0 | : | 0 |
| An <i>Urne</i> is— | 01 | : | 0 | : | 0 | : | 0 |
| A <i>Congius</i> is— | 00 | : | 2 | : | 5 | : | 0 |
| A <i>Sextarius</i> is— | 00 | : | 0 | : | 4 | : | 1 |
| An <i>Hemina</i> is— | 00 | : | 0 | : | 2 | : | 0 |

Imi,

Imi or 10 Meas. Masx. or 1 meas. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$

| | | | | | | | | | | | |
|------------|---|---|---|---|---|---|---|---|---|---|---|
| Quartarius | o | : | o | : | 1 | : | o | : | 4 | : | 1 |
| Acetabulum | o | : | o | : | o | : | 5 | : | 2 | : | o |
| Cyathus | o | : | o | : | o | : | 3 | : | 4 | : | 4 |
| Ligula | o | : | o | : | o | : | o | : | 8 | : | 6 |

More Clearly thus,

Ligula, is ————— $\frac{1}{2}$ }
 Cyathus the Quadruple of this — $\frac{1}{4}$ } Of $\frac{1}{2}$ part of a
 Acetabulum, Sextuple of Ligula $\frac{3}{4}$ } Ducal Measure.
 Quartarius, double of Acetabulum $\frac{3}{4}$ }

Hemina, double of Quartarius, $1 \frac{1}{2}$ of the Eighth part of one Measure.

Sextarius, double of Hemina, $\frac{1}{2}$ of one Measure, wanting $\frac{1}{8}$ of $\frac{1}{4}$.

IV. Liquid Measures of Grece, of the Atticks.

1. Cochlearium the least Measures of the Liquids, a Spoonful.

2. Chus, two Cochlearias, 2 Spoonfuls.

3. Cochlearia $2 \frac{1}{4}$, Chema $1 \frac{1}{4}$, Chema is 2 Drams, 1 Scruple, 4 Grains, and $\frac{1}{4}$ of a Grain of our English.

4. Κόγχη μύσα 2.

5. Cyathus is 2 Concha, and Concha is 2 Spoonfuls, or 6 Drams of English.

6. Oxybaphum, is Cyathus $1 \frac{1}{2}$, that is, 3 Concha's, or 2 Ounces and 2 Drams of ours.

7. Cyathus 3, is 6 Ounces and 6 Drams.

8. Κοτυλη, or Cotyle, ημισυλια, 2. οξυβαφα, 4. Cyathus 6. that is 1 pound, 1 Ounce, and $\frac{1}{2}$.

9. Sextarius, is 2 Cotyles, that is 2 pounds and 3 Ounces.

10. Chema, is 6 Sextarius, or 12 Cotyles, that is, 13 Pounds and $\frac{1}{2}$, almost 5 Quarts and a Pint.

11. Metreta, is Choæ 12, Sextarij 72, 'tis called also Amphora Attica.

The

*The Proportion of the Attick Measures to
the German.*

1. *Cotyle* of the *Greeks* (according to approved Authors) is $\frac{1}{10}$ part lesser than the *Roman Hemina*, so that 10 *Cotyles* make 9 *Hemins*.

2. Then since a *Roman Hemine*, is of the *Ducal Measure* $\frac{1}{1000}$, as aforesaid: 2083 multiplied by 9, produce 18747, which divided by 10, produce for the capacity of the *Cotyle* $\frac{1}{1000}$ of the *Ducal Wirtemberg Measure*.

So then take it as near as may be.

| | Imi or Meas. 10. | Masx, or Meas. 1. | $\frac{1}{12}$ | $\frac{1}{16}$ | $\frac{1}{20}$ | $\frac{1}{24}$ |
|--------------------|------------------|-------------------|----------------|----------------|----------------|----------------|
| <i>Metreta</i> | 2 | 6 | 9 | 8 | 5 | 6 |
| <i>Chas</i> | | 2 | 2 | 4 | 8 | 8 |
| <i>Sextarius</i> | | | 3 | 7 | 4 | 8 |
| <i>Cotyle</i> | | | 1 | 8 | 7 | 4 |
| <i>Quartarius</i> | | | | 9 | 3 | 7 |
| <i>Oxybaphum</i> | | | | 4 | 6 | 8 |
| <i>Cyathus</i> | | | | 5 | 1 | 2 |
| <i>Cotoba</i> | | | | 8 | 9 | 6 |
| <i>Mysrum</i> | | | | | 7 | 8 |
| ● <i>Chema</i> | | | | | 6 | 2 |
| <i>Cochlearium</i> | | | | | 3 | 1 |

Clearer, and near the Truth.

Imi

| Imi or Meaf. 10. | Meaf. of Meaf. 1. | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$ |
|------------------|-------------------|---------------|---------------|---------------|---------------|---------------|
| Metretra | 2 | 7 | 0 | 0 | 0 | 0 |
| Chus | | 2 | 1 | 0 | 0 | 0 |
| Sextarius | | | 1 | 1 | 0 | 0 |
| Cotyle | | | | 1 | 1 | 0 |
| Quartarius | | | | | 1 | 1 |
| Oxybaphum | | | | | | 1 |
| Cyathus | | | | | | 1 |
| Concha | | | | | | 1 |

A Table of Equation of English Liquid Measure.

| | Gallons. | Pottles. | Quarts. | Pints. | $\frac{1}{2}$ Pints. | $\frac{1}{4}$ Pints. |
|---|----------|----------|---------|--------|----------------------|----------------------|
| A Firkin of Ale | 8 | 16 | 32 | 64 | 128 | 256 |
| A Firkin of Beer | 9 | 18 | 36 | 72 | 144 | 288 |
| A Kilderkin of Ale | 16 | 32 | 64 | 128 | 256 | 512 |
| A Barrel of Ale | 32 | 64 | 128 | 256 | 512 | 1024 |
| A Kilder. of Beer,
or Runlet of Wine | 18 | 36 | 72 | 144 | 288 | 576 |
| A Barrel of Beer,
or 2 Runl. of Wine | 36 | 72 | 144 | 288 | 576 | 1152 |
| A Hogshead of Wine | 63 | 126 | 252 | 504 | 1008 | 2016 |
| A Tercion of Wine | 84 | 168 | 336 | 672 | 1344 | 2688 |
| A Butt or Pipe of
Wine | 126 | 252 | 504 | 1008 | 2016 | 4032 |
| A Tun of Wine | 252 | 504 | 1008 | 2016 | 4032 | 8064 |
| A Tun of Sweet Oil | 236 | 472 | 944 | 1888 | 3776 | 7552 |

This Symbol = signifies Equal to : Read the Table thus ; A Firkin of Ale equal to 8 Gallons, 8 Gallons equal to 16 Pottles, 16 Pottles equal to 32 Quarts, and so of the rest.

Note, That the Beer or Ale Gallon (now in use) contains 282 Cubick Inches, and the Wine Gallon 231, and the Corn Gallon 272 $\frac{1}{4}$.

C H A P. IV.

Of Dry Measures.

I. Of the Germans.

1. **A** Measure, containing 26 Pounds 8 Ounces, which kind of Measure hath its Divisions, to wit, an Half Part, a Fourth Part, an Eighth Part.

2. *Medimnus*, containing 8 Measures.

3. The comparing of these Measures with the Measures of Liquids is thus.

Tryal being made, it was found out, That a dry Measure, answers in Proportion to the Capacity of Ducal Measures $12\frac{1}{2}$ very nigh, exceeding only $\frac{1}{4}$ of a Measure: For a Measure exactly contains $12\frac{1}{2}$, that is, 12 Measures $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$. If, nevertheless, that near Proportion be retained for the sake of roundness, it will be thus.

Medimnus, is 100

Half *Medimnus*, is 50

A Measure, is 12 $\frac{1}{2}$

Half a Measure, is 6 $\frac{1}{4}$

$\frac{1}{4}$ of a Measure, is 3 $\frac{1}{8}$

$\frac{1}{8}$ of a Measure, is 1 $\frac{3}{8}$

Of Ducal Measures.

II. Of

II. Of the Hebrews Dry Measures.

1. *Cabus*, whose capacity is estimated 24 Eggs full, being about 3 Pints and $\frac{1}{2}$.
2. *Satum*, being 6 *Cabus*'s.
3. *Epha*, containing 3 *Satums*, or 18 *Cabus*'s.
4. *Gomer*, the Tenth Part of an *Epha*.
5. *Corus* and *Chomer*, containing 10 *Ephas*, or 30 *Satums*.
6. *Lethech*, is half a *Corus*, or 5 *Ephas*.

The Reduction of these Measures to the German Measures, may be done thus.

1. *Batus* in Liquids, and *Epha* in Dry Measures, so also *Corus* or *Chomer*, both in Liquid and Dry Measure are the same.

2. Since then the capacity of *Batus* was found to be 9 Ducal Measures in Liquids, the *Epha* will be the same: And the *Corus* or *Chomer* is 9 *Amphoras*, or 90 Ducal Measures.

3. Lastly, since a *Satum* is $\frac{1}{3}$ of an *Epha*, it will contain 3 Measures, which is also the capacity of $\frac{1}{4}$ part of a *Modius*, to wit, of Measures $12 \frac{1}{2}$: For although a Fourth part of a *Modius* contains 3 Measures, and $\frac{1}{4}$ part; Nevertheless, because this Capacity is a little greater than just, 3 whole Measures may be compared with $\frac{1}{4}$ of a *Modius* without any inconveniency: therefore let *Satum* be 3 Measures in Liquids, and $\frac{1}{4}$ of a *Modius* in Dry Measure.

In this Proportion in Dry Measure is contained.

1. *Cabus*, a fixth part of a *Modius*.

I 2

2. *Satum*,

2. *Satum*, one *Modius*, or Ducal Measures $12\frac{1}{2}$.
3. *Epha*, 3 *Modius's*, or Measures $37\frac{1}{2}$.
4. *Corus* or *Chomer*, $7\frac{1}{2}$ *Modius's*, or Measures $93\frac{1}{4}$.
5. *Latach*, that is 3 *Modius's*, or Measures $37\frac{1}{2}$.
6. *Gomer*, answers very near to one usual Measure, being somewhat less than an Eighth Part of a *Modius*.

Examples of Holy Scripture concerning Dry Measures.

I. In 1 Kings 4. 22. And Solomon's Provision for one day was 30 *Corus's* of fine Flower, and 60 *Corus's* of Meal, that is 81 *Medimnus's* of the Germans, if we attribute to *Corus* 90, and to *Medimnus* 100 Ducal Measures: But if to *Corus* we allow *Modius's* $7\frac{1}{2}$, it produces 84 *Medimnus's*, and 3 *Modius's* of German Measure.

II. 1 Kings 5. 11. And Solomon gave *Hiram* King of Tyre 20000 *Corus* of Wheat, and the same number of *Corus's* of Barley, the Sum whereof is *Corus's* 40000.

That is 36000 *Medimnus's* of Provision.

Of Grecian Dry Measures.

- | | | |
|-----------------------|---|--|
| 1. <i>Cochleare</i> . | } | The Capacities are the same in Dry Measure as in Liquid. |
| 2. <i>Cyathus</i> . | | |
| 3. <i>Oxybaphum</i> . | | |
| 4. <i>Cotyle</i> . | | |
| 5. <i>Sextarius</i> . | | |

6. *Medimnus*, contains *Chanicas* 48, answering most plainly to *Metreta*: Hence 'tis plain.

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The Reduction of these Measures to the German.

For since *Metreta* contains of German Liquid Measures 27, that is in Dry Measure, *Modius's* $2\frac{1}{4}$, the same also the *Attick Medimnus* contains.

And this *Chenicas*, *Sextarios*, and *Cotylas* being divided into the parts of *Medimnus*, constitutes the small German Measures of Dry things, rather Measuring by Weights than by small Vessels.

IV. Of the Romans Dry Measures.

1. *Ligula*.

2. *Cyathus*.

3. *Acetabulum*.

4. *Hemina*.

5. *Sextarius*.

} They are likewise the Measures
of both kinds, as well of Dry,
as of Liquid Measures.

6. And *Modius* contains *Sextarios* 16.

7. *Semimodius*, the half of it, or *Sextarios* 8.

The Reduction of these Measures to the German.

We have before made the *Sextarium* $\frac{3}{4}$ of German Liquid Measure.

Modius then being *Sextarius* 16, will be $1\frac{1}{2}$, or 6 Measures.

A Table of Equation of English Dry Measures.

| | Last | Weigh | Chald. | Quar. | Bush. | Pecks | Pints |
|--------------------------|------|-------|--------|----------------|----------------|---------------|----------------|
| One Last is | 1 | = | 2 | = | $2\frac{1}{2}$ | = | 10=80=320=5120 |
| One Weigh | | 1 | = | $1\frac{1}{4}$ | = | 5=40=160=2560 | |
| One Chaldron of Coals is | | | 1 | = | $4\frac{1}{2}$ | = | 36=144=2098 |
| One Quarter of Wheat is | | | | 1 | = | 8= | 32=512 |

| | Pecks | Gallons | Pottles | Quarts | Pints |
|-------------------------------|-------|---------|---------|--------|----------|
| One Bushel of Water Meas. is | 5 | = | 10 | = | 20=40=80 |
| One Bushel of Land Measure is | 4 | = | 8 | = | 16=32=64 |
| One Peck is | 1 | = | 2 | = | 4=8=16 |
| One Gallon is | | 1 | = | 2 | = 4=8 |
| One Pottle is | | | 1 | = | 2=4 |
| One Quart is | | | | 1 | = 2 |

A Table of Equation for English Money.

| | Marks, | Angels, | Nobles, | Crow. | Shill. | Gros. | Pen. | Farth. |
|---------------|----------------|----------------|----------------|----------------|--------|-------|------|--------|
| 1 l. Ster. is | $1\frac{1}{2}$ | 2 | 3 | 4 | 20 | 60 | 240 | 960 |
| A Mark is | 1 | $1\frac{1}{3}$ | 2 | $2\frac{1}{3}$ | 13 | 4 | 40 | 160 |
| An Angel is | — | 1 | $1\frac{1}{2}$ | 2 | 10 | 30 | 120 | 480 |
| A Noble is | — | — | 1 | $1\frac{1}{2}$ | 6 | 8 | 20 | 80 |
| A Crown is | — | — | — | 1 | 5 | 15 | 60 | 240 |
| A Shilling is | — | — | — | — | 1 | 3 | 12 | 48 |
| A Groat is | — | — | — | — | — | 1 | 4 | 16 |
| A Penny is | — | — | — | — | — | — | 1 | 4 |

A Table of Equation for Troy Weight.

| | Pounds, | Ounces, | Pen. weig. | Carots, | Grains. |
|------------------------------------|-----------------|---------|------------|----------------|---------|
| 1 Hundred Weight is | 100 | 1200 | 24000 | 28800 | 576000 |
| $\frac{1}{2}$ a Hundred Weight is | 50 | 600 | 12000 | 14400 | 288000 |
| $\frac{1}{4}$ of a Hundred Troy is | 25 | 300 | 6000 | 7200 | 144000 |
| $\frac{1}{8}$ of a Hundred is | $12\frac{1}{2}$ | 150 | 3000 | 3600 | 72000 |
| 1 Pound Troy is | 1 | 12 | 240 | 288 | 5760 |
| 1 Ounce Troy is | — | 1 | 20 | 24 | 480 |
| 1 Penny Weight is | — | — | 1 | $1\frac{1}{2}$ | 24 |
| 1 Carot Troy is | — | — | — | 1 | 20 |
| 1 Grain is | — | — | — | — | 1 |

A Table of Equation for Avoirdupois Weight in England.

Hund. Ston. Pound. Ounces, Drams. Scruples Grains

| | |
|-------------------------|--|
| 1 Tun wt. }
gross is | 20=280=2240=35840=286720=860160=17203200 |
| 1 Hund. is | 1=14=112=1792=14336=43008=860160 |
| One ½ Hund. is | 7=56=448=7168=43008=860160 |
| ¼ of an Hund. is | 3 ½=28=448=3584=10752=215040 |
| ½ of ¼ of an Hu. is | 1 ¼=14=224=1792=1376=107520 |
| 1 Stone weight is | 1=8=128=1024=3072=61440 |
| 1 Pound is | 1=16=128=384=7580 |
| 1 Ounce is | 1=8=24=480 |
| 1 Dram is | 1=3=9 |
| 1 Scruple is | 1=3 |
| 1 Grain is | 1=1 |

CHAP. V.

Of the Measure of continued Things, or of Magnitude.

I. Of the German.

1. THE least of these Measures is a Grain of Bartley: Hence they are.

2. A Digit, the breadth of 4 Grains.
3. An Inch, being 1 ¼ of a Digit.
4. A Palm, being 4 Digits, or 3 Inches.

1 4

5. A

5. A Span, of 3 Palms, 9 Inches, or 12 Digits.
6. A Foot, of 12 Inches.
7. A Cubit, of 1 Foot and an half, 18 Inches, or 6 Palms.
8. An Ell, of 6 Feet.
9. A Perch of 16 Feet, 150 of which make an Acre of Land : that is Fifteen Perches drawn into Ten.

II. Of the Hebrews.

1. *Ezba*, a Digit.
2. *Tabach*, a Palm, 4 Digits, or 3 Inches.
3. *Dsereth*, a Span, of 3 Palms, 9 Inches, or 12 Digits.
4. A *Cubit*, which is either Common, containing 2 Spans, or 24 Digits, or 18 Inches, that is a Foot and an half : Or Holy, which exceeds the Common by a Palm, to wit, containing 28 Digits, or 21 Inches.
5. *Canah*, a Measuring Pole, a Reed or Perch, which may have more or fewer Feet, or Cubits.

The Holy Rod of *Ezechiel* did consist of 6 Holy Cubits, or 10 Feet and an half. But the Common one is 9 Feet; for a Common Cubit was a Foot and an half, 6 of which made the Common Rod.

III. Of the Grecians.

1. Δακτυλός, a Digit, or the Breadth of the Finger.
2. A Palm, consisting of 4 Digits.
3. A Span, being the Interval between the Fore-finger and Thumb, when extended to the utmost, consisting of 10 Digits, or of Palms $2\frac{1}{2}$.
4. A Right Palm, is the measure from the first part of the Palm, to the upper part of the Middle finger; it is 11 Digits.
5. Σπθαμνί, 12 Digits, to wit, The space between the little Finger and the Thumb, the Fingers being extended.
6. A Foot, of 16 Digits, or 4 Palms.
7. The lesser shortned Cubit, the measure from the Elbow to the Fingers inclosed, being Digits 18.
8. The greater shortned Cubit, the measure from the Elbow to the Fingers bent, containing 20 Digits.
9. A Cubit, the Interval between the bending of the Arm, and the upper part of the Middle Finger, being 6 Palms, or 24 Digits.
10. A Pace, its Appellation is taken from spreading abroad, or separating the Hands: for it is the Extension of both Hands, with the Breadth of the Breast, called in England, Fathom, or 6 Foot.
11. An Acre, being an 100 Feet in Length and Breadth.
12. An half Acre, being half so much.

IV. Of the Romans.

1. A Digit, or Breadth of a Finger.
2. A Palm, consisting of 4 Digits, or Fingers Breadths.

3. A

3. A Foot, of 4 Palms, of 16 Digits, divided into 12 parts like *Affs*.
4. A Cubit, being a Foot and an half.
5. A Pace, 5 Feet.

Hence are Measures belonging to Husbandry.

1. The least Measure, being 120 Feet long, and 4 broad.
2. The Square, or Quadrate Measure, being 120 Feet in breadth, as well as in length.
3. The Double Measure, in length 240 Feet, and in breadth 120.

Of Comparing these Measures with the German.

There was great variety of these Measures among the Ancients, even as there is at this Day, not only by reason of divers People, but also according to the variety of Places among one People: The Foot truly of the *Grecians*, was the *Roman* Foot, and half an Inch of it, that is, a Four and Twentieth part more; so that 24 Feet of the *Grecians* contain 25 *Roman* Feet: Moreover the *Roman* Foot is otherwise assigned by some others.

Whence 'tis difficult, or rather impossible to institute a Comparison between these Measures and the *German* Measures.

For since for the most part, the Measures of the Ancients, as well as of the *Germans*, are taken from the Parts of Human Body of Just Stature, therefore it behoves there should be no great difference between them: And therefore we shall not err much from the Truth, if we use the *German* Foot for the Foot of the *Ancients*; and from it also Judge of other Ancient Measures depending from the Foot.

CHAP. VI.

Of Measures used in Measuring the Distances of Places.

I. Of the Germans.

THE Distances and Intervals of Places at this day are measured by Miles, from a word borrowed of the *Latins*, which thing is divers: For the *Germans* define Miles by the Time in which some expedite Traveller performs some space; which Time, some constitute longer, and others shorter.

2. Yet for the most part they define a *Mile* by the Journey of 2 Hours, and that, according to the Parts of Hours, is to be distinguished into Halves, Quarters, and Eighth Parts.

3. Some People of *Europe* use *Leagues*, as the *Spaniards*, and they define a *League* by an Hours Journey, which also are Travelled sooner by some, and slower by others; whence this Measure is very Fallacious and Ambiguous.

4. Among the Ancients there was a Method more accurate. For,

1. The *Romans* and the *Greeks* did begin from a Foot.

2. From a Foot was made a *Pace*, among the *Romans* 5 Feet, and among the *Greeks* 6.

3. Thence a *Stadium*, containing of the *Romans* 125 Paces, that is, 625 Feet, and of the *Greeks* 100 Paces, or 600 Feet.

4. *Stadium* was received among the *Romans* appertaining to a *Mile*, which was of 8 *Stadiums*, or 1000 Paces (whence the name arrived) that is 5000 Feet, as *Columela* testifies, although *Plutarch* and *Suidas* ascribe to a *Mile* somewhat less.

Of

Of Reducing the Ancient Miles of the Romans to the Germans, as at this day.

Since there are great variety of Opinions about this matter, let us hear first what *Lipsius* says, which was the great Admirer and Investigator of Roman Antiquity. He in his *Epistolical Institutions*, Century the 3. Epistle 59, to *Nicholas Micaultium*, writes thus :

We, saith he, for the most part compute by Leagues, and those we finish by Horary Space. Now 'tis sought how many Antient or Roman Miles are in our League ? Commonly they allow three, and not amiss, if we consider the Italick or Roman at this time ; if the Ancient, I fear they err. He that inspects and compares the Itinerary, or Distance of Places of Antoninus, will see plainly, those Miles are exceeded by these of this time : Which the Intervals of Stones which are extant at this time in the Ancient Ways, signifying Miles with their Numbers, will shew the same, and which will appear the same from ours, as follows.

For if we take only 3 Miles in one of our Leagues, the Diurnal Spaces dispatched by a Man will arise up above all Humane Strength, but that of the Stones is of it self firm and extant, with others at this time ; also in Lusitania in the Kings Highway, where all the Miles are distinguished by each having its inscribed Column, which are the Dimensions of the Spaces, surely they shew that 4 of them come into one League, therefore we follow this : Hitherto Lipsius. And according to this Computation of Lipsius is the following Calculation.

| | Hours. | Italian Miles. | Stadiums. | Paces. | Feet. |
|------------------|--------|----------------|-----------|--------|--------|
| A Spanish League | 1. | 4. | 32. | 4000. | 20000. |
| A German Mile. | 2. | 8. | 64. | 8000. | 40000. |

A Table

A Table of Equation for English Long Measure.

| | Mil. | Furl. | Perch. | Tards. | Feet, | Inches, | Grains. |
|----------------------------|------|-------|--------|--------------------|----------------------|---------|---------|
| 1 League is | 3 | 24 | 960 | 5971 $\frac{1}{2}$ | 179142 $\frac{1}{2}$ | | |
| 1 Mile is | 1 | 8 | 320 | 1760 | 5280 | 63360 | 190080 |
| 1 Furlong is | | 1 | 40 | 220 | 660 | 7920 | 23760 |
| 1 Perch, Rod or Pole is | | | 1 | 5 $\frac{1}{2}$ | 16 $\frac{1}{2}$ | 198 | 594 |
| 1 Acre is of Squ. Perch. | | | 160 | 880 | 2640 | 792 | 2376 |
| Or one } in Length | | | 40 | 220 | 660 | 7920 | 23760 |
| Acre is } in Breadth | | | 4 | 22 | 66 | 792 | 2376 |
| 1 Rod or 1 } in Length | | | 40 | | | | |
| of an Acre is } in Breadth | | | 1 | 5 $\frac{1}{2}$ | 16 $\frac{1}{2}$ | | |
| 1 Fathom is | | | | | 6 | 72 | 216 |
| 1 Ell is | | | | | 3 $\frac{1}{4}$ | 45 | 135 |
| 1 Yard is | | | | | 3 | 36 | 108 |
| 1 Foot is | | | | | 1 | 12 | 36 |
| 1 Inch is | | | | | | 1 | 3 |

C H A P. VII

Of Addition and Subtraction of divers Species or Particulars.

THe *Supputation* or counting of divers Species or Parts of any Measure; since it hath something peculiar, we must likewise admonish the studious in *Arithmetick* concerning it.

1. In *Addition* and *Subtraction*, like Species or Ranks must be placed under their like.

2. Consider well, how often the Antecedent Species or Rank contains in it self the Consequent; to wit, whether or no 'tis Double of it, or Treble, or Quadruple, &c.

3. If any intermediate Species or Rank be wanting, its place must be filled with a Cypher or 0: These being observed.

I. In

I. In Addition.

How many times you collect the Antecedent Species or Rank, so many times you must add Unity to the Antecedent Number; and under the Species or Rank in whose Numbers you occupy, you must only place that which belongs not to the Antecedent Species.

II. In Subtraction.

When you take or borrow something mutually from the Antecedent Species or Rank, resolve that first into the Species which you handle, and so from those two Numbers joined, the Subtraction must be made.

Example of Addition.

| | Hundr. | Lib. | ½ Oun. | Drams. |
|-------------|--------|------|--------|--------|
| To be added | 5 | 79 | 33 | 3 |
| | 8 | 89 | 27 | 2 |
| The Sum is | 14 | 59 | 26 | 1 |

Example of Subtraction.

| | Barrels. | Kilderkins. | Firkins. | Gallons. | Quarts. |
|----------------|----------|-------------|----------|----------|---------|
| The Minorand. | 6 | 1 | 5 | 10 | 3 |
| The Subducend. | 3 | 3 | 6 | 7 | 4 |
| The Remainder. | 1 | 1 | 1 | 2 | 3 |

C H A P. VIII.

Of Multiplication and Division of divers Species, in Ten Feet, and Sixteen Feet.

FOR this Multiplication and Division of divers Species have the chief place in the Numeration of Geometrical Measures in Germany, to wit, in Magnitude and Intervals.

I. In

Book II. Ch. VIII. of *Practical Arithmetick.* 127

I. In Multiplication of Ten Feet.

1. *Like Species or Ranks* are placed under their like.

Those in this place are *Perches* divided into 10 Feet, a Foot into 10 Digits, a Digit into 10 Grains, a Grain into 10 Scruples.

2. If either in the Multiplicand or Multiplier, some Species, either in the Beginning, or in the Middle, be omitted, it must be supplied by affixing a Cypher, or 0.

Which, altho not always necessary, yet it is useful to know the Species arising.

3. Consider how many divers Species, besides whole Perches, is contained both in the Multiplicand and Multiplier.

4. The Multiplication being finished according to the common way of working, so many figures are to be separated from the right hand by some distinction, as be apparent in the divers Species, or Ranks of Minutes, or Parts, in the Multiplicand and Multiplier.

For so the last Rank of Numbers placed before the Distinction on the left hand, signifie Perches or Poles, and the Antecedent first Figure designs the first Species or Rank after Perches: the Second shews the Second, the Third the next, and so on.

Example.

| | Ten Feet. | Feet. | Digits. |
|------------------|-----------|-------|---------|
| Let us Multiply— | 456 | 7 | 5 |
| By— | 5 | 8 | 0 |
| The Products { | 36540 | 0 | 0 |
| | 228375 | | |
| The Sum— | 264911.5 | 0 | 0 |

Then because in the Multiplicand and Multiplier (besides the Ten feet or Integers which are not numbred) there are four divers Species or Ranks: Therefore the four first Numbers, to wit, 1. 5. 0. 0. being separated, the Sum produced is,

| Ten Feet. | Foot. | Digits. | Grains. | Scruples. |
|-----------|-------|---------|---------|-----------|
| 2649. | 1. | 5. | 0. | 0. |

II. In Division of Ten Feet.

1. You must take care, that not only the omitted *Species* be supplied, but also that in the Dividend and the Divisor, the last *Species* to the right hand are alike, which if the numbers do not supply, must be effected by Cyphers.

2. Then Divide according to the common way of working, and in the Quotient you will always find the *Species* to be integers, or Perches.

3. But if there be a Remainder, add a Cypher to it, and divide again by the same Divisor, and the Quotient will be the next *Species* or Rank to Perches; to the Remainder of this, if you add another Cypher, or 0, and divide again, you will find the *Species* next to the former, and so you may proceed as far as you please.

Example.

Let us divide 92: 8. By 6. 4.

Then first, because in the Dividend, the intermediate *Species* of Feet, and in the Divisor the last *Species* of Digits is left out, therefore I supply them, that like *Species* may stand under their like.

In this manner.

| | Perches. | Feet. | Digits. |
|----------|----------|-------|---------|
| Dividend | 92 | 0 | 8 |
| Divisor | 6 | 4 | 0 |

The Division being done after the common way, there will be in the Quotient, Perches — 14.

With the Remainder — 248.

To these if I add a Cypher — 2480.

And Divide further by — 640.

I find in the Quotient, Feet — 3.

With

With the Rem.—550

The Operation.

| | | | | | | |
|----------------|---|------|---------|-----------|----------|----------|
| To which if I | { | 5600 | (24 | 280(8 Pe. | (560 Fe. | (480 Di. |
| again add 20 | | | | | | |
| And proceed | { | 640 | 8208(14 | 2480(3 | 8800(8 | 8800(8 |
| in dividing by | | | | | | |
| I find Dig. 8. | { | 480 | 8400 | 840 | 880 | 880 |
| and a Rem. of | | | | | | |
| | | | 84 | | | |

With which we must proceed in like manner, if we would cut it into the smallest parts.

Although *Ten Feet* continually divided into *Ten Parts*, in some places is not in use, but either *Twelve Feet*, or *Sixteen Feet*, as in *Germany*: Yet notwithstanding these may be assumed to be changed into *Ten Feet* and *Tenth Parts*, because of this easie way of Computation, preserving the remaining length of the *Perch*, and the Multiplication and Division being ended, you may resolve them again into the usual Parts.

But if any one will retain his own *Perch*, with its Divisions, and will institute the Multiplication and Division of them, although it be more Laborious; we will also shew him the way of working by them.

III. If then the Sixteen Feet Pole of the Germans be offered.

Which, to wit, is Divided into 16 Feet, but the Foot subdivided into 12 Inches, (for which notwithstanding, for ease in working, we will assume 16 Digits to be equal to the 12 Inches)

In Multiplication proceed thus.

1. For conveniency, make the number which hath most Species or Ranks the Multiplicand, and put it in the first place.
2. Place like Species under their like, and to this end substitute Cyphers in the middle and end, in the absent places.

K

3. Write

3. Write the Product of every Species separately, and exactly, whether it consist of one or more Figures.

4. Add the Numbers of every place into one Sum.

5. Reduce each of the Latter Species to the Antecedents, if they are contained once, or oftner in them, by taking or assuming for each 16 of the Latter, one of the preceding Species, to be added to the same.

Example.

| | Perches. | Feet. | Digits. |
|----------------------|-------------|-------|---------|
| Let | 3 | : 4 | : 6 |
| Be drawn into | 2 | : 3 | : 5 |
| <hr/> | | | |
| | 15 | : 20 | : 30 |
| Numbers of the Prod. | 9 : 12 | : 18 | |
| | 6 : 8 : 12 | | |
| <hr/> | | | |
| Sum of them | 6 : 17 : 39 | : 38 | : 30 |
| Reduced is, | 7 : 3 : 9 | : 7 | : 14 |

6. And that you may know the Species arising, note the following Rules.

Of the Species coming forth or arising.

RULE I.

Digits drawn into Digits, produce Square Digits, 16 of which constitute one of the Antecedent Species, to wit, of a Foot long, but of the Breadth of one Digit only, which Geometers and Land Measurers call a Contracted Foot.

RULE II.

Digits multiplied into Feet, make such Feet one Digit Broad, 16 of which make a Square Foot, to be transfer'd to the Antecedent Species.

RULE

RULE III.

Digits drawn into *Perches* make *Perches* one *Digit* in Breadth, and 16 of these *Perches* constitute one *Perch* having a *Foot* in Breadth, and therefore to be moved forward one Rank or Station; *Perches* of this kind a *Foot* broad, are called by Land Measurers *Contracted Perches*.

RULE IV

Feet drawn into *Feet*, make *Square Feet*, one of which is of the same quantity with a *Perch* a *Digit* broad, and therefore may be placed in one Rank or Row, and is accounted for one *Species*: For as a *Square Foot* contains *Square Digits* 256, so also a *Perch* one *Digit* Broad, each one of the 16 *Feet* contains 16 *Digits*, in all 256 *Square Digits*: Therefore as often as you have 16 *Feet*, or 16 *Perches* a *Digit* Broad, so many *Unities* must be added to the Antecedent *Species*.

RULE V.

Feet Multiplied into *Perches* make *Perches* one *Foot* broad, 16 of which constitute a *Square Perch*, and therefore by the *Unity* to be transfer'd to the first *Species*.

RULE VI.

But *Perches* drawn into *Perches* make *Square Perches*, which is the first or chief *Measure* of Fields.

These Rules being observed, we shall have in the
aforesaid Example by Multiplication,

Square Perches ——— 7.
Perches a Foot broad — 3.

K 2

Square

Square Feet or Perches 1 Foot broad—9.

Feet a Digit broad—7.

And Square Digits—14.

Others say thus.

1. *Perches* by whatsoever Species of a Fraction they are Multiplied, they restore that same Species of the Fraction.

2. *Feet* drawn into *Feet* make *Digits*, *Digits* into *Feet* make *Grains*, and *Digits* into *Digits* make *Scruples*.

So then in our Example of Multiplication in the last Product, there is ;

Perch. Feet, Dig. Gra. Scrup.

7. 3 9. 7. 14.

The Thing is the same, only the Appellations differ.

By *Perches* truly are understood *Square Perches* ; but by *Feet*, *Perches* a Foot broad : By *Grains*, *Feet* a Digit broad : Lastly, by *Scruples* are understood *Square Digits*.

Therefore the Species coming forth, you may easily know thus.

Compleating the void Spaces, or supplying the Species omitted, separate from the Product so many Species or Ranks as are found in the Multiplicand and Multiplier, besides the *Integers* : Then the First number before the Note of separation shews *Perches*, the Second *Feet*, the Third *Digits*, the Fourth *Grains*, the Fifth *Scruples*.

Example.

Let us Multiply 10 *Feet*, and 6 *Digits*, by 8 *Digits*.

Perch. Feet. Dig.

The numbers then will stand thus—0 : 10 : 6.

0 : 0 : 8.

Multi-

Perch. Feet. Digits.

Multiplicand—0 : 10 : 6.

Multiplier—0 : 0 : 8.

Products—0 : 0 : 80 : 48.

0 : 0 : 0.

Reduced—0 : 0 : 5 : 3 : 0.

By separating 4 Figures, for the 4 divers *Species* of the Multiplicand and Multiplier.

Per. Fe. Dig. Gr. Scr.

The *Species* produced will be—0 : 0 : 5 : 3 : 0.

IV. And lastly, In Division of Sixteen Feet.

1. Omitting other ways more difficult, reduce the proposed Numbers to the least *Species*, by continual Multiplication by 16.

2. They being reduced, divide them by the common way of Operation, and the Quotient will give *Perches*.

3. Multiply again the Remainder by 16, and divide by the former Divisor, and you will have *Feet* in the Quotient : So also work with the Remainder of this Division as often as you please, and you will find in the Quotient, *Digits, Grains, &c.*

Example.

Per. Fe. Dig.

Let the Dividend be—8 : 4 : 3.

The Divisor—5 : 5 : 8.

The Dividend in Digits will be—2115.

And the Divisor—1368.

The Quotient is—1 Perch.

The Remainder—747.

Drawn into 16, produces—11952.

Which Number divided by—1368.

Makes in the Quotient—8 Feet.

The Remainder—1008.

With which you may work in the same manner.

OF GEOMETRY.

The PREFACE.

I. **G**EOMETRY is a Science which considers the Principles, Kinds, Affections, and Habitudes of Magnitude,, and teaches to measure them Artificially.

The word Geometry, attributed to this Science, is called a ridiculous name by Plato in his Epinomide : For Geometry is properly the measuring of the Earth : But this Science considers not only the Earth, but all Magnitude in general ; therefore more truly it should be called by a more general word Mathelogia : But because the name of Geometry is commonly received, 'tis therefore retained, and its signification is Amplified by Synecdoche.

II. Therefore the Subject or Matter about which Geometry is conversant, is Magnitude.

(It considers this Abstractively, so that it separates it from all certain and definite matter.)

III. And we constitute the Parts of it, Six.

1. Elementary, which shews the Definitions, Axioms, and Petitions, concerning Geometrical things.
2. It treats of Circles.
3. Of Triangles.
4. Of Right Lines.
5. Of Superficial Planes.
6. It proposes divers Theorems and Problems concerning Solid Bodies.

Principles of GEOMETRY.

P A R T I.

Of Definitions, Axioms, and Petitions.

C H A P. I.

Of Magnitude, and its beginning, a Point.

1. **M**agnitude is *Quantity continued*, the common parts of which are Joined by a *Term*, or *Bound*.

2. A *Term* is the Extream of *Magnitude*.

3. A *Common Term*, is the Extream of One, together with the beginning of another.

And the first Beginning, and last Extream of all *Magnitude* is a *Point*.

4. A *Point*, is that which hath no part, and therefore is also something Indivisible.

C H A P. II.

Of the most Common Affections and Denominations of Magnitude.

The most common Affections of all Magnitude, from which they are frequently denominated, are,

| | | |
|-------------------------|----------------|---------------------------------------|
| 1. Symmetry. | | Commensurable, and the contrary. |
| 2. Proportion. | | To have Proportion. |
| 3. Proportionality. | Whence | Proportionals. |
| 4. Reason or Effability | Magnitudes are | Rational or Effable, and the contrary |
| 5. Contact. | said to be | Touching one another mutually. |
| 6. Section. | | Cutting through. |
| 7. Congruency. | | Agreeing. |

1. *Magnitudes are Commensurable*, which are measured by the same Measure : As a line of 20, and a line of 13 *Palms*, are Commensurable, because a line of one and an half, and $\frac{1}{3}$ part of a Palm, measures both.

Incommensurables are the contrary, which admit of no common Measure, as the *Diameter of a Square*, and the *Side of the same*, and many others.

2. *Proportion or Habitude*, are those things which are said to have a mutual Habitude between themselves, according to Quantity ; as two Magnitudes of the same kind, either of which being multiplied, so increases, that at length it exceeds the other.

Therefore a Finite line and an Infinite, have no Proportion to each other ; for the Finite being Multiplied never so often, will not exceed the Infinite : However the *Diameter of a Square*, and its *Side* have Proportion between themselves, although Comprehensible by no number : Likewise the *Diameter* and the *Circumference of a Circle*, although as yet not certainly known ; because the side of a *Square* twice taken exceeds the *Diameter* : And the *Diameter of a Circle* four times taken exceeds the *Circumference*.

3. *Analogy or Proportionals*, are those which are said either to have the same *Proportion*, any two of which answering mutually, are equally Multiplex of the other two, being equally multiplied ; or are equal, or in the same manner, either exceed them, or are deficient from them.

As if there be four lines in Foot Measure, *A* 12. *B* 4. *C* 9. *D* 3. these are called *Proportionals*, because *A* hath the same Proportion to *B*, as *C* to *D*, to wit, Triple : And as *A* and *C*. have *Sesquitertia* Proportion between themselves, so likewise hath *B* and *D*.

4. *Homologous Magnitudes*, are those which have Proportional Terms, the Antecedents having like Proportion to the Antecedents, and the Consequents to the Consequents.

As if there be four Proportional lines in Foot, or any other, Measure, *A* 9, *B* 6, *C* 12, *D* 8 ; these are called *Homologes*, because between the Antecedent Terms, *A* 9 and *C* 12, and between the Consequent Terms *B* 6, and *D* 8, the Proportion is alike, to wit, *Sesquitertia*, or once, and $\frac{1}{3}$ part

5. *Rational or Effable Magnitudes*, are those whose Proportions may be exprest by certain Numbers : *Irrational or Ineffable*, are the contrary.

6. *Magnitudes touching one another mutually*, are those of which, very little of one touches very little of the other.

7. *Secants or Magnitudes cutting through each other*, are those of which very little of one cuts very little of the other.

8. *Congruous or Agreeing*, are those whose Terms or Parts answer one another every where.

CHAP. III.

Of all the Kinds of Magnitude, and first of a Line.

ALL Magnitude is either a Line, or a Lineament or Diagram.

1. *A Line*, is Length without Breadth, being begat from the continual Succession of Points, and Terminated by the same.

A Line absolutely considered is,
Right, Crooked, Mixt.

2. *A Right Line*, is that which lies equally between its Points. —

This is represented by a fine Thread drawn streight, and the Instrument to draw it withal on a Plane, is a Ruler.

3. *A Crooked or Curve Line*, is that which is equally distant on every side from the middle Point, or Center of the comprehending Space, as, $\odot$.

Also 'tis called, a Spherical, a Circular, or a Round Line, that is, a Circumference ; and likewise but Abusively, a Circle : for a Circle is a Plane Figure ; and we consider here, not Figures, but Lines only, and

and indeed Crooked ones, which are drawn by the Instrument called *Compasses*.

6 4. A *Mixt Line*, is that which is compounded of both, or that which hath neither perfect Curvity, nor perfect Rectitude, See Fig. 6.

Such are *Ovaler Lines*, *Lenticular*, *Spiral Lines*, and innumerable others.

A Line, considered with respect to another, is *Perpendicular* or *Parallel*.

5. A *Perpendicular Line*, is a Right Line, falling, or cutting directly on another Line, so that it neither bows, nor inclines in any part more than another to the subjected Line, as, 

The Instruments to delineate this by, is called either a *Square*, or a *Plumb-Rule*.

6. *Parallel Lines*, are those which are equally distant to one another, and will not intersect each other, although infinitely produced, as, 

C H A P. IV.

Of Lineaments or Diagrams, and first of Angles.

1. A *Lineament* or *Diagram*, is a Magnitude more than long.

There are two Kinds of it, *An Angle*, and *a Figure*.

2. *An Angle*, is of two or more Lines touching each other mutually, by a mutual Inclination, and not lying directly in a Line.

In this, three things are considered.

1. *The Angle it self.* 2. *Its Legs.* 3. *Its Quantity.*
3. *The Angle*, is the common meeting of the Lines.
4. *The Legs*, are Lines containing the Angle.

Geometricians, express an *Angle* by three Letters, the *Middlemost* being the *Characterist*, and shews or denotes the *Angle* proposed; as if in Fig. 7. I should say *A, B, C*, is to be considered; the Letter *B* is the *Index of the Angle*, and *A, B*, and *B, C*, the *Legs* of the same.

5. The

Chap. IV. Of Lineaments or Diagrams. 139

5. The *Quantity of an Angle*, is the greater or lesser *Divarication of the Legs*.

1. Therefore, two Angles are equal, whose *Divarication* is equal, altho the *Legs* are unequal.

2. The *Measure of this Quantity*, is an *Arch of the Periphery*, divided into 360 *Degrees*, every *Degree* of which is divided into 60 *First Scruples*, and every *First or Prime Scruple* into 60 *Seconds*, and so on : Therefore when we seek the *Quantity of an Angle*, tis said to be so many *Degrees*, or so many *Degrees and Scruples*.

An Angle, according to the variety of *Subjects*, is either *Superficial* or *Solid*.

6. A *Solid Angle*, is the collection of more than two lines, touching each other mutually at one *Point*, and existing not in the same *Superficie*. See Fig. 8. 8

7. A *Superficial Angle*, is the mutual touching of two *Lines* in a *Superficie*, by mutual inclination, and not lying in a direct *Line*.

And 'tis either *Plane* or *Spherical*.

8. A *Superficial Plane Angle*, is that which is described on a *Plane Superficies*.

9. A *Spherical Angle*, is that which is described on a *Spherical Superficies*, as Fig. 9. 9

A *Plane Angle* is either *Homogene* or *Heterogene*.

10. An *Homogene*, is that which consists of *Legs*, and the concourse or meeting of the *Legs*, is of the same kind.

And 'tis either *Right Lined*, or *Curve Lined*.

11. A *Right Lined*, hath *Right Lines*.

12. A *Curve Lined Angle*, hath *Crooked* or *Circular Lines*, yet described on a *Plane*.

13. An *Heterogene Angle*, is a mixt Angle, comprehended by right and crooked lines ; of this kind is the Angle of a *Segment*, comprehended under a *Right Line*, and part of the *Circle*, as Fig. 10. & 11. 10 11

In respect of *Quantity*, an Angle is either *Right* or *Oblique*.

14. A

14. A *Right Angle*, is that whose two Legs are Perpendiculars between themselves, or are perpendicular to one another.

This always contains a Quadrant, or $\frac{1}{4}$ part of the *Periphery*, to wit, 90 Degrees.

2.13 15. An *Oblique Angle*, is, that whose Legs are not Perpendiculars, but incline from a Perpendicular to either part, as in Fig. 12 & 13.

And 'tis either *Acute* or *Obtuse*.

16. An *Acute Angle*, is less than a Right, or a Quadrant of a Circle, but an *Obtuse* is greater.

In respect of Situation, Angles obtain divers Appellations.

17. Moreover Angles posited Successively or in Order, are Angles adjoining, which Right Lines constitute, howsoever the Right Lines infist, as in Fig. 14. The Angles acb , bcd , feb , geb .

14 18. Angles at the Vertex opposite by the Cross, are those which are opposite at the Point, in which two Right Lines cut each other., as in Fig. 14. the Angles acb , and dce : Also ace , and bcd : Likewise feb , and ceg : Lastly, fec , and beg .

14 19. Angles alternate, are those which a Right Line makes alternately, by changing the Positions, in cutting two other Lines: As the Angles ace , and ceg , likewise dce , and fec . See Fig. 14.

20. Angles opposite on the same part, are those which a Right Line makes, by cutting through two other, one Angle being *Interne*, the other *Externe*; As the Angles acb , and ace : And bcd , and ecd .

21. Angles not opposite on the same part, are the two *Intern Angles*, made by a Right Line falling on two Right Lines on the same part: As the Angles ace , and fec : Also dce and geb .

CHAP. V.

Of Figures, and their Parts and Denominations, taken from the most common Affections or Dispositions.

1. A Figure, is a Diagram or Lineament, Terminated on every side.

For

For not every Magnitude which consists in Terms is a Figure, but those only which is every where inclosed by Terms; of which Terms, by how many the more they are, by them the Figure is the larger.

The Parts of Figures are :

- | | |
|--------------------------|----------------------|
| 1. <i>Sides.</i> | 7. <i>Diagonal.</i> |
| 2. <i>The Area.</i> | 8. <i>Axis.</i> |
| 3. <i>The Center.</i> | 9. <i>Poles.</i> |
| 4. <i>The Perimeter.</i> | 10. <i>Vertex.</i> |
| 5. <i>Diameter.</i> | 11. <i>Base.</i> |
| 6. <i>Semidiameter.</i> | 12. <i>Altitude.</i> |

1. *Sides*, are the Extrems of Figures next the Terminants, and those are either Lines or Superficies.

2. *Area*, is the Space comprehended by the Sides.

3. *Center*, is the middle Point in a Figure.

4. *Perimeter*, is the Ambit and Comprehension of a Figure.

5. *Diameter*, is a Right Line drawn thro' the Center, and Terminated on both sides at the Perimeter.

6. *Semidiameter*, or *Radius*, is a Right Line drawn from the Center to the Perimeter.

7. *Diagonal*, is a Right Line drawn thro' the Center, and Terminated at the two opposite Angles.

8. *Axis*, is a Right Line drawn thro' the Center, about which, the Figure is understood to be turned about.

9. *Poles*, are the two extream Points of the Axis.

10. *Vertex*, is a Point on the top of the Figure, from which a Right Line may be drawn Perpendicular on the subjected Base, either True or Protracted.

11. *Base*, is a Side, on which the Figure is understood to insist, or lye on.

12. *Altitude*, is a Perpendicular drawn from the Vertex, either to the True or Protracted Base.

13. *The most common Affections of Figures, denominating the same, are,*

- | | | |
|--------------------------------|------------|-----------------------------|
| 1. <i>Order or Regularity,</i> | Whence | <i>Ordinate or Regular.</i> |
| 2. <i>Primacy,</i> | Figures | <i>Primes.</i> |
| 3. <i>Equal Altitude,</i> | are called | <i>Of Equal Height.</i> |

4. *Like-*

| | | |
|-------------------|---------------------------------|--------------------------|
| 4. Likeness, | Whence
Figures
are called | Like. |
| 5. Like Position, | | A like Scituated. |
| 6. Agreeableness, | | Congruant, or Agreeing. |
| 7. Compass equal, | | Equal Periphery. |
| 8. Affcription, | | Adscribed. |
| 9. Kinds, | | Homogene, or Heterogene. |

14. *Ordinate Figures*, are those, all whose Terms (as Lines in *Superficies*, and *Superficies* in *Bodies*) are equal between themselves, and are *Equiangled*.

Understand this of the Ordination and Regularity assumed, which consists in the Equality of Sides, and of Angles: Of which Regular Figures in all kinds of Planes, there are not more than one: For in *Triangles* only the *Equilateral* is Ordinate or Regular: In *four sided Figures*, only the *Quadrate*: In *Many Angled Figures*, those which are *Inscribed*, or *Circumscribed* about a *Circle*, with *Equal Sides*: All the rest in this acception of *Special Ordination*, are *Inordinate*: In more general Signification, all *Figures* are said to be *Ordinate*, which have *Ordinate Disposition*, and may be reduced or recalled to certain kinds, as *Triangles*, *Quadrangles*, *Oblongs*, *Rhombus's*.

15. *The First* or *Prime*, are not to be divided into others more simple. Of this kind of Figure in Angles there are only two; the *Triangle in Superficies*, and the *Pyramid in Bodies*: For altho' these may be divided into more *Triangles*, or more *Pyramids*, yet they cannot be resolved into any other Figure (as the *Quadrangle* into two *Triangles*) more simple than the former it self.

16. *Of Equal Height*, are those which consist in the same *Parallels*.

17. *Like Figures*, are *Equiangled* and *Proportional*, from the Legs of the Angles.

18. *Like Scituate Figures*, are those whose *Proportional Terms*, being alike Scituated, answer to one another.

19. *Agree-*

Chap. V. Of Figures and their Parts. 143

19. *Agreeable Figures*, are those which fill up the place without any opening. Also we have before spoke of *Agreeableness*, but there it is to be understood such, as the Parts of one being applied, or laid upon the Parts of the other, they answer every way; which congruous Magnitudes, must of necessity be like and equal: But in this place, we understand that agreement, whereby many Magnitudes, altho of divers Kinds may be so fitted, that there be left no Opening or Gaping between them; As the *Duodec-angle*, and the *Equilateral Triangle*: The *Octangle*, and the *Quadrate*: The *Circle*, and the *Spherical Triangle*, the Agreement of which we see in Windows.

20. *Isoperimetrical Figures*, are those whose Ambits or Compasses are Equal: As the *Triangle*, the *Quadrangle*, the *Circle*; the Ambit of each of them being three Feet.

21. *Adscribed Figures*, are, when the Terms of one Terminate the Terms of the other: Of these the Interior are said to be *Inscribed*, and the Exterior to be *Circumscribed*.

22. *Homogene Figures*, are of the same kind, as to the number of Sides: *Heterogene* the contrary.

CHAP. VI.

Of the Kinds of Figures:

THE Kinds of Figures are two, *Superficies* and *Bodies*.

1. *A Superficies*, is a Figure having Length and Breadth only, as Fig. 15. (the express Image of this, the Shadows of our Bodies shew.)

In respect of Scite or Scituation, a Superficie is either *Convex* or *Concave*.

2. *A Convex Superficie*, is that which encompasses

16 *16* fes any Body without : *Concave*, whereby Hollow Bodies are Terminated on the inside. See Fig. 16.

In respect of Form, a Superficie is either Plane or Gibbous.

3. *A Plane Superficie*, is that which lyes equally between its Lines, and is usually said to be even : *Gibbous*, is that which lies unequally between its Terms or Lines, and is rough or uneven.

A Plane Superficie is either Curve-lined or Right-lined, or Mixt.

4. *Curve-lined*, is that which is contained by *Crooked Lines*, *Right-lined* by *Right Lines*, and *Mixt* by *Mixt Lines*.

C H A P. VII.

Of a Circle.

A *Plane Superficie Curve-lined*, is called a Circle.

17 *17* 1. *A Circle*, is a Plane Figure contained by one line drawn about (which is called *Periphery* or *Circumference*) to which all *Right Lines* drawn from the middle Point are equal, as in Fig. 17.

In a Circle, besides the aforesaid, common to all Figures, the following are considered.

1. *An Arch*. 2. *A Quadrant*. 3. *A Semicircle*.
4. *A Chord*. 5. *A Segment*. 6. *A Sector*.

18 *18* 2. *An Arch*, is any Portion of the Circumference, as in Fig. 18. *b h*, or *c i*, to which, that which is wanting to 90, or to 180 Degrees, is called the *Complement*, and what is above 90 Degrees, is called the *Excess*.

3. *A Quadrant*, is a Plane Figure, comprehended by two *Semidiameters* constituting a Right Angle, and a fourth part of the Circumference of the Circle.

Such

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Such is bg , a fourth part of the Circle, comprehended by the Semidiameters ba , and ga , constituting a Right Angle at a .

4. A *Semicircle*, is a Plane Figure, which is contained under the Diameter, and an half part of the Circumference, as bgc , the half part of the Circumference, comprehended under the Diameter bac , as in Fig. 18.

5. A *Chord*, or *Subtense*, is a Right Line drawn through the Circle without the Center, as the Line fke .

6. A *Segment*, is a Figure contained under the Chord, and part of the Periphery : and 'tis the greater Segment in which the Center of the Circle is found, and the lesser, which is without the Center.

Under the Chord fke is the lesser Segment, the Portion fge : But the greater Segment is the Portion of the Circle $fbice$.

7. A *Sector*, is a Figure comprehended by Right Lines, constituting an Angle in the Center, and from the Periphery assumed by them, as abi .

C H A P. VIII.

Of Superficial Planes, being Right Lined.

And First of Triangles.

OF Superficial Right Lined Planes, there are two Kinds, the *Triangle*, and the *Triangulate*.

1. A *Right Line Triangle* having three Corners, is a Superficies comprehended by three Lines, as Fig. 19.

The Parts of it are,

- | | |
|----------------|--------------------------------|
| 1. The Angles. | 4. The Hypotenuse or Subtense. |
| 2. The Legs. | 5. The Perpendicular. |
| 3. The Base. | 6. The Area. |

L

2. An

2. *An Angle*, is the meeting of Lines.

3. *Legs*, are Sides comprehending any Angle, as in Fig. 19. ab , and bc , are the Legs of the Angle b : So ac and ab , are Legs of the Angle a .

4. *The Base*, is a Side on which an Angle is understood to insist; So ac , is the Base of the Angle b , and ab the Base of the Angle c .

5. *A Subtense*, is any side opposite to an Angle, and subtending it; as ab is the Subtense of the Angle c .

6. *A Perpendicular*, is a Right Line falling on another Right Line, and constituting a Right Angle with it:

As the Right Line bc in the Triangle abc , in Fig. 19. And the prick'd Perpendicular in the Triangles def , Fig. 24. and la , Fig. 20. that falling within, this without the Triangle, on the Base, protracted.

Therefore in any Triangle, if you make one side the Base, the other two are Legs: Yet in the Right angled especially, the side opposite to the Right Angle, is called the *Hypotenuse*, and of the Remainers, one is called the *Base*, and the other being Perpendicular to it, is called the *Perpendicular*.

7. *The Area of a Triangle*, is the middle space comprehended by three Lines.

The Parts of Triangles inscribed in a Circle, are.

1. The whole Sine, Radius, or Semidiameter ac , in Fig. 18.

2. *The First Right Sine*, ci . Fig. 18.

3. *The Second Right Sine*, or the Right Sine of the Complement, ke .

4. *The Versed Sine*, or Arrow, lc .

5. *The Tangent*, ch .

6. *The Secant*, ah .

7. *The whole Sine or Radius*, is the Semidiameter it self of the Circle, as ac .

8. A

8. *A Sine, in general*, is half the Right Line subtending the double Arch of the Circle, as $k e$, is half the Right Line $f k e$, subtending the double of the Arch $g e$, to wit, $f g e$.

9. *A First Right Sine*, is half the Subtense of double the Arch in the First given; as if there be given in the first place the Arch $e c$, the double of it is the Arch $e c i$, and its Subtense $e l i$, half of which Subtense $e l$, is the *First Right Sine*: Where Note, that the Sine of the Arch given in the first place $e c$, and of its complement to a Semicircle $e g f b$, is the same, to wit, $e l$.

10. *A Second Right Sine*, or the Sine of the Complement, is half the Subtense of the Double of the Arch of the Remainder of the First given to a Quadrant.

Such is $k e$, half of the Subtense $f k e$, subtending the double $f g e$, of the Arch $g e$, which is the Remainder to the First given $e c$, or the Complement to a Quadrant $g a c$.

11. *A Versed Sine*, is the Segment of the Diameter intercepted between the Right Sine and the Circumference.

So $l c$ is the *Versed Sine*, being a Segment of the Diameter $b c$, intercepting the Arch $e c$, as being between the right Sine $e l$, and the Arch $e c$.

And a *Versed Sine*, is one Greater, the other Lesser.

12. *A Versed Sine Lesser*, is the Sine of an Arch less than a Quadrant, as $l c$, of the Arch $e c$.

The *Greater Versed Sine*, is of an Arch greater than a Quadrant, as $b l$, is the Versed Sine of the Arch $b g e$.

13. *A Tangent*, is a Right Line without the Ambit or Periphery of the Circle, Perpendicular to the Extream of the Diameter, and Terminated at the other end by the Secant; as the Right Line $c h$.

14. A *Secant*, is a Right Line drawn from the Center through the Term of the Arch, even to the Tangent, as the Right Line *a e b*.

A Right Lined Triangle, in respect of the Sides, is Equilateral, Equicrural, and Various.

21 15. *Equilateral*, is, which hath three equal Sides, as the Triangle *d e f*, in Fig. 21.

22 16. *Equicrural*, is, which hath two equal Sides, but the third unequal, as *g b i*, in Fig. 22.

23 17. *Various*, which hath three unequal Sides, as *a b c*, in Fig. 23.

In respect of Angles, they are either Rectangled, or Obliqueangled.

19 18. *Rectangled*, is, which hath only one Rectangle, as the Triangle *a c b*, in Fig. 19.

19 19. *Obliqueangled*, is, which hath no Right Angle, and this is two fold,

24 20. *Acuteangled*, which hath three Acute Angles, as the Triangle *d e f*, in Fig. 24.

21. *Obtuseangled*, which hath one Obtuse Angle, as the Triangle *k m l*, in Fig. 20.

C H A P. IX.

Of Triangulate Planes.

1. A *Triangulate Plane*, is a Right Lined Plane, which may be resolved into many Triangles.

And 'tis either *Quadrangular*, or *Multangular*.

2. A *Quadrangular*, is that which is comprehended by four Right Lines.

And 'tis either a *Parallelogram*, or a *Trapezium*.

3. A *Parallelogram*, is *Quadrangular*, having the opposite Sides Parallel.

A Tra-

A *Trapezium*, is the contrary, as abc in Fig. 25.

A *Parallelogram*, is Rectangled or Obliquangled.

4. *Rectangled*, is that which hath all the Angles right; *Obliqueangled*, hath all oblique.

Rectangled, is either a Square or an Oblong.

5. A *Square*, is a Rectangled Figure of equal Sides, Fig. 2.

An *Oblong*, is a Rectangled Figure of unequal Sides, and longer on one part. Fig. 15.

The Parts of a *Parallelogram*, are,

1. *The Diagonal.* 3. *The Complements.*

2. *Two Diagonals.* 4. *The Gnomons.*

6. *The Diagonal*, is the Diameter of the *Parallelogram*, touching the two Angles, as ac , Fig. 28.

7. *Diagonally*, is a particular *Parallelogram*, of a common Angle, and Diagonal with the whole *Parallelogram*: as the small *Parallelograms* $fgic$, and $ahge$, in the same Square.

8. *The Complement*, is a particular *Parallelogram* comprehended by the Conterminate Sides of the Diagonal, as are the *Parallelograms*, $dhgf$, and $egbi$.

9. *Gnomon*, is the Diagonal of either, with two Complements, as if $gfic$, be understood to be cut off, the Remainder of the whole *Parallelogram* is the *Gnomon*; or if $ahge$ be cut off, the *Gnomon* also will be the Remainder.

An *Obliquangled Parallelogram*, is either a Rhombus, or Rhomboides.

10. A *Rhombus*, is an Obliquangled Figure of equal Sides: as $abcd$, in Fig. 29.

A *Rhomboides*, is of unequal Sides: as $cabd$, in Fig. 30.

11. *Multangled*, is, which is comprehended by more than Four Lines.

And it is either Regular or Irregular.

12. *Regular*, is Multangled, of equal Angles, and equal Sides : *Irregular* is the contrary.

These are Denominated by the Number,

| <i>Of Angles.</i> | | <i>Of Sides.</i> | |
|-------------------|----------------------|----------------------|---------------------|
| <i>Greek.</i> | <i>English.</i> | <i>Greek.</i> | <i>English.</i> |
| <i>Pentagon,</i> | <i>Five Angles.</i> | <i>Penta Pleura,</i> | <i>Five Sides.</i> |
| <i>Hexagon,</i> | <i>Six Angles.</i> | <i>Hexa Pleura,</i> | <i>Six Sides.</i> |
| <i>Heptagon,</i> | <i>Seven Angles.</i> | <i>Hepta Pleura.</i> | <i>Seven Sides.</i> |
| <i>Octagon,</i> | <i>Eight Angles.</i> | <i>Octo Pleura,</i> | <i>Eight Sides.</i> |

See Fig. 31, and 32.

CHAP. X.

Of Gibbous Superficies.

1. **A** *Gibbous Superficies*, is, that which lyes unequally between its Terms.

And 'tis either Sphærical or Various.

2. *A Gibbous Sphærical Superficies*, is that which is equidistant from the middle of the containing Space.

In this are delineated Sphærical Triangles.

3. For a *Sphærical Triangle*, is that which is described on a *Sphærical Superficies*, as a *Plane Triangle* on a *Plane* : as Fig. 9.

4. Its parts are the same as of a *Plane Triangle*, to wit :

1. *Angles.*

4. *Hypotenuse* or *Subtense.*

2. *Legs.*

5. *Perpendicular.*

3. *Base.*

5. *The Kinds*, in respect of the Sides, are the same as of *Planes*, to wit,

Equilateral, Equicrural, Various.

6. In

Chap. X. Of Gibbous Superficies. 151

6. In respect of *Angles*, they are also the same as in *Planes*; to wit:

1. *Rectangled*. } Which is either { *Acute Angled*, or,
2. *Obliquangled*. } *Obtuse Angled*.

7. A *Right Angled* hath either { One Right, } Two *Acute*.
or more { Two *Obtuse*.
Right an- { *Acute and Obtuse*.
gles, or— { Three.
Two with the Third { *Acute*, or
{ *Obtuse*.

8. *Obliquangled*, { Pure, from { *Acute*.
is either { Three, either { *Obtuse*
Or,
{ Mixt, either { Two *Acute*, and
of— { One *Obtuse*, or
{ Two *Obtuse*, and
{ One *Acute*.

9. *Various*, which is either not equidistant from the middle, as *Ovaler*, *Lenticular*; or *Mixt*, of Plane and Gibbous, as *Conical* and *Cylindrical*, Fig. 6. 35. 36.

10. *Conical*, is that which is raised to a sharp top equally, from a subjected Plane Circle.

11. *Cylindrical*, is that which is equally raised upright to the top, from a subjected Circle, being equidistant, and equal on the Top, to the subjected Circle or Base.

CHAP. XI.

Of Plane Bodies.

1. A *Solid Body*, is that which hath Length, Breadth and Depth, or Thickness, and is Terminated by the next Superficies.

Bodies are either *Plane* or *Gibbous*.

L 4

2. *Plane*

2. *Plane Bodies*, are comprehended by *Plane Surfaces* : *Gibbous*, by *Gibbous* or *Crooked Surfaces*.

A *Plane Body*, is a *Pyramid* or *Pyramide*.

3. A *Triangular Pyramid*, is a *Solid Plane*, raised up to a *Point* at *Top*, from a *Right Lined Triangular Base* : as *Fig. 8*.

There are infinite kinds of these, which from the *Figure* of their *Bases* are called *Triangular*, *Quadrangular*, *Multangular*.

4. *Pyramide*, is a *Solid Plane*, contained by *Pyramids*.

And 'tis a *Prisme*, or a mixt *Polyhedron*, or *Polygon*.

5. A *Prisme*, is a *Solid Plane*, of which, two opposite *Sides* are *Equal*, and *Like*, and *Parallel* ; the other are *Parallelograms*.

And 'tis of *Five Sides*, or *Pentaedrum* ; or *Six Sides*, or *Hexaedrum* ; or many *Sides*, or *Polyedrum*.

6. Of *Five Sides*, or *Pentaedrum*, is that which hath a *Triangular Base*, and is contained by five *Sides*, *Fig. 38*.

7. *Hexaedrum*, (which is also called *Parallelepipedon*) is that which hath a *Quadrangular Base*, and is comprehended by six *Sides* or *Bases*.

And an *Hexaedrum* is either *Right Angled*, or *Obtuse Angled*.

8. A *Right Angled Hexaedrum*, is that which hath all *Right Angles*.

And 'tis either a *Cube* or an *Oblong*.

9. A *Cube*, is a *Rectangled Parallelepipedon*, contained under six equal square *Sides*. *Fig. 5*.

10. An *Oblong*, is a *Parallelepipedon*, contained by six *Sides*, two being equal, the rest longer on the other part. *Fig. 40*.

11. An *Oblique Angled Parallelepipedon*, is that which hath all *Oblique Angles*.

And 'tis either *Rhombus*, or *Rhomboides*,

12. *Rhombus*, is a *Parallelepipedon* obliquangled, comprehended by Six *Equilateral Sides*, or *Bases*, *Fig. 41*,

13. *Rhom:*

13. *Rhomboides*, is an Obliquangled *Parallelipipedon*, contained by six Sides or Bases, two of which being Opposites, are Equilateral, the rest are longer on one part, Fig. 42.

14. *Polyedrum*, is that which hath a Multangled Base, and is contained by more than six Sides.

Of these there are an Innumerable Multitude, which are named from their Sides, *Heptaedra*, *Octoedra*, *Enneaedra*, &c.

15. A *mixt Polyedron*, is that which hath the Vertex in the Center, and the Base bearing outwards.

And 'tis *Octaedron*, *Dodecaedron*, *Icosaedron*.

16. *Octaedron*, is a mixt *Polyedron*, contained under eight equal Triangles, Equiangular, and Equilateral. Fig. 48.

17. *Dodecaedron*, under twelve *Pentagons*, Equilateral, and Equiangular. Fig. 49.

18. *Icosaedron*, is comprehended under twenty Equiangular, and Equilateral Triangles. See Fig. 46, 47, 48, 49, 50, 51.

CHAP. XII.

Of Gibbous or Crooked Bodies.

A *Gibbous Body*, is either a Sphere, or Various.

1. A *Sphere*, or *Globe*, is a Solid Body, of which all the Extrems are touched by like *Radii*'s from the Middle: See the *Diagram* of it in Fig. 51.

2. *Various*, is contained by a various Superficie.

The Kinds of this are Innumerable, but the chief are the *Cone* and *Cylinder*.

3. A

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3. *A Cone*, is a Solid, contained by a Circle, and a Conical Superficie, which is placed between the Vertex and the Circumference of a Circle.

And it is either *Right* or *Scalene*.

35 4. *A Right Cone*, is that which hath the Axis at Right Angles to its Base, Fig. 35.

And 'tis either *Orthogon*, or *Oxygon*, or *Amblygon*.

5. *Orthogon*, is when the Axis is equal to the Semidiameter of the Base.

6. *Oxygon*, whose Axis is greater than the Semidiameter of the Base.

7. *Amblygon*, whose Axis is lesser than the Semidiameter of the Base.

43 8. *A Scalene Cone*, is that whose Axis is oblique to the Base, Fig. 43.

36 9. *A Right Cylinder*, is that which hath its Axis insisting at right Angles to the Base, Fig. 36.

45 10. *A Scalene Cylinder*, is that whose Axis is oblique to the Base, Fig. 45.

C H A P. XIII.

Of the Five Regular Bodies.

Bodies are either Regular or Irregular.

1. *Irregular*, are those which fall under no Rules.

2. *Regular Bodies*, in general, are those which are finished by such Superficies, of which the Rules are definite, and may be comprehended by certain Species or Kinds.

Of these, some are less Principal, but others more Principal, and Regular.

3. *The less Principal*, are those which consist of divers unlike Superficies, either Plane or Spherical, as Pyramids, Prismes, Cylinders, and Cones.

4. *The*

Chap. XIII. Of Regular Bodies.

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4. *The Principal Regular*, are so called, because all the Planes are Equal, Equilateral, and Equi-angled.

5. Of such, there are found but 5 Planes in the whole nature of things; to wit: 1. *The Tetraedron*. 2. *The Cube*. 3. *The Octaedron*. 4. *The Dodecaedron*. And 5. *The Icosaedron*.

6. *The Tetraedron*, is comprehended under four Triangles: *The Cube*, under 6 Squares: *The Octaedron* under 8 Triangles: *The Dodecaedron* under 12 Pentagons: *The Icosaedron* under twenty equal Triangles; all Equilateral and Equiangular: See Fig. 46, 47, 48, 49, 50.

7. *And the Sphere*, is the receptacle of all these, in which each of them being inscribed, every one of their Angles touch it; but being Circumscribed, each of their Planes touch at the Centers.

C H A P. XIV.

Of Axioms and Petitions of Geometricians.

I. Petitions.

1. **T**HIS required to be granted, that a *Right Line* may be drawn from any Point to any Point.

2. That a *Streight Line* may be infinitely continued.

3. Also that a *Circle* may be described from any *Center*, and with any *Interval*.

4. Likewise to any *Magnitude* being given, we may assume another *Magnitude*, either greater or lesser.

5. That

5. That any *Magnitude* may be so often Multiplied, until it exceed any *Magnitude* of the same kind.

II. A X I O M S.

1. **T**Hose Things or Quantities which are equal, or unequal to the same, are equal or unequal between themselves.

2. If equal things be added to, or taken away from equal things, the Total also, and the Remainder will be equal.

3. If equal things are added to, or taken from unequal, the whole also, and the remainder are unequal.

4. Those Things or Quantities which are the half, or the double of the same Quantity, and which agree mutually between themselves, are equal between themselves.

5. The Whole is greater than its Part, and is equal to all its Parts taken together.

The End of the First Part.

OF GEOMETRY.

PART II.

Of CYCLOMETRY.

THEOREMS.

I. **A** Circle, of all Plane Figures, is the Chief or Prime, the most Simple, the most Perfect, and the most Admirable.

Chief, because 'tis not compounded of others, Prior to it, either Figures, or Parts of Figures; as a Triangle from 3 Lines, and a Quadrangle from two Triangles.

Most Simple, because 'tis comprehended by one Line only, every way conform to it self.

Most Perfect, because it wants beginning and end, unless that humane weakness be attributed to it: and since it gives the Place and Measure to all others, it is not Circumscribed and Measured, but of it self.

Most Admirable, for many other causes, as well as for because the parts of the Diameter of the Circle being equally moved about the Center, dispatch or finish divers Spaces in the same time, the least, next the Center, the greatest, more remote from the same.

II. A Circle, is the most Capacious of all Plane Figures.

What *Isoperimetral* Figures are, is said before, to wit, which when they are divers, yet they have equal Ambit or Periphery, of which the Terms taken together are equal, as if the three Sides of a Triangle taken together, are equal in length to the four Sides taken together of a Quadrangle.

There-

Therefore since between Regular Figures, Planes, many Sided Figures; a Figure of most Sides is most Capacious, as we shall shew hereafter in the Fifth Part; a Circle must needs be the most Capacious of all other Figures of the same Ambit: which hath no Sides, yet may be understood to have infinite Sides.

III. *If in a Circle, some Right Line cut any other in the middle, and at Right Angles; the middle Point of the Secant, or Cutting Line, is the Center of the same Circle.*

In Fig. 52. As because the Line db , cuts the Line fg , in two in the middle, at Right Angles, therefore also it cuts the whole Circle in the middle, and is the Diameter of the Circle, whose middle point a is the Center.

IV. *A Circle cannot cut a Circle in more than two Points, and cannot touch it in more than one Point.*

This is evident, if any one shall describe one Circle thro' another, or join one contiguous to the other; And the cause seems to be the perfect Roundity, by reason of which, all the Points bend from each other: And by the same reason also a Circle can touch a Line or perfect Plane, but in one Point only.

V. *If two Angles insift of the same parts of the Periphery in a Circle; one at the Center, the other at the Periphery; that which is at the Center is double of the Angle at the Periphery; and this subduple of that, or an half part.*

Example.

In Fig. 53. The two Angles acb , described from the Center c , and adb from the Periphery, insift on the same Periphery acb ; I say the Angle c is double of the Angle d , and this subduple, or an half part of that; which Euclid demonstrates, Book 3. Prop. 20.

There-

Therefore of whatsoever part of the Periphery, any Angle drawn from the Center insists, 'tis always double of the opposite Angle, and this subduple of that, and an half part; whence also it follows, that,

VI. *Angles at the Periphery, which are in the same Segment, are all equal between themselves.*

For they insift on one and the same part of the Periphery, of which they are half parts; and those things which are halves of the same, are equal between themselves.

VII. *Angles likewise which insift on the Diameter, are all Right Angles.*

For they insift of half the Periphery, which is 180° , of which since they are subduple, they must needs be 90 Degrees, and therefore Right Angles.

VIII. *But if two Plane Triangles, be inscribed in the same Segment of a Circle, on the same Base, and Joined at Top, that from thence a four sided Figure be made, intersected by Diagonals; the Rectangle of the Diagonals is equal to the Rectangles taken together of the opposite sides.*

As in Fig. 54. there are two Triangles abd , and acd , in the same Segment of the Circle $abcd$, inscribed on the same Base ad , and joined at top by the Right Line bc : The Rectangle of the Diagonals ab and cd , is equal to the Rectangles of the opposite Sides ac and bd , also to bc and ad taken together, as Pitiscus hath demonstrated in his *Trigonometry*, Book 1. Pag. 21.

IX. *The Semidiameter of a Circle, is the side of an Hexagon, inscribed in the Circle.*

The Demonstration of this Theorem is evident from two equal Circles, one of which passes through the Center of the other: for the Semidiameter's being drawn from the Centers, the Semidiameter of one, will appear to be the side of an Hexagon of the other, inscribed in the Circle. See *Euclid*, Book 4. Prop. 15.

X. The

X. The Diameter, the Equilateral Triangle, the Quadrate, the Pentagon, the Hexagon, the Pentagonal, and Octogonic Star, inscribed in a Circle, cut it Harmonically. See Fig. 55, 56, 57, 31, 32, 61, 60.

For if a Circle by imagination be conceived, as it were, a String stretched out in an Instrument or Lute; and in the first place understand it to Vibrate, the whole being free from all Contact, but afterwards, as it were toucht by the Finger, *One Half Part*; Secondly, *a Third*; Thirdly, *a Fourth*; Fourthly, *a Fifth*; Fifthly, *a Sixth*; Sixthly, moreover *Three Eighth Parts*; and Seventhly, *Two Fifths*: To wit, like as, 1. The Diameter. 2. The Equilateral Triangle. 3. The Quadrate. 4. The Pentagon. 5. The Hexagon. 6. The Octogonic Star: And 7. The Pentagonal Star, cut the Circle: All these Sections shew Harmony, or the Agreement of Musick; to wit, $\frac{1}{2}$ Diapason or Octava, Quinta, Diapente, $\frac{2}{4}$ Quarta, Diatessaron, $\frac{4}{3}$ Tertia Major, $\frac{3}{4}$ Tertia Minor, $\frac{5}{6}$ Sexta Minor, $\frac{3}{2}$ Sexta Major.

And these are thought to be the genuine causes of Harmonical Proportion, or Musick, because the Sides of Figures, cutting the Circle in the said manner, have a notable Prerogative beyond others, to wit, *Scientifical Demonstrability*, with respect to the Diameter of the Circle; since the Terms of the Disagreeing Intervals of Musick, are either not hifing, or making a Sound, or unfitly making a Sound; therefore they conspire together friendly Sustainance to the Mind, and the Ears.

II. The Diameters of Circles, or the Semidiameters, are to one another, as their Circumferences, and the Contrary.

For since a Circle is described by the Circumvolution of the Semidiameter, one Extream being fixt,

'tis manifest, that those Circles are equal, of which either the Semidiameters, or the whole Diameters are equal : But the Diameters, or Semidiameters being unequal, makes also unequal Circles, lesser or greater, &c.

XII. *The Areas of Circles are to one another, as the Squares of their Diameters, and the contrary.*

XIII. *The Proportion of the Periphery of any Circle to its Diameter is $3 \frac{1}{7}$, very near ; that is, Triple Sesquiseptima.*

The least Terms of this Proportion, are, 7 and 22 ; the next lesser, but nearer the Truth, are, 113 and 355.

XIV. *The Proportion of the Square of the Diameter to the Area of the Circle, is $1 \frac{1}{11}$ very near, that is, Supertripartiens undecima.*

The least Terms of this Proportion, are 14 and 11.

XV. *The Proportion of the Square of the Circumference of a Circle, to the Area of the same, is very near $12 \frac{1}{7}$, that is 12 times, and four Seventh Parts over and above.*

In the least Terms, as 88 to 7.

XVI. *The Area of any Circle is equal to a Rect-angle Triangle, one side of which about the Rect-angle is equal to the Semidiameter of the Circle, and the other is equal to the Periphery, or which is the same, to the Diameter, and half the Periphery.*

In Fig. 62. The Area of the Rectangle Triangle CBA, made upon the Semidiameter AB, and the Side BC, which is assumed equal to the Periphery of the Circle, is equal to the Area of the Circle.

X. The Diameter, the Equilateral Triangle, the Quadrate, the Pentagon, the Hexagon, the Pentagonal, and Octogonic Star, inscribed in a Circle, cut it Harmonically. See Fig. 55, 56, 57, 31, 32, 61, 60.

For if a Circle by imagination be conceived, as it were, a String stretched out in an Instrument or Lute, and in the first place understand it to Vibrate, the whole being free from all Contact, but afterwards, as it were toucht by the Finger, *One Half Part*; Secondly, *a Third*; Thirdly, *a Fourth*; Fourthly, *a Fifth*; Fifthly, *a Sixth*; Sixthly, moreover *Three Eighth Parts*; and Seventhly, *Two Fifths*: To wit, like as, 1. The Diameter. 2. The Equilateral Triangle. 3. The Quadrate. 4. The Pentagon. 5. The Hexagon. 6. The Octogonic Star: And 7. The Pentagonal Star, cut the Circle: All these Sections shew Harmony, or the Agreement of Musick; to wit, $\frac{1}{2}$ Diapason or Octava, *Quinta*, *Diapente*, $\frac{2}{3}$ *Quarta*, *Diateffaron*, $\frac{3}{4}$ *Tertia Major*, $\frac{4}{5}$ *Tertia Minor*, $\frac{1}{3}$ *Sexta Minor*, $\frac{2}{5}$ *Sexta Major*.

And these are thought to be the genuine causes of Harmonical Proportion, or Musick, because the Sides of Figures, cutting the Circle in the said manner, have a notable Prerogative beyond others, to wit, *Scientifical Demonstrability*, with respect to the Diameter of the Circle; since the Terms of the Disagreeing Intervals of Musick, are either not hushing, or making a Sound, or unfitly making a Sound; therefore they conspire together friendly Sustenance to the Mind, and the Ears.

II. The Diameters of Circles, or the Semidiameters, are to one another, as their Circumferences, and the Contrary.

For since a Circle is described by the Circumvolution of the Semidiameter, one Extreame being fixt,

'tis manifest, that those Circles are equal, of which either the Semidiameters, or the whole Diameters are equal: But the Diameters, or Semidiameters being unequal, makes also unequal Circles, lesser or greater, &c.

XII. *The Areas of Circles are to one another, as the Squares of their Diameters, and the contrary.*

XIII. *The Proportion of the Periphery of any Circle to its Diameter is $3\frac{1}{7}$, very near; that is, Triple Sesquiseptima.*

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XVI. *The Area of any Circle is equal to a Rect-angle Triangle, one side of which about the Rect-angle is equal to the Semidiameter of the Circle, and the other is equal to the Periphery, or which is the same, to the Diameter, and half the Periphery.*

In Fig. 62. The Area of the Rectangle Triangle CBA, made upon the Semidiameter AB, and the Side BC, which is assumed equal to the Periphery of the Circle, is equal to the Area of the Circle.

PROBLEMS.

I. *To cut a given Circle in the Middle.*

1. **D**raw a Right Line through the Center, terminated at both ends by the Circumference, the Circle will be divided in the middle.

2. Or if no Center appear, draw any Right Line whatever, terminating on both sides at the Circumference, which cut in the middle, then draw another Right Line through the Point of Section at Right Angles, terminating on both sides at the Circumference, which will cut the Circle in the middle into two equal Portions: In the same manner also a part of the whole Periphery, under a Line drawn, joining the Extrems, and cutting it in the middle at Right Angles, may be divided into two equal Portions, as in Fig. 52.

II. *To find the Center of a given Circle.*

A Right Line cutting the Circle into two equal Portions, may be found by the foregoing Problem: again divide it into two equal Parts, and you will have in the point of Intersection, the Center of the given Circle.

III. *To divide a Circle given into four Quadrants Accurately.*

Draw a Right Line through the Center, and it divides the Circle in the middle; then opening the Compasses a little more than the interval of the Semidiameter; fix one Foot in the Extream of the proposed line, and describe, without the Periphery, an occult Arch, the Compasses being at the same distance, set one Foot in the other Extream of the proposed line, and intersect the other by a little occult Arch: If again you draw a line from the point of Intersection through the Center, the Circle will be divided into four equal parts.

IV. A

IV. A Circle being given, and divided into four Quadrants: To divide it into 12 parts, the Compasses remaining at the same distance, whereby the Circle was described; and it being divided into 8 parts, you may divide it into 24, with the Radius of the Circle, or the same distance.

I. The division of the Circle into 12 Parts.

1. In Fig. 63. let the 4 Quadrants of the Circle be *ABCD*, numbring from the top towards the left hand.

2. Then taking the Interval of the Semidiameter, begin from *A*, and proceed towards *B*, through the whole Periphery, noting the divisions.

3. Then begin from *B*, and proceed towards *C D*, beyond *A*; and in like manner, if you will, you may affix the Notes of Division, and the Circle will be divided into 12 parts.

II. The Division of the Circle into 24 Parts.

1. Again, in Fig. 64, the Circle is divided into 8 parts, numbring from the top towards the left hand, *abcdefgh*.

2. Taking the Semidiameter of the Circle, begin first from *a*, and proceed towards *b c*, through the whole Circle; then begin from *b*, then from *c*, also from *d*, and again and again go over the whole Circle, always leaving the marks of Division; so at length you will divide the Circle, (the Compasses remaining at the same distance) into 24 parts.

V. From three Points given, not being in the same Right Line, to describe a Circle, that shall pass thro' the three given Points.

From the first Point to the second draw a Right Line, also from the second to the third; divide both these Lines in the middle at Right Angles, by Lines produced to any length: The Point of intersection of these two lines will shew the Center of the Peri-

phery of the Circle passing through the three given Points.

56 VI. *To find the Center of any Arch, and from it to finish the Circle.*

In the aforesaid manner, assume in the given Arch 3 points, join them by Right Lines, cut the Right Lines in the middle at Right Angles, the point of intersection of the two Secants produced, is the Center of the given Arch, from whence it, and also the whole Circle may be described.

57 VII. *In a Circle given, to describe a Square.*

Draw the two Diameters, cutting each other at right Angles in the Center; join the Extrems of the Diameters, and a Square will be inscribed in the given Circle.

60 VIII. *To describe a Circle in a given Square.*

Divide the Sides of the Square in the middle, and join each opposite point of intersection by right lines, the point of intersection of these two right lines will be the Center of the Circle to be inscribed, to touch the sides of the Square.

IX. *To describe a Square about a given Circle.*

Draw in the Circle the two Diameters, cutting each other in the Center at right Angles; through the Extrems of each Diameter make Perpendiculars touching the Circle; the meeting of which Perpendiculars, will make the Square circumscrip to the Circle.

61 X. *To describe a Circle about a given Square.*

In the Square, draw two Diagonals, cutting each other at right Angles, from the point of Intersection, with the Interval of half the Diagonal describe a Circle, 'tis that you sought.

XI. *In a given Circle to describe an Æquilateral, and Æquiangled Hexagon.*

Assume, or retain the Semidiameter (which is the side of an Hexagon) it will divide the Circle into
into

into six parts; join the points of Division, and you will have the Hexagon required.

XII. *To inscribe an Equilateral Triangle in a given Circle.*

Divide the Circle into six parts, and omitting one intermediate point of Division, join the rest by right lines, and you will have an Equilateral Triangle inscribed in a Circle.

XIII. *To inscribe an Equilateral and Equiangular Pentagon, and in the same manner also a Decagon, in a given Circle.*

Fig. 65. From the Center e describe the Circle $abcd$, draw the Diameter bd , and the Perpendicular ae ; cut the Semidiameter eb in the middle, in f , make the Interval fa in the Diameter bd from f , and let this interval be fi : the right line ia , will be the side of a Pentagon, and ei , the side of a Decagon inscribed in the Circle: The right line then ia , being five times assumed, and ei , ten times in the Periphery, will comprehend points, which if you join by right lines, will be the Pentagon and Decagon desired.

But we must note in this place, that to inscribe many Angled Figures in a Circle, we shall inscribe in the same Multangular Figure, another which hath doubly more Sides and Angles, if we divide the Arches which the Sides subtend in the middle: So in an inscribed Square, we may inscribe an Octagon; in a Sexagon, a Duodecagon; in a Decagon, an Icosagon.

XIV. *In a Circle given, to inscribe, and Circumscribe any Regular Multangle.*

Divide the Circle into 4 Quadrants, subdivide one Quadrant into so many equal parts, as the Multangle to be inscribed hath Sides and Angles: Example, Into 7 parts if you would inscribe a Septangle, into 8 parts if an Octangle: And a right line, which subtends 4 of the same parts, will be the side of the Multangle to be inscribed: For of how many parts

of the Quadrant one of those is, into which the Quadrant is divided, so many is the Arches of Four of such parts of the whole Periphery.

But if in a Circle, any Regular Multangle whatever be inscribed in this manner, and at the Extreams of the Diameters, Perpendicular lines be raised from the Center to the Angles of the inscribed Figure, these Perpendiculars joyned, will constitute a Regular Multangle Figure, of the same number of Sides and Angles, circumscribing the Circle.

To be Noted concerning the Heptagon, or 7 Sided Figure.

Although the Heptagon may in some reason be inscribed in a Circle Mechanically, nevertheless it wants a Geometrical Description, as well without the Circle as within; since the quantity of the Sides, in respect either of the Diameter of the Circle, or of any other Figure, (although necessary) is yet unknown: Neither, as *Kepler* says in his *Harmonicks*, can it be known by Humane Understanding: Therefore, says he, no one at any time knowingly and willingly, hath required of any one the Construction of a Regular Septangle from any thing proposed, neither can it be made from any thing proposed, but by meer Chance; and yet 'tis necessary to doubt, whether it may be constructed, or not?

XV. *To determine the Sides of all Regular Planes, in Parts of which the Semidiameter of the Circle is 100000, in a Plane circumscribed of the Circle.*

With the side of the Figure to be inscribed in a Circle, enter the following Table, which comprehends the Proportion of the Sides, with respect to the Semidiameter, from a Triangle even to one of 20 Angles: for the side being found on the left hand, over against it, is the Quantity of the same, in respect to the Semidiameter of Parts 100000.

Examp.

Example. Of how many such parts will the side of a *Pentagon* be, to be inscribed in a Circle? *Ans.* To the number of Sides of Figures over against 5, which is the Note of a *Pentagon*, answers 117557; therefore so many parts of the Semidiameter of Parts 100000, will the side of a *Pentagon* have, which ought to be inscribed in a Circle of this Diameter.

A Table of the Quantity of Sides, in respect of the Semidiameter of Parts 100000, from the Triangle, even to one of 20 Angles, or Sides.

| Numb. of Sides. | Quant. of Sides. | Numb. of Sides. | Quant. of Sides. |
|-----------------|------------------|-----------------|------------------|
| 3 | 173205 | 12 | 51764 |
| 4 | 141422 | 13 | 47863 |
| 5 | 117557 | 14 | 44503 |
| 6 | 100000 | 15 | 41582 |
| 7 | 86776 | 16 | 39018 |
| 8 | 76536 | 17 | 36750 |
| 9 | 68404 | 18 | 34729 |
| 10 | 61803 | 19 | 32918 |
| 11 | 56346 | 20 | 31268 |

The way of Making and of Continuing this Table, is thus.

1. Divide 360 Degrees of a Circle, by the number of Sides of the given Figure: Example, by 4, the Sides of a Quadrate or Square, it makes 90.

2. Seek the Sine of half of the Quotient, viz. of 45, which is 70711.

3. The double of it, 141422, shews the side of the Square sought; in parts, of which the Semidiameter is 100000.

XVI. *The Side of any Figure being assumed, of Parts 1000; To find the Semidiameter, or Radius of the circumscribing Circle in the same Parts.*

Enter the following Table, and seek the number of the Sides of the Figure (in the first Column) to which the Circle is to be circumscribed, over against which answers the Quantity of the Semidiameter in the same parts.

As if we were to find the Semidiameter of a Circle to circumscribe an *Heptagon* : If the side of the *Heptagon* be supposed, or put of Parts 1000 ? To the Number of 7, the side of an *Heptagon*, over against it we find 1152, the quantity of the Semidiameter in those parts.

A Table of Radius's of a Circle, circumscribing Regular Figures, assuming the Sides of any Figure, of Parts 1000.

| Number of Sides. | Quantity of Radin. | Number of Sides. | Quantity of Radij. |
|------------------|--------------------|------------------|--------------------|
| 3 | 577 | 12 | 1932 |
| 4 | 707 | 13 | 2089 |
| 5 | 850 | 14 | 2247 |
| 6 | 1000 | 15 | 2405 |
| 7 | 1152 | 16 | 2563 |
| 8 | 1307 | 17 | 2721 |
| 9 | 1462 | 18 | 2879 |
| 10 | 1618 | 19 | 3038 |
| 11 | 1775 | 20 | 3196 |

The manner of making and continuing this Table, is thus.

1. Take from the Table of Problem XV, the Quantity of the Side of any Figure, to a *Radius* or Semidiameter of Parts 100000 ; to wit, of a Square Figure, the quantity of whose Side is 141422.

2. Then make it as the $\left\{ \begin{array}{l} \text{To the Radius} \\ \text{Side of the Square 141422} \end{array} \right\}$ of Parts 100000 $\left\{ \begin{array}{l} \text{So the Side of a} \\ \text{Square 1000.} \end{array} \right.$

To the *Radius* 707, omitting without sensible Error, the adhering Fractions.

XVII. *A Circle being given, to Double or Triple it, or to increase or Diminish it, in any Multiplex Proportion.*

If you will proceed in a Geometrical Proportion, continually double.

1. Take the subtense of the first Quadrant of the Circle to be increased, for a Semidiameter, whereby describe a Circle doubly greater to the first given: Of this double Circle, assume the subtense of the Quadrant, instead of a Semidiameter, it will give a Circle 4 times as great as the first: If you make the subtense of the Quadrant of this, a Semidiameter, it will make a Circle 8 times as great as the first; and so you may proceed in *infinitum*.

But in returning or going back again, take the Semidiameter of the greatest Circle, instead of a subtense of a Circle doubly lesser, or as little again; from which subtense you may easily discover the Semidiameter of this Circle doubly lesser, in the Semidiameters of lessened Circles, cutting each other at Right Angles in the Center: And so also you may do, for finding Circles 4, and 8 times lesser.

But if you would ascend in Arithmetical Progression, in Double to Triple, Quadruple, Quintuple, &c.

1. You may let the point of the Compass, remain in the extream point of the Semidiameter of the first Circle, *a*, in Fig. 66. from which, if you take the subtense of the Quadrant *a c*, and transfer it into the line *b l*, you will have *b d*, the Semidiameter of the double Circle.

2. Then if you transfer the subtense *a d*, into the same line *b l*, you will have *b e*, the Semidiameter of a Circle Threefold greater than the First given.

From *a e*, if you make *b f*, you will produce the Semidiameter of a Circle Fourfold greater than the first

first given, and so on; you may ascend to Quintuple, Sextuple, &c.

In going back again, the Ratio is reverse, as if from the Semidiameter bf , of a Circle Fourfold greater than the first given, you would inquire the Semidiameter of a Circle only Triple of the first: Take with your Compasses the Semidiameter bf , and make from it the subtense ae , and it will be be , the Semidiameter of a Circle, only Triple greater of the first given: Then from the Semidiameter be , make the subtense ad , and you will have db , the Semidiameter of a Circle only doubly greater than the first given, &c.

67 XVIII. To constitute or make a Rectangle Triangle, equal to a given Circle. Fig. 67.

Divide the Diameter into seven equal parts, and at the Extream of it, B , raise the Perpendicular BC , whose length must contain the Diameter three times, and over and above a seventh part of the Diameter: If then you join either the Extream of this Perpendicular C , with the Center A ; or which is the same, the half of it, or the middle point, with the other Extream of the whole Diameter, as the prick line shews, you will have a Rectangle Triangle, equal to the given Circle, very near.

XIX. To Square the Circle; or to make a Square equal to a Circle.

Because the Rectangle Quadrangle, contained under half the Periphery and the Semidiameter, is equal to the Circle: If then we find a mean Proportional, between the Right Line which is equal to half the Periphery, and the Radius or Semidiameter of the same Circle, it will be the side of a Square equal to the Circle.

If

If the *Area* of the Circle be given in Numbers, the Square Root of them, will be also the side of a Square equal to the Circle.

XX. To describe a Circle equal to a given Square.

In Fig. 68. Let the Square given be $abcd$, in which the Diameters ad and bc , cut each other mutually in the Center of the Square; then cut off $\frac{1}{4}$ part of the side dc from d , which let be df ; the remainder fe is the Semidiameter of a Circle, equal to the Square $abcd$, *proxime*.

XXI. The Diameter of a Circle being given, to find its Circumference.

Make it as 7 to 22, or more near the Truth, as 113 to 355: For Example, so 14 to 44 the Circumference, yet a little greater than the Truth; but by the last Proportion, 'tis $43\frac{11}{13}$, being yet a small matter too great.

XXII. The Circumference given, to find the Diameter.

Make it as 22 to 7, so the Circumference given: For Example 44, to the Diameter sought 14, which is somewhat less than the Truth.

XXIII. Given the Diameter of a Circle, to find its Area.

Make it as 14 to 11, so for Example, the Diameter 7, its square is 49, (which is had by drawing the Diameter into it self) To the Area of the Circle $38\frac{1}{2}$, a little too much.

XXIV. By knowing the Periphery of the Circle, to find its Area.

Suppose the Periphery, for Example, 22, its Square 484; and make it as 88 to 7, so the Square of the Periphery 484 to the Area sought, $38\frac{1}{2}$ *proxime*.

XXV. Given

XXV. *Given the Semidiameter and Circumference of a Circle, to find the Area.*

Draw half the Circumference into the Semidiameter, or half the Semidiameter into the whole Circumference, and it produces the Area of the Circle; as for example, if the Semidiameter be 7, the Semicircumference 22, the Area of the Circle will be 154.

XXVI. *Given the Area of a Circle, to find its Diameter.*

Make it as 11 to 14, so the Area 154 to the Square of the Diameter 196, whose Square Root 14 will be the Diameter sought, *verà paulò minor.*

XXVII. *From the Area of a Circle given, to find its Periphery.*

Make it as 7 to 88, so the Area of the Circle 154, to the Square of the Periphery 1936, whose Square Root 44, will be the Periphery, *Paulo tamen major vera.*

For the true and exquisite Proportion of the Circumference of a Circle to its Diameter, to wit, of a Right Line, and a Crooked, is not as yet found out, nor perhaps will ever be found; wherefore what we have delivered hitherto, we are content in this place with the Proportions Invented and Demonstrated by *Archimedes*, and others.



O F
G E O M E T R Y.
P A R T I I I.

O f T R I G O N O M E T R Y.

The P R E F A C E.

Of the Canon of Sines.

1. **T***rigonometry*, is the Doctrine of Solving or Measuring of Triangles.

2. An help to it, is the *Canon of Sines*, whereby assuming in a Circle a *Radius*, or Semidiameter of certain parts, the Quantity of the Sines, and also of the Tangents, and the Secants, and Versed Sines, are assigned to every Arch of one Quadrant; to this end, that from or by these, the Angles and Sides of Triangles being known, we are brought to the knowledge of others unknown, by the benefit of the *Golden Rule of Proportion*.

3. *In this Canon two things are to be regarded.*

1. Its *Structure* : And 2. Its *Use*.

4. *The Structure*, is easie, if we have the known Proportions, which the sides of Rectangle Triangles make between themselves; also of Plane Figures inscribed in a Circle, of the *Trigon*, *Tetragon*, *Pentagon*, *Hexagon*, *Decagon*, &c. with the Semidiameter: for such sides are subtenses of Arches, the half of which, constitute the Sines of half the Arches; from one of which many may be had, as *Pitiscus* plainly teaches in many places in his *Trigonometry*.

5. *The*

5. The Use of the Canon of Sines in General is:

1. That either from an Arch given; which being sought in the Extrems of the Canon, the Sines, Tangents, Secants, and Versed Sines, may be had every one under their Titles.

2. Or from given Sines, Tangents, or Secants, being sought and found in the Area of the Canon, is found the Arches answering to them.

6. Where notwithstanding we must Note.

I.

1. The Canon of Sines is only made to the Arches of a Circle less than a Quadrant, or to Acute Angles.

2. That if therefore there be offered an obtuse Angle, or an Arch greater than a Quadrant, or 90° , it must first be taken from 180° , or which is the same, we must take its Complement to a Semicircle, and with this find the Sine, Tangent, or Secant, which will also be the Sine, Tangent, and Secant of the obtuse Angle proposed, or of an Arch greater than a Quadrant.

Example.

| | | | | | | |
|-------------------------------|-------|-----|---|----|---|-----|
| An Arch being proposed of | _____ | 125 | : | 13 | : | 15. |
| For it, I take its Complement | _____ | 54 | : | 46 | : | 45. |
| Or from 180 , or | _____ | 179 | : | 59 | : | 60. |
| I take away | _____ | 125 | : | 13 | : | 15. |
| And there is left the Arch of | _____ | 54 | : | 46 | : | 45. |

II.

1. Sines, Tangents, and Secants also, are sometimes computed to First and Second Scruples, but some Canons are content only with Degrees.

2. And if therefore such a brief Canon be offered to you, and you would use also Scruples, as well Primes as Seconds in your Calculation, you must seek the part Proportional, and that in two Cases.

1. If

1. If to an Arch given, you are to seek the Sine, Tangent, Secant, or Versed Sine.

The Process is thus.

Let us seek the right Sine to a given Arch of $10^{\circ} : 15' : 30''$.
Take the Sine of the Arch, next greater, 11° , which is 19081.
Likewise the Sine of the next lesser Arch, 10° , which is 17365.
The difference of these is 1716 .

Say then by the Golden Rule.

If 11° make 19081, what makes $15' : 30''$.
Or reducing the Extreams to the least Terms,
If 3600' give 1716, what gives 930'
The operation being made, it produces 443, or 444 almost.
Which added to the Sine of 10° 17365,
Makes 17809.
The Sine of the Arch sought $10^{\circ} : 15' : 30''$

2. If to a right Sine given, we seek the Arch answering to it.

Proceed thus.

| | | | |
|------------------------------------|----------------|--------|-------------------|
| The Sine next greater to the given | 11° . | 19081. | The given, 17809. |
| The next lesser | 10 | 17365. | 17365. |
| The difference | 1716. | | The differ. 444. |

Say then:

If 1716 give 1° , or 3600', what 444? The operation being made, it produces 931', or $15' : 30''$, to be added to 10° , that you may have the Arch answering to the Sine given.

But 'tis not always necessary that we should be troubled with second Scruples, but in most Calculations we may be content with Minutes, or First Scruples: And what in this Example is said of Sines, the same also is to be understood concerning Tangents, and Secants, and Versed Sines.

III.

Versed Sines are seldom added to the Tables of Sines, (yet to save trouble I have added them) because

because they may be had from the Table of Sines with a great deal of ease : For,

1. *The lesser Versed Sine of an Arch less than a Quadrant, may be had ;*

If you take away the Sine of the Complement of the given Arch from the Radius, or whole Sine.

2. *The greater Versed Sine, being an Arch greater than a Quadrant may be obtained.*

If to the Radius you add the Excess of the right Sine, to wit, whereby the given Arch exceeds a Quadrant.

Thus much of the Canon of Sines, now follows

THE Doctrine of Trigonometry. PART III.

Of TRIANGLES.

THEOREMS.

Of Triangles in General.

1. **A**LL Right Angles are equal between themselves, to wit, each to 90° .
2. When a Line is insisting on a Line, or cutting the same ; it makes Angles that are either Right, or equal to two Right Angles, as in Fig. 12 and 13.

In

In this Scheme, the Perpendicular Line, with the subjected Line, makes two Right Angles; but the Line falling obliquely makes one Acute, and the other Oblique, and both equal to two Right Angles: For by how much one wants of a Right Angle, by so much the other exceeds the same Right Angle.

3. *Angles at the Vertex, or opposite by the Cross, are equal between themselves.*

As in Fig. 14. the Angles $b c d$, and $a c e$: Also $f e h$, and $c e g$ are equal. 14

4. *Angles whose Legs agree, are equal between themselves.*

As if an Angle be placed, or put on an Angle, and each of the Legs so agree, that never a one bends or deflects from the other, although perhaps one may be longer than the other, those Angles are accounted equal.

5. *In any Triangle whatever, a Perpendicular let fall from an Angle contained by equal Legs, cuts both the Base and the Angle in the middle, or into two equal parts.*

In Fig. 24, in the Equilateral Triangle $d e f$, the Perpendicular $e c$ being let fall on the Base $d f$, cuts it in the middle. 24

6. *In all Triangles, two sides taken together are greater than the third, or remainder.*

Therefore if any one commands a Triangle to be made, whose sides contain 3. 4. 8. it may be perceived to be impossible.

7. *In every Triangle the greater side subtends the greater Angle, the lesser side the lesser Angle, and equal sides subtend equal Angles.*

For by how much greater or lesser the divarication of the sides are, that constitute the Angle, from whence the Angle is made greater or lesser; by so much greater or lesser also the subtense is made.

N

8. Hence

8. Hence every Equilateral Triangle is also Equi-angled, and the contrary.

9. But an Isosceles, or Equicrural Triangle hath the Angles at the Base equal.

Of Plane Triangles.

10. If a Right Line falls on Parallel Lines, it makes like Angles, and being scituate in like manner, or alternate, it makes equal Angles, and the contrary.

124 So in Fig. 14. Like Angles, and alike scituated, are the Angles bcd , and ceg ; and alternately scituated are ace , and ceg ; and as these, so also those, are equal between themselves.

11. If Right Lines are cut by Right Lines that are Parallels, the Intersegments are Proportional.

72 In Fig. 72. The two Right Lines ab , and ac , being intersected by the Parallel Lines de , and bc ; The Intersegments then, ae and ad , also db and ec are Proportional: that is, as ad , to ab , so ae to ac . And as ae to de , so ac to bc .

12. Hence, in a Plane Triangle, a Parallel to the Base, cuts the Legs Proportionally.

As if in the Triangle bac , you take bc for the Base, and a Parallel to it de , these Legs ad , and ae , will be cut Proportionally.

13. Equiangled Triangles have their Sides about the equal Angles Proportional, and the contrary.

Example, Fig. 72.

72 Let there be two Triangles ade , and abc , equi-angled, so that the Angles at d and b , likewise at a and a , and at e and c are equal: I say the sides of them about the equal Angles are Proportionals, that is, they be:

As ad to de ; so ab to bc . And as ad to ae ; so ab to ac .

And as ae to de ; so ac to cb , &c.

14. If

14. If many Plane Triangles are compounded, and intersected by Parallel Right Lines, the Segments are Proportional.

For Example in Fig. 73, the two Triangles eaf , and fag are compounded, and are intersected by the Parallel Right Lines bcd , and efg ; those Intersegments are Proportional: and it is,

As bc to ef , so cd to fg .

As bc to cd , so ef to fg .

15. If of any Plane Triangle whatsoever, the Side be continued, the outer Angle made by that continuation, is equal to the two inward opposite Angles, as in Fig. 74.

If the side ac of the Triangle abc be continued in d ; the outer Angle bcd , will be equal to the two inner opposite Angles bac , and abc : for the Angle ecd is equal to the Angle bac , for they are situate alike between Parallels.

Also the Angle bce , the other part of the Angle bcd , is equal to the Angle abc , because alternately situate to it; whence it follows also, that the whole Angle bcd , is equal to the two opposite Angles, bac and abc .

16. Every Plane Triangle hath three Angles equal to two Right.

For the Angles bcd and bca , are equal to two Right, therefore since they are also equal to the three Angles in the Triangle abc , it must necessarily follow, that these three Angles also are equal to two Right: For bca is a common Angle, and equal to it self, but bcd is equal to the Angles abc and bac , as appears by the precedent Problem; therefore the Triangle abc hath three Angles equal to two Right: Because those things which are equal to one and the same, are also equal between themselves.

17. Hence, in no Plane Triangle, can there be more than one Right or Obtuse Angle.

For otherwise, the Angles would exceed two Right Angles, which is contrary to the Antecedent Problem.

18. And one Angle being Right or Obtuse, the other two must needs be Acute.

19. And either of the two Acute, is the complement of the third to two Right Angles.

20. Moreover, if two Triangles have pairs of Angles equal, they are wholly equal.

For if you take both the pairs of equal Angles from 180, the remaining complement of both the Triangles must needs be equal, according to the Rule, viz.

When Equal things are taken from Equals, the Remainders are also Equal.

21. In a Rectangled Plane Triangle, the Squares of the Sides including the Rectangle, taken together, are equal to the Square of the Hypotenuse.

Example.

75 Fig. 75, In the Triangle abc , Right Angled at b ; I say the sides ab , and bc , including the Right Angle, if of them you make Squares; equal to those Squares is the Square of the Hypotenuse ac : As if the side ab , be 3 Feet, its square will be 9 Feet; and if the side bc , be 4 Feet, its square is 16 Feet; which 2 Squares taken together, make 25 square Feet, and is as much as the square of the Hypotenuse: For if it be 5 Feet, its square also will be 25 Feet, as the two former taken together: The Demonstration of this most noble Theorem, you may see in *Euclid*, Book 1. Prop. 47, and also in *Pitiscus*.

22. In a Rectangle Plane Triangle, one of the Acute Angles, is the Complement of the other to a Quadrant.

For the Right Angle is always 90° , if therefore one of the Acute be 36° , the other of the Acute Angles

OF TRIANGLES. 181

gles must needs be its Complement to a Quadrant, that is 54° ; to wit, that the 3 Angles together, may be made equal to two Right Angles, or 180° .

23. *The Sides are to one another, as the Sines of their opposite Angles.*

For since the sides are to one another, as the subtenses of their opposite Angles: But they make the subtenses of the opposite Angles, if you circumscribe a Circle about the 3 Points of the 3 Angles, they will be also to one another as the halves of their subtenses, that is, as the Sines of the opposite Angles; for that which is the *Ratio* of the whole to the whole, the same also is the *Ratio* of the half to the half, as nature it self Dictates; for Example, The *Ratio* of 10 to 6, is also the same as 5 to 3.

24. *As the Sum of two Sides, to the difference of the same, so is the Tangent of half the Sum of the two opposite Angles, to the Tangent of the Difference, beneath or above the half.*

Fig. 76. As in the Oblique Triangle abc , if it be,
 The side ac , 5. } The Sum of the Angles at a & b , 140° .
 The side bc , 3. } The Half ————— 70° .
 The Sum — 8 } The Tangent of this Half, is 274747.
 Difference, 2. } I lay then.

As the Sum } to the Dif- } So the Tangent of the
 of the sides 8. } ference 2. } half, 70° , being 274747.

To 6868593, the Tangent of the Arch $34^\circ : 28' 59''$; which is the Difference of the Angle a , beneath, and b , above the half:

Whence if from the half $70^\circ : 0' : 0''$
 We Subtract ————— $34 : 28 : 59$

It leaves the Angle at a $35 : 31 : 1$

N 3

But

Likewise the Leg bc is a mean Proportional between the Base ac , and the conterminatè Segment ec .

Of Sphærical Triangles.

27. *The Sides of Sphærical Triangles are Arches of the greatest Circles, dividing the whole Sphere into two Hemispheres, each less than a Semicircle.* Fig. 77, 78 and 79.

28. *If a great Circle of the Sphere pass through the Poles of a great Circle, it will cut it aright, or at Right Angles.*

In Fig. 80. The Arch of the Circle ab , cuts the Arch of the Circle gb rightly, the Pole of which is the Point a .

29. *The Measure of a Sphærical Angle, if it be taken in a great Circle, is an Arch of a great Circle described from the Angular Point between the Intercepted Legs in some measure continued : See Fig. 80.*

So the Measure of the Angle cab , is the Arch gb , distant from the Angular Point by a Quadrant.

30. *If the Legs of a Sphærical Triangle being continued, do concur or meet, they make Semicircles, and they comprehend an Angle equal and opposite to the aforesaid Angle.*

As the Legs of the Angle bac , being continued to d , make the Semicircles abd , and acd , and comprehend the Angle bdc , equal to the Angle bac , because the same Arch gb measures both the Angles, a and d .

31. *Any Sphærical Triangle, from any Angular Point, hath another Triangle opposite to it, whose Base is the same, and the Angle opposite to the Base ; the remaining parts are the Complements of the Parts of the former Triangle.*

So in the Triangle bac , from the Angular Point a , it hath the Triangle bdc opposite to it self, whose Base is the same bc , and the Angle opposite to the Base bdc ; But the sides bd and cd are complements of the Sides ab and ac to a Semicircle: And the Angles dbc , and dcb , are complements of the Angles abc , and bca , to two Right, or 180° .

32. A Rectangled Spherical Triangle, with two Acute Angles, hath from the Right Angle, a Rectangle Triangle, opposite to it, with two Obtuse Angles; and the contrary.

As if the Angles at a and d are supposed to be right, it hath the Rectangle bac , with two Acute at b and c ; and the opposite Rectangle bdc , hath two Obtuse Angles at b and c .

33. Rectangle Spherical Triangles, with two Obtuse Angles, two of the Sides are greater than Quadrants, as bd and dc , and the third less than a Quadrant, as bc , in the Triangle bdc .

34. In a Rectangle Spherical Triangle, with an Acute and an Obtuse Angle, it hath opposite from the Acute Angle, a Rectangle Triangle, with two Acute Angles.

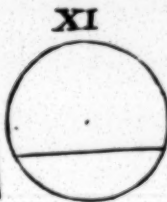
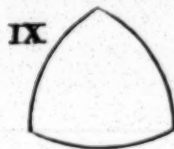
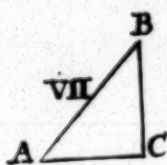
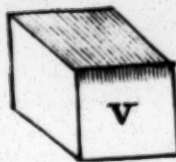
As in the Rectangle Triangle cde , with the Acute ecd , and the Obtuse ced , it hath the opposite Triangle Rectangled edf , with two Acute at e and f .

35. Spherical Triangles having more Right Angles than one, the Sides subtending, the Right Angles are Quadrants.

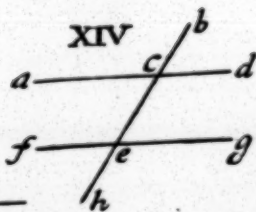
So if the Triangle agb is a Rectangle Triangle, with three Right Angles, all its Sides are Quadrants.

36. If of a Spherical Triangle, having two Right Angles, the third Angle be Acute, the third Side is less than a Quadrant: But if Obtuse, 'tis greater.

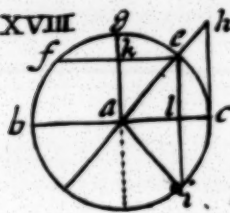
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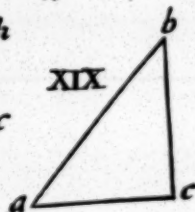
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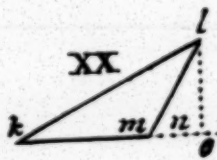
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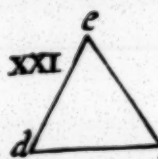
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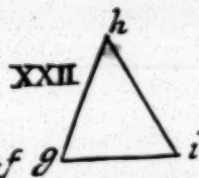
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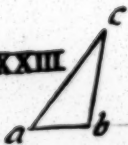
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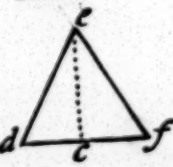
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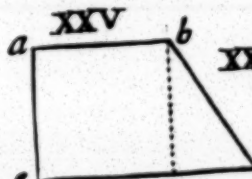
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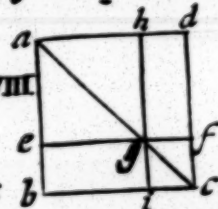
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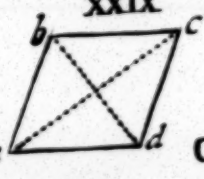
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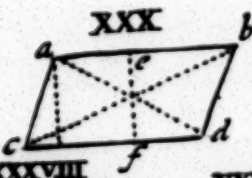
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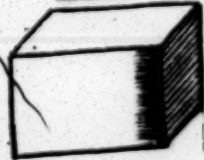
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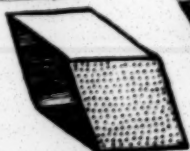
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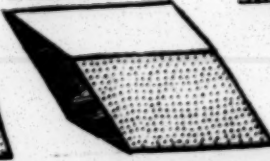
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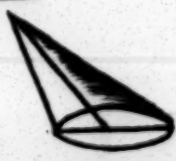
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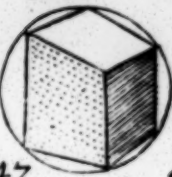
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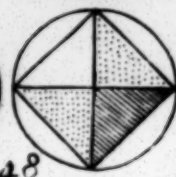
Tetraedron



Cube



Octaedron



Dodecaedron

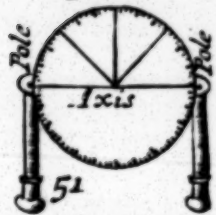


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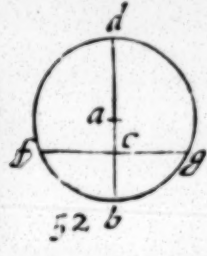


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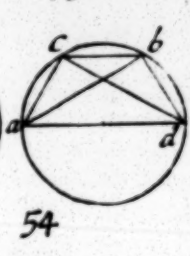
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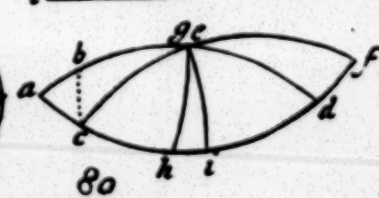
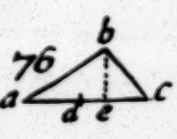
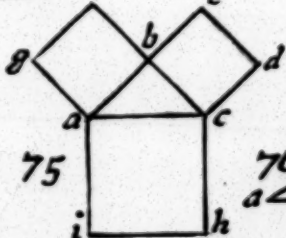
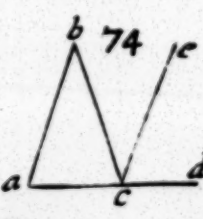
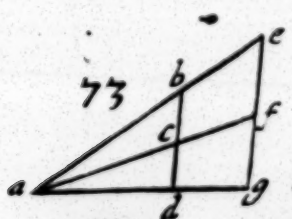
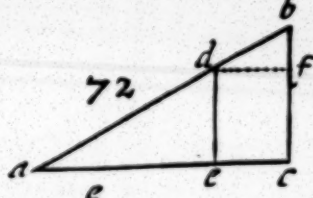
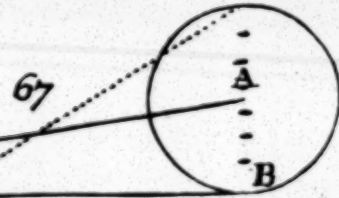
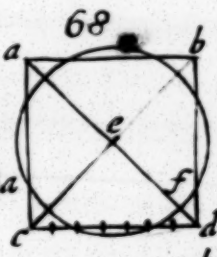
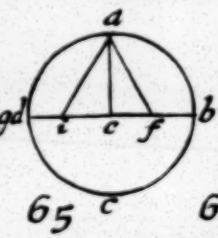
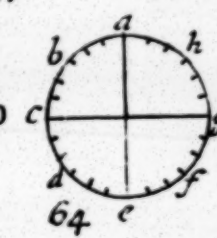
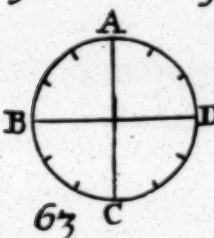
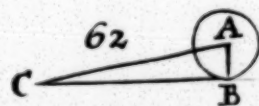
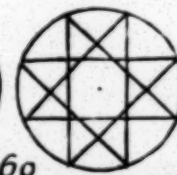
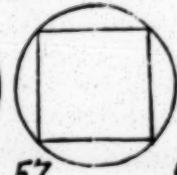
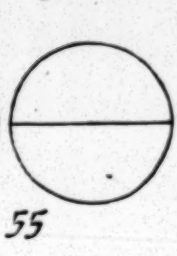
48



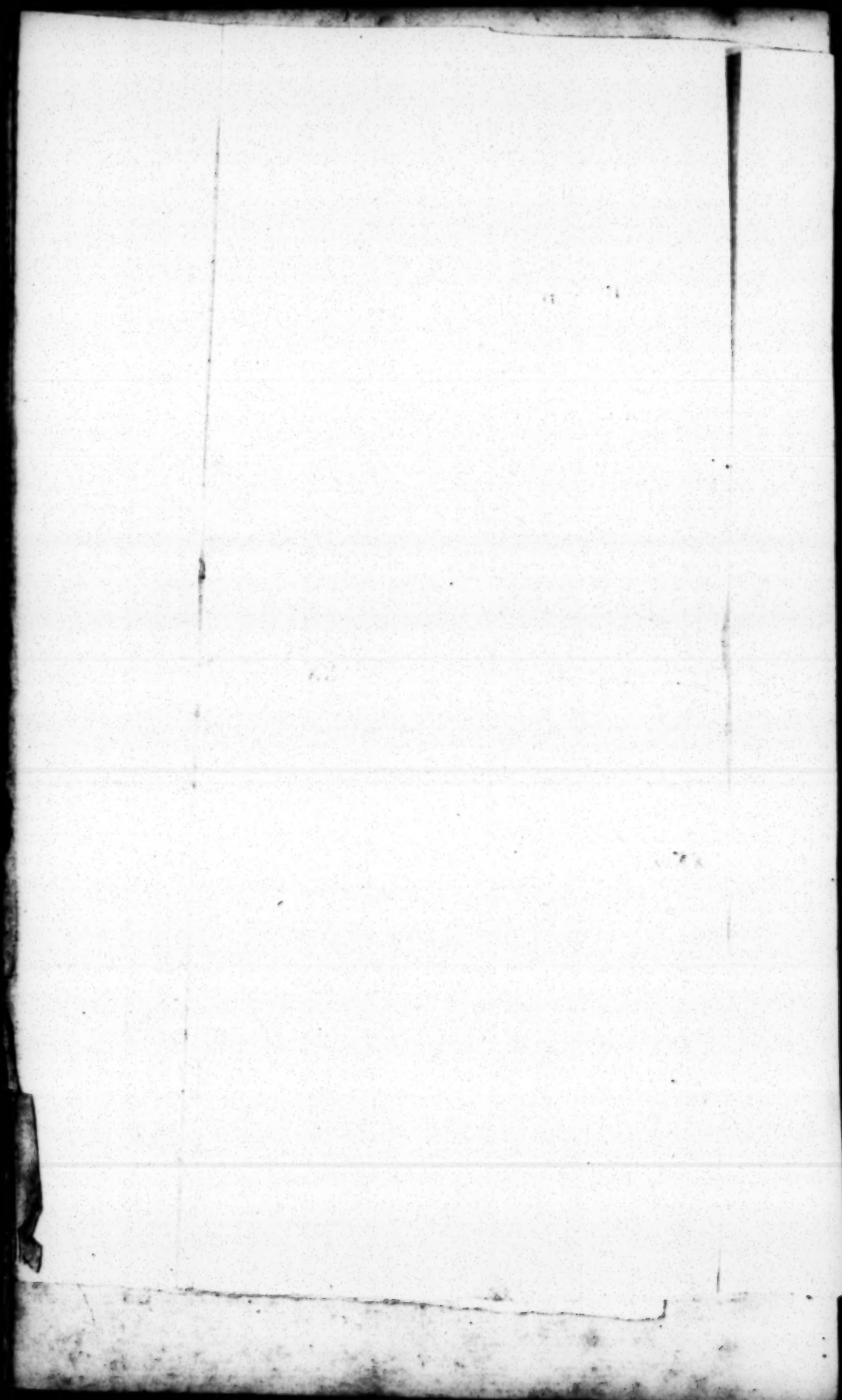
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Place this at Page 185. to Fold out.



As in the Triangle bgi , Acuteangled at g , the third side bi is less than a Quadrant : But in the Triangle agi , Obtuseangled at g , the third side is greater than a Quadrant.

37. In a Spherical Triangle, purely Acuteangled, its opposite Spherical Triangle hath two Obtuse, and one Acute Angle, and the contrary.

As if the Angles at a , and at d , are supposed Acute, To the Triangle abc , purely Acute, (that is to say, having three Acute Angles) there is opposite the Triangle bdc , with two Obtuse at b and c , and one Acute Angle at d .

38. In a Spherical Triangle, purely Obtuse, there is an opposite Spherical Triangle, with two Acute, and one Obtuse Angle, and the contrary.

As if the Angles at a and d are made or supposed to be obtuse ; To the Triangle, purely Obtuse Angled, bdc , there is opposite the Triangle abc , with two Acute Angles at b and c , and one Obtuse at a .

39. The three Angles of any Spherical Triangle, are greater than two Right Angles.

Therefore the thing is otherwise than in Plane Triangles, in which the three Angles taken together, are equal to two right.

40. In Rectangled Spherical Triangles, the same having Acute Angles at the Bases ; The Sines of the Hypothenufes, and of all the Perpendiculars, are Proportionals between themselves. Fig. 81.

Let there be two Rectangle Spherical Triangles, abc and $ae f$, in which the Hypothenufes are ab , and ae , and the Perpendiculars bc and ef , and the Bases ac and af ; and the Angle at a common to both Bases : I say the Sines of the Hypothenufes, and of the Perpendiculars, are Proportionals between themselves, to wit, as the Sine of the Hypothenufe

use ab , to the Sine of the Perpendicular bc : So is the Sine of the Hypothenufe ae , to the Sine of the Perpendicular fe .

Likewise as the Sine of the Hypothenufe ae , to the Perpendicular ef ; so is the Sine of the Hypothenufe ab , to the Perpendicular bc : And as the Perpendicular ef , to the Sine of the Hypothenufe ae ; so is the Perpendicular bc , to the Sine of the Hypothenufe ab .

41. In Rectangled Spherical Triangles, having more Acute Angles than one at the same Bases; the Sines of the Bases, and all the Tangents of the Perpendiculars, are Proportionals between themselves: that is,

1. As the Sine of the Base af , to the Tangent of the Perpendicular ef ; so the Sine of the Base ac , to the Tangent of the Perpendicular bc .

2. As the Sine of the Base ac , to the Tangent of the Perpendicular bc : so the Sine of the Base af , to the Tangent of the Perpendicular ef .

3. As the Tangent of the Perpendicular ef , to the Sine of the Base af ; so the Tangent of the Perpendicular bc , to the Sine of the Base ac .

42. In all Spherical Triangles, the Sines of the Sides are direct Proportionals to their opposite Angles.

Example.

1. As the Sine of the Angle acb , To the Sine of the Side ab :

So the Sine of the Angle abc , To the Sine of the Side ac .

2. As the Sine of the Side ab , To the Sine of the Angle acb :

So the Sine of the Side --- ac , To the Sine of the Angle abc .

3. As

3. As the Sine of the Angle acb , To the Sine of the Side ab .

So the Sine of the Angle bac , To the Sine of the Side bc .

4. As the Sine of the Side ab , To the Sine of the Angle cab :

So the Sine of the Side bc , To the Sine of the Angle bac .

5. As the Sine of the Angle bac , To the Sine of the Side bc :

So the Sine of the Angle abc , To the Sine of the Side ac .

6. As the Sine of the Side bc , To the Sine of the Angle bac :

So the Sine of the Side ac , To the Sine of the Angle abc .

Notwithstanding here also we must attend to the Species or Kind of the Angle, whether it be Acute or Obtuse? Since the Sine may be the same of both.

43. In all Spherical Triangles, two Sides being each less than a Quadrant: If you first compound them between themselves; then the lesser Side with the Complement of the greater; and from, or to the Sine of the latter compounded Arch, you Subtract the Sine of the Complement of the first compounded Arch, or add the Sine of the Excess; It will be, as the Radius to the Half of the Right Line made by that Subtraction or Addition: So the Versed Sine of the Angle comprehended by the said two Sides, to a Right Line; which Subtracted from the Sine of the latter compounded Arch, leaves the Sine of the Complement of the third Side: Or from which Subtracting the Sine of the latter compounded Arch, it leaves the Sine of the Excess of the third Side.

Example.

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Example, Fig. 81.

Given in the Triangle $b a c$,

The Ac. Ang. a , of $50^\circ : 4'$ and let it be contained
by the Side $ab = 26 : 20$ The same $26^\circ : 20'$

And the Side $ac = 59 : 58$ Its Comp. $30 : 02$

* The Sum $86 : 18$ The Sum $56 : 22$ Its Sine 8325991

Its Compl. $03 : 42$ Its Sine 645323

Subtracted, leaves 7680658

Half of it is 3840334

An Angle given of $50^\circ : 4'$ The Radius 10000000.

Its Complement $39 : 56$ Its Sine --- 6418958.

The Verfed Sine by Subtraction - - - 3581032.

Having these, I say ;

As the Radius, To the Half : So the Verfed Sine.

10000000. 3840334. 3581032.

To the Right Line 1375239. This subtracted from the Sine of the
latter comp. Arch 8325991.

Leaves --- 6950752. The Sine of the Arch $44^\circ : 2'$:
whose Supplement $45^\circ : 48'$, is in the aforesaid Triangle, the
Arch or Side $b c$.

N O T E,

At Sign * If this Sum be greater than a Quad-
rant, we must take instead of it the Excess of the
Complement, and the Sine of it must not be sub-
tracted but added.

*More Examples may be seen in Pitiscus's Trigo-
nometry, from whence these were taken.*

44. The Sides of Spherical Triangles may be chan-
ged into Angles, and the contrary : The Comple-
ments to a Semicircle being taken on every side for
the greatest Side and Angle.

Trigo-

Trigonometrical Problems.

Of the Construction of Plane Triangles.

Problem I.

UPon a Terminate Right Line given, to make any Triangle.

1. Equilateral. Fig. 82.

From the two extream Points of the given Right Line, with the Interval of the given Line, describe with the Compasses two occult small Arches, cutting each other; from which Point of Section, Right Lines being drawn to the Extrems of the given Line, will make an Equilateral Triangle.

2. An Isosceles, or one of Equal Legs.

'Tis made in the same manner, except only in describing of the little Arches, we must take either a greater or lesser Interval, than is the Right Line given, upon which the Triangle is to be made.

3. A Scalene, or of various Sides.

It will be made, if we take not only a greater or lesser Interval than the given Line; but also in the description of those little Arches, you must alter the Compasses, that so you may produce 3 unequal Sides, Fig. 83.

4. A Rectangled Triangle.

Will be made, if at the extream of the given line *ac*, you raise a Perpendicular, and join the Extream of this, with the Extream of the given Line: Of which hereafter.

I. Of Finding of Angles in Plane Triangles.

In any Triangle, besides the Area, there are six things: to wit, 3 Sides, and 3 Angles: from one or another of these 3 given, the rest may be found
divers

divers ways : only one Case excepted, that is, if only 3 Angles be given, for from these no Side can be found, because 3 Angles of one Triangle may be equal to 3 Angles of another, although the Sides be altogether unequal : Therefore how either the Angles or the Sides are to be found Artificially, we shall now declare.

II. *To find the Angles of any Triangle by Instruments Mechanically, may be done several ways.*

1. *By a Square*, with which, if the Sides of the given Angle agree, the Angle is a Right Angle ; if they fall without the Square, the Angle is Obtuse, if within, Acute, by Theorem the 4th of Trigonometry, in Pag. 177. of this.

2. *By a Measuring Rod or Rule*, by measuring in the given Plane Angle, if it may be done, in one side 3, in the other 4, either Feet or Inches : If the Hypothenuse contain 5, the given Angle is a Right Angle ; if less than 5, 'tis Acute ; if more, 'tis Obtuse, by Theorem 21 of this.

3. *By a Quadrant, a Semicircle, an Astrolabe, a Geometrical Quadrate, or the like Instrument*, having the Divisions of the Circle, and furnished with Sights or Perpendiculars, by which not only the kinds of the Angles is had, but also their Measure, as in the description of the Making and Use of such Instruments may be seen ; concerning which, we shall say no more here.

III. *In a Plane Triangle, from one Angle given, to find the Rest, either singly, or the Sum of them.*

1. *In a Rectangled Triangle* ; The Right Angle is known by it self to consist of 90 Degrees ; the Sum of the rest are as many Degrees, to wit, 90 ; one
of

Of Plane Triangles.

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of which being given, the other is also known, if you Subtract the given Angle from 90, by Theorem the 16th, and the following.

2. In an *Equilateral Triangle*, which is also Equiangled, each of the Angles contains 60 Degrees ; for 60 multiplied by 3, is equal to 180.

3. In an *Isofcele*, one of the Equal Angles being given, the other is also given, the Complement of which two to a Semicircle, is the third unequal Angle ; and the unequal Angle being given, the Sum of the Rest is given, if you subtract it from 180, the half of which Sum denotes either of the two.

4. In all *Triangles*, one Angle being given of a Plane Triangle, the Sum of the Rest are given, if you subtract the given and known from 180, or if you take the Complement of the same to a Semicircle by Theorem the 16, of this.

IV. From two Sides of a *Rectangled Triangle* given, to find the two *Acute Angles*.

For the declaration of this, and some of the following Problems, we make use of the Rectangled Triangle acb , in Fig. 84. which we have inscribed in 4 Quadrants of a Circle, that it may more plainly appear, how each Side may be had, either for Radij, or for Sines, or for Tangents and Secants.

And we have ascribed to ac the lesser side 3.

But to the greater bc ——— 4.

And to the greatest, or Hypothenufe ab ——— 5.

And the *Ratio* of finding the Angles is thus, as in Example.

The Angle at a is sought.

1. From the Hypothenufe given ab , and from the

the side bc , opposite to the Angle sought. Let it be done in the first Circle.

As the Hypoth. ab } To the Side bc } So the Rad. ab
 5, 4; 100000.

To bc } The Sine of the Angle at a : whose Arch
 80000 } $53^\circ : 8'$ is the Angle at a sought.

2. From the Hypothenufe ab , and from the Side ac , adjacent to the Angle a , sought. Make it in the second Circle.

As the Side ac } To the Hypoth. ab } So the Rad. ac
 3, 5; 100000.

To ab } The Secant of the Angle at a , whose Arch
 166666 } $53^\circ : 8'$, is the Angle at a , sought.

3. From 2 Sides given, the Angle at a opposite to the Side bc , may be found: If you make it in the second Circle.

As the Side ac } To the Side bc } So ac the Radius
 3, 4; 100000.

To bc } The Tangent of the opposite Ang. to a , whose
 133333 } Arch $53^\circ : 8'$, is the Angle at a sought.

The Ratio is altogether the same, only changing, if the Angle at b be sought: For,

1. From the Hypothenufe ab , and the Side ac , let it be done in the fourth Circle.

As the Hypoth. ab } To the Side ac } So the Rad. ab
 5, 3; 100000.

To ac } The Sine of the Angle opposite to b , whose
 60000 } Arch $36^\circ : 52'$, is the Angle at b , sought.

2. From the Hypothenufe ab , and from the Side bc , let it be in the third Circle.

As

As the Side bc } To the Hypoth. ab } So the Rad. bc
 4, } 5; } 100000.
 To ab } The Secant of the Angle at b , whose Arch
 125000 } $36^\circ : 52'$, is the Angle at b sought.

3. From two Sides, the Angle at b , opposite to the Side ac is found, if you make it in the third Circle.

As the Side bc } To the Side ac } So the Radius bc
 4, } 3; } 100000.
 To ac } The Tang. of the Angle opposite to b , whose
 75000 } Arch $36^\circ : 52'$, is the Angle at b sought.

V. In any Triangle, two Sides being given, with the Angle opposite to one of the Sides, to find the Angle opposite to the other Side, by Theor. 23. of this, at Pag. 181.

In the Triangle abc in Fig. 85. From the Side given ab 34, with the Angle opposite to it at c , which let be $60^\circ : 23'$; also there is given the Side bc $13\frac{1}{2}$.

For the Angle at a : Say then,
 As the Side ab } To the Side bc } So the Sine of the
 34, } $13\frac{1}{2}$; } Angle at c $60^\circ : 23'$
 } } 86935.

To the Sine of the Angle at a , opposite to the Side bc , which is 34475, whose Arch $20^\circ : 10'$ fere, is the Angle sought.

Note, Here notwithstanding we must mind, whether the Angle be Acute or Obtuse? Since the Sine is the same of both: Therefore also the kind of the Angle sought, ought to be known.

VI. In any Triangle, from two Sides given, and from an Angle contained by them: To find the other two Angles, by Theorem the 24th, Pag. 181.

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The Process is thus. In Fig. 84.

In the Triangle $a b c$.

One side given, is the greatest, — $a b$ — 10.

And the other side given, is the lesser, $a c$ — 6.

The Sum of the sides is ————— 16.

The Difference of them is ————— 4.

The Angle at a given, is ————— $53^{\circ} : 8'$.

The Sum of the Remainders ————— 126 : 52.

Half the Sum of this ————— 63 : 26.

The Tangent of this Half ————— 199986.

Then make it :

As the Sum of the sides } To the Differ. of the same
16, } 4 ;

So the Tangent of the Half } To the Tangent
199986, } 49996.

Of which the Angle, or Arch, is $26^{\circ} : 34'$.

Which Subtracted from the Half, makes $36^{\circ} : 52'$, viz. the lesser Angle at b .

The Addition of the same, makes $90^{\circ} : 00$, viz. the greater at c .

Hitherto of the Invention of Angles, now follows,

The Finding of the Sides of Plane TRIANGLES.

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VII. In a Right Angled Triangle, from two Sides given, to find the third. Fig. 84.

1. If the sides given be $a c$ and $b c$, comprehending a Rectangle, and the Hypothenuse $a b$ is sought.

And for Example, let $a c$ be 6.

And $b c$ ————— 8.

1. Take

Of Plane Triangles.

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1. Take the Squares of them—36 and 64.
The Sum of which is—100.
2. The Root of this Sum—10.
Is the third Side $a b$, to wit, the Hypothenufe.

2. If there be given the Hypothenufe, with one of the fides $b c$, and the fide $a c$ is fought.

Example.

- The Hypothenufe $a b$ —10.
With the fide $b c$ —8.
1. Take the Squares of them—100 and 64.
 2. Subtract the Square of the lesser from the }
greater, and there Remains—36.
Its Root 6, is the Side fought, $a c$.

In the same manner also you may find the fide $b c$, from the Hypothenufe, and from the fide $a c$.

VIII. In a Rectangle Triangle, given one Side and one Angle ; To find the other Side.

In Fig. 84. The fides of the Rectangle Triangle, are, 3, 4, and 5. Then,

1. The Hypothenufe $a b$ is had from the fide $a c$, and from the Angle a adjacent to it ; if you make it in the second Circle.

As the Radius $a c$ } To $a b$, the Secant of the given
100000, } Angle at a $53^{\circ} : 8'$
166679.

So the Side $a c$ } To the Hypothenufe fought,
3, } 5,

2. The same Hypothenufe $a b$ may be found from the Side $a c$, and from the Angle opposite to it, at b in the fourth Circle, if you make it :

As $a c$ the Sine of the Angle } To the Radius $a b$,
given at b $36^{\circ} : 52'$ }
59995. } 100000.

O 2

So

84

So the Side given ac } To the Hypotenuse ab sought,
3, 5.

3. The Side bc is had from the Hypotenuse ab given, and from the Angle a , opposite to it, if you make it in the first Circle.

As the Rad. ab } To bc the Sine of the Ang. a , $53^\circ : 8'$
100000, 80003.

So the Hypotenuse ab } To the Side bc
5, 4.

4. The same also may be had from the Hypotenuse, and from the Angle b adjacent to it, if you make it in the third Circle:

As the Secant ab of the } To the Radius bc
Angle at b $36^\circ : 52'$ }

124995, 100000.
So the Hypotenuse ab } To the Side bc
5, 4.

5. Also either Side may be found, from one Side, and the Angle opposite to the same, if in the second Circle you make it:

As the Tangent bc of the op- } To the Radius ac
posite Angle at a given* }

133349, 100000.
So the Side given bc } To the Side sought ac
4, 3.

IX. From one Side given in any Plane Triangle, and from the two Angles opposite to the given, and the sought Side, To find either of the other Sides, by Theorem 23, of Trigonometry, at Pag. 181.

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In Fig. 85. In the Triangle abc , whose given Side is ab 34, the opposite Angle at c $60^\circ : 23'$, the Angle at a opposite to the other sought bc $20^\circ : 10'$: make it:

As

So the Sine of the Angle } To the Side sought $b c$
 at $a, 20^{\circ} : 10'$,
 34475. } 13 $\frac{1}{2}$

1. If the Perpendicular fall within the Triangle, which it doth in any *Oxigonium*; in *Amblygoniums*, and *Rectangled* ones, 'tis drawn from the Right Angle, or the Obtuse.

1. Assume the greater side $b c$ ——— 20.
2. The lesser side ——— $a b$ ——— 13.
3. The Difference of both ——— 7.
4. The Sum of the sides ——— 33.
5. The Basis, or the greatest Side $a c$ 21.

As the Base or great-
est fide *a c*,
21,

To the Sum, of the fides
33.

So the Difference $\left\{ \begin{array}{l} \text{To the Segment of the Base} \\ 7, \qquad \qquad \qquad \text{II.} \end{array} \right.$

This subtracted from the Base 21, leaves ea 10.
Whose half fe or af ————— 5.

Is the Segment of the Base, which subtracted }
from the whole Base ac ————— } 21

Leaves the other Segment of the Base $f_c - 16$.

2. If the Perpendicular fall without the Triangle, on the Base protracted, as happens in *Ambligoniums*, if it be given from an Acute Angle : Proceed thus.

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In Fig. 20. In the Triangle $k l m$

1. Assume the greater side $k l$ ——— 40.
2. The lesser side ——— $l m$ ——— 16.
3. The Difference of both ——— 24.
4. The Sum of the sides ——— 56.
5. The Base, or greater side $k m$, pro- } 32.
tracted in o , receiving the Perpend.

Then make it :

| | | |
|-------------------|---|-------------------------|
| As the Base $k m$ | } | To the Sum of the sides |
| 32, | | 56. |
| So the Difference | } | To the Segment of the |
| 24, | | Base protracted $m o$ |
| | | 42. |

From hence, take away the Base $k m$ ——— 32.

It leaves $m o$ ——— 10.

Half of which Remainer $m n$ ——— 5.

Is the Segment of the protracted Base, receiving the Perpendicular $l n$.

And in this manner, by means of the Perpendicular, and of the Base, either True or Protracted, is now made a Rectangle Triangle, from an Obliquangled Triangle : Whence by taking away what is to be taken away, you have the Quantity of the Side, and of the Angles unknown, in the given Obliquangled Triangle.

Of Spherical Triangles.

XI. Some peculiar things requisite to be Known in the Solution of Spherical Triangles.

1. A Rectangled Spherical Triangle, with three Right Angles, wants no Solution, for the 3 Right Angles,

Angles, are known by themselves ; and the Bases of them are Quadrants of a Circle.

2. Nor that which hath two Right Angles, for the Sides opposite to the Rectangles are Quadrants: The third Angle may be either Acute or Obtuse, the Measure of which is the third Side, in some measure always opposite to it ; and is greater than a Quadrant, if it subtend an Obtuse Angle, but less, if an Acute : Therefore whether this be a third Angle, or a third Side, 'tis given, for both are understood to be given, since this third Side is no other than the Measure of the Angle.

3. For a Rectangle Triangle, with two Obtuse Angles, you must Solve the Triangle opposite to the Rectangle, with two Acute Angles : For from any three given, in either of these Triangles, you cannot be ignorant of those in the other, since the parts of one Triangle, are the Complements of the Parts of the other, to a Semicircle.

4. For a Rectangle Triangle, with an Obtuse and an Acute Angle, you must Solve a less Rectangle Triangle, opposite to the Acute Angle, which hath two Acute Angles ; and whatsoever they are in either of those given, in the other you must needs know, for the opposite Angles are equal, and the acute Angle is the Complement of the Obtuse to two Right Angles ; moreover the Sides are Complements to a Semicircle : As for Example.

If there be given to be Solved, the Rectangle Triangle bdc in Fig. 80, being Rectangled at d , Obtuse angled at b and c ; for it we must Solve the Triangle abc , opposite to the given, from the Angle d , in which the Angles at a and d are equal : The Sides ab and ac , are the Complements of the Sides bd and cd , to a Semicircle : And the Obtuse Angles at b and c , are the Complements of the Acute Angles at b and c : Therefore whatsoever Three are given in the Triangle bdc , they Three will likewise be given in the Triangle abc . But if we are offered a Rectangle Triangle ced , to be Solved, which is Right Angled at d ,

Obtuse at e , and Acute at c ; for it, we must Solve the Triangle edf , opposite to the Triangle ecd , from the Acute Angle c .

5. There remains then to be Solved in Rectangled Triangles only, that which hath a Right Angle with two Acute Angles, whose Solution follows.

XII. To Resolve a Rectangled Spherical Triangle given, with two Acute Angles.

1. The Angles are to be found.

1. From the Base of the Right Angle, or Hypotenuse; and from the Side or Leg adjacent to the Angle sought: Make it thus,

As the Radius to the Tangent of the Complement of the Base: So the Tangent of the Leg given, to the Sine of the Complement of the Angle sought:

For Example.

89. In Fig. 89, In the Rectangle Triangle abc , the Angle at c is sought.

There is given the Base ac ————— $30^\circ : 9'$
 Its Complement is ————— $59 : 51$
 The Tangent of this Complement is ————— $172162, A.$
 The Leg adjacent to the Ang. sought, bc $11^\circ : 34'$
 Its Tangent ————— $20466, B.$

Make it then.

As the Radius { To the Tangent A } So the Tang. B.
 $100000,$ { $172162.$ } $20466.$

To the Sine of the { Whose Complement $69^\circ : 22'$
 Angle $20^\circ : 38'$ { Is the Angle at c sought.
 $35238,$ }

2. From

2. From the Base, and from a Leg opposite to the Angle sought.

Make it, As the Radius, to the Sine of the Base: so the Secant of the Complement of the given Leg, to the Secant of the Complement of the Angle sought :

Example.

Let's seek the Angle at a, in Fig. 89.

Let the Base given be ac ————— $30^{\circ} : 9'$

Its Sine ————— 50227 , A.

The Leg cb opposite to the Angle a — $11 : 34$

The Secant of the Complement of it 498733 , B.

Make it then,

| | | | | |
|---------------|---|---------------|---|----------------------|
| As the Radius | } | To the Sine A | } | So the Secant of the |
| 100000, | | 50227 : | | Complement B. |
| | | | | 498733. |

| | | |
|--------------------------|---|---------------------------------------|
| To the Secant of the | } | The Complement of which |
| Angle $66^{\circ} : 28'$ | | $23^{\circ} : 32'$, is the Angle a |
| 250498 , | | sought. |

3. From the Base, and from one Angle.

Make it, As the Radius, to the Sine Complement of the Base ; so the Tangent of the given Angle, to the Tangent of the Complement of the Angle sought :

Example.

The Angle at c is sought.

There is given the Base ac ————— $30^{\circ} : 9'$

Its Complement is ————— $59 : 51'$

The Sine of this Complement is ————— 86471 , A.

The Angle at a ————— $23^{\circ} : 32'$

Its Tangent ————— 43550 , B.

Make

Make it then :

As the Radius { To the Sine A } So the Tangent B.
 100000, { 86471 : } 43550.
 To the Tang. { Whose Arch is ——— 20° : 38'
 37658. { Its Compl. the Ang. sought 69 : 22

4. From both Legs of the Right Angle.

Make it, As the Radius, to the Sine of the Leg adjacent to the Angle sought : So the Tangent of the Complement of the other Leg, to the Tangent of the Complement of the Angle sought.

For Example, we will seek the Angle at c.

And let the given Leg be *bc*, adjoining to it, 11° : 34'
 The Sine of it ——— 20051, A.
 The other Leg *ab* ——— 28° : 2'
 Its Complement ——— 61 : 58'
 The Tangent of this Complement is ——— 187809, B.

Make it then :

As the Radius { To the Sine A } So the Tangent B,
 100000, { 20051 ; } 187809.
 To the Tangent { Whose Arch ——— 20 : 38,
 37657, { Is the Complement of } 69 : 22.
 { the Angle sought ——— }

5. From an Angle, and from either Leg opposite to the Angle sought.

Make it, As the Radius, to the Sine of the Angle given : So the Sine of the Complement of the given Leg, to the Sine of the Complement of the Angle sought.

Example.

The Angle at *a* is sought.

And let the given Angle at *c*, be ——— 69° : 22
 Its Sine ——— 93585, A.
 The Leg opposite to the Ang. sought *bc*, 11 : 34
 Its

Of Spherical Triangles.

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Its Complement ————— $78^\circ : 26$.

The Sine of it ————— 97969, B.

Say then :

As the Radius } To the Sine *A* } So the Sine *B*,

100000, } 93585; } 97969.

To the Sine } Whose Angle is ————— $66^\circ : 28'$

91684. } Its Compl. the Angle fought 23 : 32

6. From one Angle, and a Leg adjacent to the Angle fought. Fig. 89. 89

Make it As the Radius, to the Secant of the given Leg ; So the Sine of the Complement of the Angle given, to the Sine of the Angle fought.

Example.

The Angle at c is fought.

Let the Leg adjacent be ————— $11^\circ : 34'$

Its Secant ————— 102073, A.

The Angle given at *a* ————— 23 : 32

Its Complement ————— 66 : 28

The Sine of this Complement ————— 91683, B.

Say then :

As the Radius } To the Secant *A* } So the Sine *B*,

100000, } 102073; } 91683.

To the Sine } Whose Angle is ————— $69 : 22$.

93583, } The Angle fought at *c*.

II. To Find the Base.

1. From either Leg given, and from an Angle adjacent to it.

Make it, As the Radius, to the Sine of the Complement of the Angle given ; so the Tangent of the Complement

Complement of the given Leg, to the Tangent of the Complement of the Base sought.

Example.

The Base a c is sought.

The Angle given at *a* is ———— $23^{\circ} : 32'$
 Its Complement ———— $66 : 28$
 The Sine of the Complement ———— $91683, A.$
 The Leg adjacent to the Angle given *ab* $28 : 2.$
 Its Complement ———— $61 : 58$
 The Tangent of this ———— $187809, B.$

Say then :

As the Radius { To the Sine *A* } So the Tangent, *B.*
 $100000, \quad 91683 ; \quad 187809.$
 To the Tangent { Whose Arch is ———— $59 : 51.$
 $172190, \quad$ Its Compl. the Base sought $30 : 09.$

2. From a Leg given, and from an Angle opposite to this Leg.

Make it, As the Radius, to the Secant of the Complement of the Angle given ; so the Sine of the Leg given, to the Sine of the Base sought.

For Example.

Let's again seek the Base a c.

And the Angle given at *a* is ———— $23^{\circ} : 32'$
 Its Complement ———— $66 : 28.$
 The Secant of this Complement ———— $250449, A.$
 The opposite Leg *bc* ———— $11 : 34.$
 Its Sine ———— $20051, B.$

Say then :

As the Radius { To the Secant *A* } So the Sine *B.*
 $100000, \quad 250449 ; \quad 20051.$
 To the Sine { Whose Arch $30^{\circ} : 9'$, is the Base
 $50217, \quad$ sought.

3. From

3. From both Legs given.

Make it, As the Radius, to the Secant of either Leg; so the Secant of the other Leg, to the Secant of the Base.

The Base $a c$ is sought in Fig. 89.

And the Leg given is $a b$ ————— $28^{\circ} : 2'$
 Its Secant is ————— $113292, A.$
 The other Leg $c b$ ————— $11 : 34$
 And its Secant ————— $102073, B.$

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Make it then :

As the Radius $\left\{ \begin{array}{l} \text{To the Secant } A \\ 100000, \end{array} \right. \left\{ \begin{array}{l} \text{So the Secant } B, \\ 102073. \end{array} \right.$
 To the Secant $\left\{ \begin{array}{l} \text{Whose Sine ————— } 30^{\circ} : 09' \\ 115640; \end{array} \right. \left\{ \begin{array}{l} \text{Is the Base sought.} \end{array} \right.$

4. From both of the Acute Angles given.

Make it, As the Radius, to the Tangent of the Complement of the former Angle: so the Tangent of the Complement of the latter, to the Sine of the Complement of the Base.

Example.

Let's seek the given Base $a c$.

The Angle given of the former at c is $69^{\circ} : 22'$
 Its Complement is ————— $20 : 38.$
 The Tangent of it is ————— $37654, A.$
 The Angle of the latter at a is ————— $23 : 32.$
 Its Complement is ————— $66 : 28.$
 The Tangent of it ————— $229619, B.$

Say then,

As the Radius $\left\{ \begin{array}{l} \text{To the Tangent } A \\ 100000, \end{array} \right. \left\{ \begin{array}{l} \text{So the Tang. } B. \\ 229619. \end{array} \right.$
 37654;

To

To the Sine, Whose Arch is ————— $59^{\circ} : 51'$
 86460, { And its Comp. the Base sought, $30 : 09$

III. The Sides are found.

1. From the Base given, and an Angle opposite to the Leg sought : If you make it, As the Radius, to the Sine of the Base ; so the Sine of the Angle given, to the Sine of the Leg sought.

Example.

Let's seek the Leg bc in Fig. 89.

The Base ac ————— $30^{\circ} : 09'$
 Its Sine is ————— $50227, A.$
 The Angle given at a ————— $23 : 32$
 Its Sine ————— $39928, B.$

Say then :

As the Radius { To the Sine A } So the Sine B .
 100000, { 50227 ; } 39928.
 To the Sine { Whose Arch $11^{\circ} : 34'$
 20064, { Is the Leg sought bc .

2. From the Base, and from an Angle contiguous, or adjacent to the Leg sought : Make it,

As the Radius, to the Sine Complement of the Angle given ; so the Tangent of the Base, to the Tangent of the Leg sought.

Example.

The Leg cb is sought.

The given Angle at a is ————— $23^{\circ} : 32'$
 Its Complement is ————— $66 : 28$.
 The Sine of this is ————— $61683, A.$
 The Base ac is ————— $30^{\circ} : 09'$
 Its Tangent is ————— $58084, B.$

Say

Say then:

As the Radius { To the Sine *A*, } So the Tangent *B*.
 100000, { 91683 ; } 58084.
 To the Tangent { Whose Arch ———— 28° : 2'
 53253, { Is the Leg sought, *a b*.

3. From the Base, and either of the Legs:

Make it, As the Radius, to the Secant of the Leg given; so the Sine of the Complement of the Base, to the Sine of the Complement of the Leg sought.

Example.

Let's seek the same Leg *a b*.

Given the Leg *b c* ———— 11° : 34'
 Its Secant is ———— 102073, *A*.
 And the Base *a c* ———— 30 : 09
 Its Complement ———— 59 : 51
 The Sine of this is ———— 86471, *B*.

Say then:

As the Radius { To the Secant *A*, } So the Sine *B*.
 100000. { 102073 ; } 86471.
 To the Sine { Whose Arch is ———— 61° : 58'
 88263, { And its Compl. is the Leg, *a b* 28 : 02

4. From one Leg, and an Angle adjacent to the Leg sought.

Make it, As the Radius, to the Tangent of the Complement of the Angle given; so the Tangent of the given Leg, to the Sine of the Leg sought.

Example.

The Leg *a b* is sought.

From the adjacent Angle given at *a* 23° : 32'
 The Complement of which is ———— 66 : 28.
 The Tangent of this is ———— 229619, *A*.
 And there is given also the Leg *b c* — 11 : 34.
 The Tangent of it is ———— 20466, *B*.

Say

Say then :

As the Radius $\left\{ \begin{array}{l} 100000, \\ 46993, \end{array} \right. \begin{array}{l} \text{To the Tangent } A \\ \text{To the Sine} \end{array} \left\{ \begin{array}{l} 229619; \\ \text{Whose Arch} \end{array} \right. \begin{array}{l} \text{So the Tang. } B. \\ 20466. \\ 28^\circ : 2' \end{array}$
 Is the Leg fought, $a b$.

5. *From either of the Legs, and from an Angle opposite to the Leg fought :*

Make it, As the Radius, to the Sine of the Leg given ; so the Tangent of the Angle given, to the Tangent of the Leg fought.

Example.

The Leg b c is fought.

And let the Leg given be $a b$ ——— $28^\circ : 2'$
 And the Sine of it ——— $46999, A.$
 The Ang. at a opposite to the Leg fought $23 : 32$
 And the Tangent of it ——— $43550, B.$

Say then :

As the Radius $\left\{ \begin{array}{l} 100000, \\ 20468, \end{array} \right. \begin{array}{l} \text{To the Sine } A, \\ \text{To the Tangent} \end{array} \left\{ \begin{array}{l} 46999; \\ \text{Whose Arch} \end{array} \right. \begin{array}{l} \text{So the Tangent } B. \\ 43550. \\ 11^\circ : 34' \end{array}$
 Is the Leg fought.

6. *From both the Acute Angles.*

Make it, As the Radius, to the Secant of the Complement of the Angle given, adjacent to the Leg fought ; so the Sine of the Complement of the other Angle given, to the Sine of the Leg fought.

Example.

The same Leg b c is fought

Let the Ang. given, being adjacent at c be $69^\circ : 22'$
 Its Complement is ——— $20 : 38$
 And the Secant of it is ——— $106854, A.$
 The other Angle at a is ——— $23 : 32$
 The Complement of which is ——— $66 : 28$
 And the Sine of this Complement ——— $91683, B.$

Say

Say then :

As the Radius } To the Secant A } So the Sine B,
 100000, } 106854. } 91683.

To the Sine } Whose Arch is $78^{\circ} : 28'$
 97969, } And its Compl. 11 : 34 the Leg sought.

And there are yet more ways of finding all these things hitherto sought, which you may see in *Metius*, or in *Bonaventura Cavallerius* his *Directory*, or in his *Trigonometry*; and others: But these seem to us to be sufficient in this place.

XIII. To Solve an Oblique Spherical Triangle given.

Oblique Angled Spherical Triangles may be Solved by reducing them to Rectangles, by letting fall a Perpendicular Arch from the Vertex of the Triangle on the opposite and subjected Base: And this Perpendicular Arch either falls within or without the Triangle, to wit, at the continued part of the Base, making the Oblique Angled Triangle into two Rectangled Triangles: Whence Noting and Perceiving the Solution of Rectangled Triangles, you cannot be ignorant of the Solution of Oblique-angled ones; for by Addition and Subtraction you may easily find whatsoever you desire in adjoined Obliqueangles, especially if you continue each side to Quadrants, (and that at once or twice, if one continuation doth not suffice) and the whole Figure be inclosed by a Quadrant; Where at length the Proportion will be found in the Complements of the given, and the sought.

Example, Fig. 90.

There is given the Oblique Triangle abc , and in it.

P

1. The

90

1. The Angle at a ————— $23^{\circ} : 32'$
2. The Angle at b ————— $56 : 07$
3. The Side $a b$ ————— $15 : 56$

To find the Rest.

Let fall from the Angle at b , a Perpendicular Arch, which let be $b d$, forming the Rectangle Triangle $b d a$, containing within it another lesser $b d c$.

1. *For finding the Leg $b d$, say:*

As the Radius } To the Sine of the Side $a b$ $15^{\circ} : 56'$
 100000. } 27452.

So the Sine of the Angle at a $23^{\circ} : 32'$, 39928
 To the Sine } Whose Arch — $6 : 18$, is the
 10961, } Leg $b d$.

By Problem 12, Member 3. Num. 1. in Pag. 206.

2. *For the Leg $a d$, say,*

As the Radius ————— 100000,
 To the Sine of the Complement of the }
 Angle at a , $66^{\circ} : 28'$ ————— } 91683;

So the Tangent of the Side $a b$ $15^{\circ} : 56'$ 28549,
 To the Tangent ————— 26174.

Whose Arch $14^{\circ} : 20'$ is the Leg $a d$ sought: By the same at Number 2.

3. *For the Angle $a b d$, say,*

As the Radius ————— 100000.
 To the Sine of the Complement of }
 the Side $a b$ $74^{\circ} : 4'$ ————— } — 96158.

So the Tang. of the Angle at a $23^{\circ} : 32'$, 43550.
 To the Tangent ————— 41876.

Whose Arch is $22^{\circ} : 43'$, and its Complement $67^{\circ} : 17'$, the Angle $a b d$ sought.

By Prob. 12, Memb. 1, Numb. 3, in Pag. 201.

4. *For the Angle $c b d$, in Fig. 90.*

From the Angle found $a b d$ ————— $67^{\circ} : 17'$,

Take away the Angle given, $a b c$ ————— $56 : 07$.

It leaves the Angle $c b d$ ————— $11 : 10$.

5. *For*

5. For the Side cb , the Base of the Right Angle at d , say:

As the Radius ————— 100000,
 To the Sine of the Complement of }
 the Angle cbd , $78^\circ : 50'$ ————— } 98107,
 So the Tangent of the Complement of }
 the Angle bd — $83 : 42$, ————— } 905789
 To the Tangent ————— 888642
 Whose Arch is $83^\circ : 35'$, and its Compl. $6^\circ : 25'$,
 is the Base cb sought: By Probl. 11, Memb. 2,
 Numb. 1, in Pag. 199.

6. For the Leg cd , say:

As the Radius ————— 100000,
 To the Sine of the Leg bd , $6^\circ : 18'$ 10974,
 So the Tang. of the Ang. cbd . $11 : 10$ 19740,
 To the Tangent ————— 2166.
 Whose Arch is $1^\circ : 14'$, and the Leg sought cd .
 By Problem 12, Memb. 3, Numb. 5, Pag. 208.

7. For the Side ac .

From the Leg ad ————— $14^\circ : 40'$,
 Take away the Leg found cd ———— $01 : 14$,
 It leaves the Leg or Side ac ———— $13 : 26$.

8. For the Angle dcb , say, in Fig. 90.

As the Radius ————— 100000,
 To the Side bd , the Sine of the Com- }
 plement $83^\circ : 42'$ ————— } 99396,
 So the Sine of the Angle cbd $11^\circ : 10'$ 19366,
 To the Sine ————— 19249.
 Whose Arch is $11^\circ : 6'$, but the Complement
 $78^\circ : 54'$, the Angle dcb .

9. For the Angle $a c b$.

From the two Right Angles ——— $179^{\circ} : 60'$
 Take away the Angle $d c b$ — — $78 : 54$,
 It leaves the Angle $a c b$ ——— $101 : 06$,

Therefore in the Oblique Angled Triangle $a b c$,

There is given,

The Angle $c a b$ ——— $23^{\circ} : 32'$
 The Angle $a b c$ ——— $56 : 07$.
 The Side - - $a b$ ——— $15 : 56$.

There is found by the Perpendicular,

The Angle $a c b$ ——— $101^{\circ} : 06'$
 The Side - - $a c$ ——— $13 : 26$.
 The Side - - $c b$ ——— $6 : 25$.

You may also proceed in the same manner in other Oblique Angled Triangles ; which often Use and Exercise will teach.

Also you may use the Artificial Sines, Tangents, and Secants, if you please, instead of the Natural ones, which will shorten the Operations.

The End of the Third Part.

O F

O F
G E O M E T R Y.
P A R T I V.

Of Measuring the Length of Right Lines.

T H E O R E M S.

1. **T** *Here cannot be given a Line streighter than a Right Line.*

For a Right Line is like to it self every where, from which if it err the breadth of an hair, it is not a Right Line.

2. *If a line be a Right Line, it will be also the shortest between the same Terms.*

Hence 'tis said, Nature acts by Right Lines, which doth nothing that tends to long Circumstances, which may be done by a nearer way.

3. *Two Right Lines do not contain a Space, nor constitute a Figure.*

For if two Right Lines grow closer to each other on one part, they always grow farther and farther asunder, (if they are produced) on the other part; nor will they ever Terminate, unless some third line be Joined, whence it will be a Triangular Space, which is the first of all Right Line Figures.

4. *Two Right Lines have not one and the same Common Segment.*

It must be understood of two divers Right Lines, and not of them which are so Joined, that they constitute one and the same Right Line: For these may have a common Segment, but being divers, they have

not a common part, besides one point in which they convene or meet.

5. *Two Right Lines meeting in one point, if both are produced, they must needs cut each other mutually in that point.*

For unless it be so, they would run together into one line, and have a common Segment, and would not be any more two, but one Right Line.

P R O B L E M S.

I. *TO cut a Right Line given in the middle, that is, into two equal parts. Fig. 91.*

From the two Extrems of the given line a and b , with the Compasses at any Interval, yet exceeding half of the given line, describe two Arches, cutting each other mutually above and beneath, or on one part of the line only, but with divers opening of the Compasses, as in the lines $a b$, and $b c$.

A Right Line drawn thro' the points of Intersection of these Arches, will cut the given line in the middle.

II. *From a Point given, to raise a Perpendicular Line.*

1. From the Point f or g , of the given Right Line $a b$, or $b c$, at which point the Perpendicular is to be drawn, cut off with the Compasses, equal parts on both sides, to wit, $f a$, and $f b$, or $g b$ and $g c$.

2. From the points of Section a and b , or b and c , describe little Arches cutting each other, as you see in $d e$ and b .

3. Join the point of Section of the Arches d or h , with the given point f or g , to which the Perpendicular is to be drawn, and you will have a Perpendicular to the given point.

III. To

III. To draw a Parallel to a Distance given, of a Right Line given. Fig. 92.

In the line given assume two points, somewhat remote from one another : from which, with the Interval of the given distance, with the same opening of the Compasses, describe two parts of a Periphery, then draw a line to the contact of the Peripheries, and it will be a Parallel to the given line.

IV. To find a third Proportional, to two Right Lines given. Fig. 76.

Let the two given lines be ae and be , to which a third Proportional is to be found :

1. Join the two given lines ae , and be , at a Right Angle aeb , and let ea be the Antecedent, which continue from the part e .

2. Then join the Right Line ab , to which, from b let fall the Perpendicular bc , making a Right Angle at b , with ab , it will happen on the continued line of ae in c ; then ec will be the third Proportional line sought, to the two given lines ae and be , and it will be, As ae to be : So be to ec .

V. To find a part Proportional to three Right Lines given. Fig. 72.

The three given lines are ae , ec , and ad , to which a fourth Proportional is to be found.

1. Make one line ac , of ae and ec , and let ad be appointed at any Angle.

2. Join de , to which from c draw the Parallel bc , intersecting the Right Line ad , prolonged in b ; and db will be the fourth Proportional : And it will be, As ae , to ec : So ad , to db .

VI. To find a mean Proportional to two Right Lines given. Fig. 52.

The two lines are dc and cb , between which a mean Proportional is to be found.

1. Dispose the two lines given in one Right Line db .

2. From the middle point a , as a Center, describe the half Periphery dgb , whose Diameter is db .

3. From c to db , raise the Perpendicular cg , cutting the Periphery in g ; and cg will be a mean Proportional between the lines dc and cb : That is, As dc , to cg ; So cg , to cb .

VII. To find two mean Proportionals to two Right Lines given. Fig. 93.

ab and ac are two Right Lines, between which we are to find two mean Proportionals.

1. From ab and ac make a Rectangle Quadrangle, $abcd$, draw the Diagonals ad and bc , cutting each other in the Center e .

2. Transfer the Right Line ab , into the Right Line ac , continued beyond the Quadrangle, which let be af ; also continue ab towards b , as far as needs.

3. Join the points f and b , by drawing the Right Line fb , from which take away the Diagonal bc , from b even to g , but the remainder gf take in cf , to wit, from c even to o .

4. From o in the same line af continued, assume again the Right Line ac , which let be oi .

5. If you draw a Right Line from i through d , it will cut the line ab continued in b ; and i and b will be points equidistant from e , the Center of the Rectangle Quadrangle: Whence the Right Line bb will be the second, and ci the third Proportional, between the given lines ac and ab : And it will be, As ac the lesser, to bb ; so the same bb , to ci ; and ci , to the greater ab .

VIII. To measure the Altitude of any line, being Perpendicular to the Horizon.

Such are the Altitudes of Walls, of Towers, of Houses, and of other Erect things; for the measuring of which there are divers ways.

1. By a Looking-glass, Mirror, or Speculum.

1. The Looking-glass, or Mirror, must be placed on the Plane of the Horizon, over against the accessible line to be measured.

2. To this the Measurer goes backwards and forwards, until he espies the Top of the line to be measured in the Looking-glass; which being done:

As the Distance between the Measurer and the Looking-glass, to the Height of the Measurer Erect, from his Eye to his Foot: So

is the Distance between the Looking-glass and the Line to be measured, to the Altitude of the measured line; which proportion is found by the *Golden Rule of Proportion*.

2. *By a Shadow.*

1. The Sun shining, Erect a Rod or Rule of certain known feet, on some Plane.

2. In the same measure of Feet, take the length of the shadow which the Rod or Rule casts.

3. Likewise in the same Moment of Time, take the length of the shadow cast by the Altitude of the Thing to be measured: Then,

As the shadow of the Rod, to the Rod: So the shadow of the Thing to be measured, to the height of the same.

3. *By any Rod or Ruler whatsoever.*

1. Erect the Measuring Rod of a known length, on a Plane over against the Thing to be measured.

2. The Measurer must go backward from the Rod, until the Visive Ray, from the Eye of the Measurer, extend by the Top of the Rod, to the Top of the Thing to be measured.

3. Measure one and the same measure of Space, on both sides between the Measurer and the Rod, and between the Measurer and the thing to be measured; Then:

As the Distance between the Eye of the Measurer and the Rod, to the length of the Rod:

So also is the Distance between the Eye of the Measurer, and the Thing to be measured, To the Altitude of the Thing to be Measured.

N O T E,

The thing is the same, if to the thing it self to be measured, you Erect Perpendicularly, a Perpendicular on a Plane, that the subjected point may be approached unto, and the Measurer lying down on the ground, must look by the Extream of the Rod: And if he stands erect, he must take only that length of the Rod which appears above the Eye: Or the Difference between the Rod and the Stature of the Measurer, from his Eye to his Foot: And the Operation being done, to the Altitude found he must add the height of the Measurer, from his Eye to his Foot: Which Observation takes place, not only in this, but also in the following ways of Measuring,

4. *The*

4. *The Trigonometrical way by the Canon of Sines, and indeed,*

1. *From one Station, if we may approach to the thing to be measured. Fig. 94.*

The Tower to be measured is bc , distant from the Measurer 200 Feet.

1. Take by a Quadrant, or such like Instrument, the Angles at b and a , one of which being given, the other is given also, if we Subtract the given Angle from 90° : And let the Angle at a be $29^\circ : 40'$, at b $60^\circ : 20'$.

2. Make it, As the Radius 100000, to the Tangent of the Angle at a 569620: So the Distance ae , or dc 200 Feet; to the Altitude of the Tower be , Feet 113 $\frac{1}{2}$: To which, if you add ec , or ad , the height of the Measurers Eye, you will have the whole height bc .

2. *From two Stations, if the thing to be measured be inaccessible, Fig. 95.*

Let the same Tower bc be to be measured from the first Station d , and the other f .

1. In the first Station d , take the Angle abe 44° , whose Tangent ae is 96569.

2. In the second Station f , find the Angle gbe , $60^\circ : 20'$; whose Tangent ge is — 175556.

3. Take also the Distance between the first and second Station df , which let be Feet 90.

4. From the greater Tangent ge — 175556.

Take away the lesser ae — 96569.

The Difference of them is ga — 78987.

5. Hence I collect the Distance unknown between the first Station, and the Tower, thus:

As the Difference of the Tangents ga 78987: To the less Tangent ae 96569.

So the Distance of the two Stations known, ga 90 Feet, To the Distance ae of the first Station, and of the Tower unknown ge 110 Feet.

6. These

6. These two Distances, 90 and 110 Feet, I join; so that the whole Distance from the second Station to the Tower, is made 200 Feet.

7. Then say at last.

As the greater Tangent $g e$ ————— 175556.

To the Radius ————— $b e$ ————— 100000.

So the whole Distance — $g e$ Feet ————— 200.

To the Altitude of the Tower from
the Eye, $b e$ ————— } 113 $\frac{16117}{175556}$.

To which add $e c$, or the Stature of the Measurer, from the Eye, $g f$.

IX. Given the Distance between the Eye, and some Line erected Perpendicularly, To find the Hypothenuse.

In the Station d , let the Hypothenuse to be found be $a b$.

1. Observe the Angle at the Eye of the Measurer a .

Which is in Fig. 94. ————— $29^{\circ} : 40'$.

Whose Secant is $a b$ ————— 115085.

2. The Distance given $a e$ is Feet ————— 200.

Say then:

As the Radius $a e$ 100000, to the Secant $a b$ 115085, of the Angle at a $29^{\circ} : 40'$.

So the Distance $a e$ 200 Feet; To the Hypothenuse $a b$ 230 $\frac{1}{11}$.

(These Hypothenuses shew the lengths of Ladders to be used to Walls or Towers.)

X. To measure a Line before us, projected on a Plane. Fig. 72.

Among divers ways this is one:

1. The line to be measured is $e a$: Then standing in one of its Extrems e , leave there a sign; advance in a Right Line, and at Right Angles, to some certain Distance in d , and there again put some sign in d .

2. This

2. This being done, Repeat the first Station at e , and from its Sign, go backwards to some longer distance, yet so that you always remain in the same measured line ae ; and there likewise place a third sign c , and measure the Distance between c and e .

3. Then again turn your self from c , according to the line aec , at a right Angle, and go forward until the sign placed at d , incur between thy sight, and the extream of the line to be measured a .

4. That will happen at b : Then take away the Interval de from the Interval bc , that the difference fb may be had.

Then say :

As fb , the difference of the Intervals bc and de , to the Interval ce , or df .

So de is to ea , the line to be measured.

N O T E :

If de be some Staff erected of a known length : bc the height of the Measurer, from Eye to Foot ; fb , the difference between the Staff, and the height of the Measurer, from his Eye ; ce , the distance between the Staff and the Measurer, the thing will be performed the same way.

O R,

If in dc you erect a Rod or Staff from d , and take the Angle at a , you may make it,

As the Tangent of the Angle dae , to Radius :

So the length of the Rod de , to the line to be measured ac .

(In this manner the distances of Ditches, and of Rivers : Likewise the distances of the Position of things to be looked at, may be measured.)

XI. To measure a Line declining downwards Perpendicularly from the Eye. Fig. 19.

The line bc is to be measured, by the Eye constituted at b , falling downwards Perpendicularly.

1. Note

1. Note the distance of the point a , from the point c , in the other Extream of the line to be measured.

2. Look from the point b to $a c$, and take the Angle $b a c$: Then it will be,

As the Radius, to the Tangent of the Angle $b a c$:
So the distance $a c$, to the line to be measured $b c$.

By this Artifice may be measured the Depth of Wells, in which $a c$ represents the Diameter of them at the bottom ; which may be had by the Diameter at Top.)

But these, and such like, which instruct to the measuring of things, are not learned rightly, but by daily use and exercise, and by the right handling of Instruments, especially the Geometrical and Astronomical Quadrants, of whose make and use several have writ ; which the studious in the Mathematicks ought not to want, for by this Instrument Angles are found ; which if we have, with one thing or other given, and a Canon of Sines, there is nothing almost but we may measure.

O F

GEOMETRY.

PART V.

Of the Dimension of Planes.

T H E O R E M S.

1. **T**HE number shews the order of right line Figures, as well of the Sides, as of the Angles, by taking away two.

So a Triangle is the first of right line Figures, containing

taining 3 Sides, or 3 Angles ; from which, if we cast away two, there remains one : *The Quadrangle is the second Figure*, having 4 Sides and 4 Angles, from which, two being taken away, there remains two.

2. *All the Angles of any right line Figure, are equal to twice so many right Angles, as there is between the right line Figures.*

So the three Angles of the first Triangular Figure, are equal to twice one right Angle, that is, two right Angles : And the Angles of the second right lined Figure, being Quadrangular, are equal to twice two right Angles, that is, to four right Angles ; and so of the rest.

3. *All the Angles of any right lined Figure, are equal to twice so many right Angles, abating four right Angles, as it contains Sides or Angles.*

So the Angles of any Triangle are equal to twice 3, or 6 right Angles, 4 being taken away, that is, to two right Angles : The Angles of a Pentagon Figure are equal to twice 5 or 10 right Angles, abating 4 ; therefore together 'tis equal to 6 right Angles.

4. *The opposite Sides of a Parallelogram are equal between themselves, also the opposite Angles are equal, and the whole is cut in two in the middle by the Diagonal.*

5. *In a Quadrate, and Rhombus also, the opposite Angles are cut in two in the middle by the Diagonals.*

'Tis otherwise in those that are longer on one part than the other, and in the Rhomboid.

6. *In the Quadrate, and that which is longer on one part, the Diagonals are equal : In the Rhombus and Rhomboids unequal.* See Fig. 2, 29, and 15, and 30.

7. *A right line cutting the Diagonal any how in the middle, divides also the Parallelogram in two in the middle, as the right line e f in the Rhomboid, in Fig. 30.*

8. If

8. If from any Angle of a Triangle, a right line be drawn Perpendicularly, it divides the opposite Side in the middle, also it will cut the Triangle in two in the middle.

In Fig. 24. So as ec cuts the Base df in the middle, it also cuts the Angle at e in the middle. 24

9. If a right line be the Double of another right line, the Square of it will be Quadruple of the Square of that, and the contrary.

So the Square made from the right line ab $a \text{ --- } b$
Will be Quadruple of the Square of the right line $c \text{ --- } d$
And this Subquadruple of that.

10. If in a Square, any Diagonal be drawn, the Square described from the Diagonal, will be the Double of the aforesaid Square. Fig. 28. 28

In the Square $abcd$, the Square of the Diagonal ac , is the Double of the Square $abcd$: for 'tis divided by the Diameter into two Rectangled Triangles, and it is the Subtense of them, the Square of which is equal to the Squares of the Sides.

11. The Square of the Diagonal of an Oblong, or of a Figure longer on one part, is equal to two Squares of the unequal Sides.

To wit, for this reason, because also the Diagonal of the Oblong, divides it into two Rectangle Triangles.

12. Among all Equilateral and Equiangular Figures, only the Triangle, the Square, and the Hexagon can fill the place; that is,

In a Plane, they may be so fitted mutually between themselves, that between the Angles of them there is no vacuity, but all of them constitute one Plane Superficies.

13. The Area of any Regular Figure, is equal to the Rectangle contained under the Perpendicular, derived from the Center of the Figure to one Side, and under the half of the Ambit or Periphery of the same Figure.

For in this manner Rectangled Triangles are constituted

stituted, one side of which is the Perpendicular, including the Rectangle, which multiplied into the half of the other, produces the Area; and that which is the Ratio of the Perpendicular, and one of the Sides, is the same of all the other.

14. *Figures (which have equal Ambits, or Peripheries, and of which the Terms taken together are equal) are not of the same Capacity: But among those whose Sides are equal in number, the Regular Figures are the most capacious, that is, those which are Equilateral and Equiangular: And among those Regulars, the Multilateral are the most Capacious, therefore the Circle is the most Capacious of all.*

Geometricians say, by how much a Figure is more Terminated, it is the more Capacious, and that is said to be more Terminated, which hath most Terms or Sides: Hence a Circle is the most Capacious, because it hath, as it were, infinite Sides, and infinite Angles, because of the multitude of points, of which the Periphery consists. See Fig. 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107.

P R O B L E M S.

I. **U**pon a given Right Line, to describe a Square. Fig. 28.

To make a Square upon the Right Line $a b$.

1. On the given line $a b$, from one of its Extrems raise the Perpendicular $a d$, equal to the given line, that you may have two Sides of the Square to be described.

2. From the point d , with the Interval $d a$, or $a b$, describe a small Arch, and the like from the point b , cutting the former in c .

3. Join the Right Lines $b c$ and $d c$, and the Square will be made. (You proceed in the same manner also in describing an Oblong Rectangled Quadrangle, only in raising the Perpendiculars, the opening of the Compasses must be accommodated to the unequal lines given.)

II. *Many Squares being given, either Equal or Unequal, to make one Equal to all.*

Let the Sides of 4 Squares be A, B, C, D .

Act thus.

A ———
B ———
C ———
D ———

Fig. 108.

Fig. 108, To the line ef , equal to A , raise a Perpendicular eg , equal to B , and join fg ; to which again raise a Perpendicular gh , equal to C , and draw fh : Then, to fh raise the Perpendicular hi , equal to the Right Line given D , and draw fi .

The Square made upon fi will be equal to the four Squares made upon $ABCD$.

III. To make a Square equal to a given Triangle.

Fig. 109.

The given Triangle is acb , equal to which a Square is to be made.

1. Continue the Base ab from the part a towards b .
2. By the opposite Angle c , make a Parallel to ba continued to b , which is iec .
3. By the Angle at A , erect a Right Line Perpendicular to the Base, which is ea , cutting the Base in a .
4. Cut the Base ab in the middle in d , and set the half dd , from a to f .
5. Cut fe in the middle in g .
6. From g as a Center, with the Interval gf or ge , describe the Semicircle fhe , which will cut ab continued in b .

The Square made upon ah , will be equal to the Triangle acb given.

IV. To make a Square equal to any Rectilinear Figure given, whether it be Quadrangular or Multangular.

1. Resolve the Rectilinear into Triangles, to wit, by drawing Right Lines from each of the Angles into the Rest.

2. Then by the Antecedent Problem find the Sides, whose Squares are equal to these Triangles in Order.

3. Then by the second Problem, make a Square equal to all the Squares of the Sides, which will be

Q

equal

stituted, one side of which is the Perpendicular, including the Rectangle, which multiplied into the half of the other, produces the Area; and that which is the *Ratio* of the Perpendicular, and one of the Sides, is the same of all the other.

14. *Figures (which have equal Ambits, or Peripheries, and of which the Terms taken together are equal) are not of the same Capacity: But among those whose Sides are equal in number, the Regular Figures are the most capacious, that is, those which are Equilateral and Equiangular: And among those Regulars, the Multilateral are the most Capacious, therefore the Circle is the most Capacious of all.*

Geometricians say, by how much a Figure is more Terminated, it is the more Capacious, and that is said to be more Terminated, which hath most Terms or Sides: Hence a Circle is the most Capacious, because it hath, as it were, infinite Sides, and infinite Angles, because of the multitude of points, of which the Periphery consists. See Fig. 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107.

P R O B L E M S.

I. **U**pon a given Right Line, to describe a Square.
Fig. 28.

To make a Square upon the Right Line *a b*.

1. On the given line *a b*, from one of its Extrems raise the Perpendicular *a d*, equal to the given line, that you may have two Sides of the Square to be described.

2. From the point *d*, with the Interval *d a*, or *a b*, describe a small Arch, and the like from the point *b*, cutting the former in *c*.

3. Join the Right Lines *b c* and *d c*, and the Square will be made.
(You proceed in the same manner also in describing an Oblong Rectangled Quadrangle, only in raising the Perpendiculars, the opening of the Compasses must be accommodated to the unequal lines given.)

II. *Many Squares being given, either Equal or Unequal, to make one Equal to all.*

Let the Sides of 4 Squares be *A, B, C, D*.

Act thus.

A ———
B ———
C ———
D ———

Fig. 108.

Fig. 108, To the line ef , equal to A , raise a Perpendicular eg , equal to B , and join fg ; to which again raise a Perpendicular gh , equal to C , and draw fh : Then, to fh raise the Perpendicular hi , equal to the Right Line given D , and draw fi .

The Square made upon fi will be equal to the four Squares made upon $A B C D$.

III. *To make a Square equal to a given Triangle.*

Fig. 109.

The given Triangle is acb , equal to which a Square is to be made.

1. Continue the Base ab from the part a towards b .
2. By the opposite Angle c , make a Parallel to ba continued to b , which is iec .
3. By the Angle at A , erect a Right Line Perpendicular to the Base, which is ea , cutting the Base in a .
4. Cut the Base ab in the middle in d , and set the half ad , from a to f .
5. Cut fe in the middle in g .
6. From g as a Center, with the Interval gf or ge , describe the Semicircle fhe , which will cut ab continued in h .

The Square made upon ah , will be equal to the Triangle acb given.

IV. *To make a Square equal to any Rectilinear Figure given, whether it be Quadrangular or Multangular.*

1. Resolve the Rectilinear into Triangles, to wit, by drawing Right Lines from each of the Angles into the Rest.

2. Then by the Antecedent Problem find the Sides, whose Squares are equal to these Triangles in Order.

3. Then by the second Problem, make a Square equal to all the Squares of the Sides, which will be

Q

equal

equal also to all the Triangles, and consequently will be equal to the given Figure; for because all the parts taken together is equal, it also agrees to the whole.

V. *To increase or diminish a given Square in any Multiplex Proportion, Fig. 66.*

1. Let bc and ab be the Sides of a Square to be increased.

Then having produced one Side of the given Square to be increased bc , from the other Extream of the Side a , draw the Diagonal ac , which will be the Side of a Double Square, or a Square as great again.

2. Apply this Diagonal to the produced Side, it makes bd : Again, draw from a , the Transverse Line ad , which will be the Side of a Triple Square, or three times as great.

3. Place this Transverse Line ad on the produced Side, and to its Extream make another Transverse Line from the former point a , which is ae , and you will have the side of a Square four times greater.

And thus you may proceed to Quintuple, Sextuple, &c. in *infinitum*: And likewise you may diminish, by returning back again: For Example, if from the Side of a Sextuple Square, you make a Transverse Line, you will have the Side of a Quintuple: If from the Side of a Quintuple you make a Transverse Line, it will be the Side of a Quadruple Square, and so on: Altogether in the same manner as is explained before, for the increasing or diminishing of a Circle, whose Figure also serves here, if, as is said, from two Semidiameters of a Circle you make a Square.

VI. *To make a Table of Multiplication of Squares.*

If you make the Radius or Side of the first Square of 100 parts, so that the Square of it be 10000 of the same parts, and then double this Sum, Triple, Quadruple,

Quadruple, Quintuple it, you will find the Roots of each Multiplex, *proxime*, and acquire the Sides Multiplex of the same, in parts of which, the Sides of the first Square is 100.

A Canon of the Sides of Squares, in Parts 10000.

| Measures. | Parts. | Measures. | Parts. |
|-----------|--------|-----------|--------|
| 1 | 100 | 6 | 245 |
| 2 | 141 | 7 | 264 |
| 3 | 173 | 8 | 283 |
| 4 | 200 | 9 | 300 |
| 5 | 224 | 10 | 316 |

Thus you may increase this Table as far as you please.

VII. *From the Side of any Multilateral Regular, consisting of Parts 100000; To find the Perpendicular falling from the Center of the Figure to one Side.*

1. The whole Circle being of Parts 360, divide by the number of Sides, of the given Multilateral Regular.

2. Take the half of the Quotient, which is the Angle of the Figure, and the Complement of this half to a Quadrant.

3. Find the Tangent of this Complement, and make it, As the Radius, To half of the given Side: So the Tangent of the Complement of half the Quotient, or of the Angle of the given Figure, To the Perpendicular sought.

Example.

Given an Equilateral Pentagon, whose Perpendicular is to be found.

The Parts of the Circle are ———— 360°.

Divided by the Number of Sides ———— 5.

Produces in the Quotient ———— 72.

Q. 2

The

The half of this is ————— 36.

Its Complement to a Quadrant is ————— 54.

The Tangent of this Complement is — 137638.

The Side of the Pentagon in Parts — 100000.

The half of it ————— 50000.

Say then:

As the Radius } To half } So the Tangent of the Compl.
 100000 } the Side } of half the Quotient
 50000 } 137638.
 To the Perpendicular 68819.

In this Proportion also you may find a Perpendicular to any Side given in other Numbers.

VIII. To find the Area of a Rectangled Quadrangle.

We measure Areas, by the Superficies of any Square measure: Therefore in a Rectangled Quadrangle, the Length drawn into the Breadth, being estimated by a certain measure; the Product will be the Area of the given Quadrangle:

Example.

A Plane whose Length is 40 Feet, and its Breadth 20 Feet, the Area will be 800 Square Feet.

IX. To find the Area of a Rectangled Triangle.

Since every Rectangled Triangle is the half of that Rectangled Quadrangle: Therefore of the Legs comprehending the Right Angle, draw one into half the other, the Product is the Area of that Triangle: Or draw one of the Legs into the other, and the half of the Product will be the Area of the Triangle.

X. To find the Area of an Equilateral Triangle.

1. Take the Square of one Side.
2. Multiply that by 13.

3. Divide

3. Divide the Product by 30; the Quotient is the Area of the given Triangle.

Note, Because an Equilateral Triangle, whose Side squared is 1, hath its Area $\frac{1}{4}$; 'tis just so, as if by the Golden Rule of Proportion, you say,

The Triangle, the Square of whose Side is 1,
Contains in its Area ————— $\frac{1}{4}$.

What will a Triangle contain, whose
Side squared is ————— } 25

The Operation being performed, it produces $\frac{25}{4}$.
That is, $10\frac{1}{4}$, or $10\frac{1}{4}$.

XI. To find the Area of any Triangle, Fig. 20 and 24.

I. *W A T.*

1. Find the Perpendicular falling from some Angle to the Base, either true or protracted.

2. Draw this Perpendicular into half the true Base; or on the contrary, draw the true Base into half this Perpendicular, and the number produced will be the Area of the given Triangle.

II. *W A T.* Fig. 83.

1. Collect the three Sides of the given Triangle, which are, for Example, 10, 17, 21, into one Sum, and they make 48.

2. Take the half of the Sum, which is — 24.

3. From this half subtract each Side, that you may have the three differences, 14, 7, 3.

4. Multiply continually each into the Half, to wit, 14 into 24, whence is made 336; then 7 into 336, whence is produced 2352: Lastly, 3 into 2352, whence is made 7056.

5. The Square Root of this last product, to wit, 84, is the Area of the Triangle.

III. *W A Y*, by *Trigonometry*.

From two Sides given, and an Angle comprehended by them, in this manner.

1. Take the right Sine of the given Angle.
2. Multiply the two Sides comprehending this Angle, and the given, in themselves.
3. Take half the Product, and say, As the Radius, To the Sine of the Angle given : So half the Product, To the Area sought.

XII. *To find the Area of a Rhombus.* Fig. 29.

Draw one Diagonal into the other, and half the Product will give the Area of the Rhombus.

XIII. *To find the Area of a Rhomboides.* Fig. 30.

From some one Angle draw a Perpendicular to the greater side, multiply it into the whole side, and it will produce the Area of the Rhomboid.

XIV. *To produce the Area of a Trapezium, whether Rectangled or Oblique Angled, having two Sides Parallel.* Fig. 25.

Draw the Sum of the two given Parallel Sides into half the Perpendicular, the Product is the Area sought.

Example.

Add cc 15, and ab 10, they make 25, multiply it by half the Perpendicular 5, it makes 125, the Area.

XV. *To find the Area of a Multilateral Regular Figure.*

Multiply half the Ambit into the Perpendicular, drawn from the Center of the Figure, to one Side.

Example.

A Pentagon is to be measured, whose Perpendicular

lar contains of a certain known measure of Parts 4.

Any Side hath of the same Parts ——— 6.

So that the Ambit or Sum of the Sides make 30.

The Half of which is ——— 15.

Which drawn into the Perpendicular ——— 4.

Produces the Area ——— 60.

Which may also be produced, if you draw the Sum of the Ambit into half the Perpendicular: And how to find the Perpendicular is taught at the 7th Problem, at Page 227.

XVI. To find the Area of a Multangular Irregular Figure.

Divide the Figure as Compactly as you can, either into Quadrangles, or into Rectangled Triangles, of which seek the Area of each, and add them into one Sum, they will make the Area of the whole Figure.

XVII. To divide a Triangle into equal Parts, meeting in one Angle. Fig. 111.

1. Divide the Base ac of the Angle b , from which you desire to institute the Division, into so many parts as the Triangle is to be divided, as in this place into 3, ad, de, ec .

2. Join the points of this Division by Right Lines with the Angle, as db, eb , and the Triangle will be divided into three equal Parts.

XVIII. To divide a Triangle into three or more Parts, so that the Lines of Division meet not in one Angle. Fig. 112.

Let the Triangle be abc , to be divided into three equal parts: From the Base ac , first cut off a third part ad , and join it with the point b , by the line db : Again divide this line bd in the middle in e , and draw the Right Line ec , and the Triangle will be divided, as desired.

113
XIX. To divide a Triangle into 2 or 3 equal Portions from some given point, so that that point being assumed in any Side may be common to all the Parts. Fig. 113.

Let the Triangle be abc , to be divided from the point g into three equal parts, so that the point g , may be common to all, and that there may be access from all parts to it.

1. Divide the side ac in which the point g is posited, into some equal parts; for Example, into three, which let be ah, hi, ic .

2. From the point g , draw a Right Line into the opposite Angle b .

3. From h and i draw two Parallel Lines hk, il , parallel to gb , even to the Sides of the Triangle abc .

4. Join the Extrems kl , with the given point g , and so the Triangle will be divided into three equal parts, as was required.

114
XX. To divide a Parallelogram in the middle by any given point, so that the dividing line may include that point. Fig. 114.

The Parallelogram $abcd$, is to be divided into two equal parts by the point e .

1. Draw the two Diagonals ad and bc , that their intersection may shew the Center o .

2. By this Center and the given point e , draw another Right Line, which will divide the Parallelogram in the middle by the given point.

115
XXI. To divide any Regular Figure into as many Parts as you please. Fig. 115.

Circumscribe the Circle of the given Figure, which divide into the given parts; in the same manner also the inscribed Figure will be divided, by drawing Right Lines from the Center to the points of Section of the Circle.

XXII. To divide any Irregular Figure in the Middle.

Let any Irregular Oblong Figure be to be divided into two equal parts, according to the Breadth.

1. Divide it into two parts whatsoever, equal to one another by Estimation.

2. Mea-

2. Measure both, and if you find them unequal ; for Example, one of 2656 Square Perches, the other of 8 Square Perches 2592.

3. Take the Difference of the Perches 64.

4. The half of this Difference 32, must be taken from the greater part, and added to the lesser.

5. Then measure the line which divides the Figure by Estimation according to the Breadth, and let it be 46 Perches, and 8 Feet.

6. Then since if I add to the lesser part of the Parallelogram, one Perch in Breadth, and 46 Perches and 8 Feet in Length, it will make it greater than Just, for it wanted only 32 Perches : Therefore we shall find the true part to be added, if we say :

If 46 Perches and 8 Feet give 1, what doth Perches 32 give ?

Or if 468 Feet give 10 Feet, What doth ——— 320 give ?

Answer, 6 Feet 8 Inches, that is, the Perch of 10 Feet, being divided into 10 Feet, and a Foot into 10 Inches or parts.

Wherefore if you constitute a Parallel line, distant from the former line of Division 6 Feet 8 Inches, towards the lesser part, it will divide the Figure proposed into two equal parts.

By the like manner of working we may cut off a third part, a fourth part, or any other part, from any proposed Figure.

AN APPENDIX.

Of Land Measuring.

1. **F**ROM these Rules of Measuring of Areas, (which I have briefly run over, because the Fundamentals of them are already delivered before) depend Land Measuring, and Measuring and Dividing of Fields, which was first found out by the *Egyptians*, afterwards the name of *Geometry* was given to it.

2. Of this, some make a peculiar Discipline, which notwithstanding may conveniently be included more generally ; that is to say, which teaches to measure the Areas of Plane Figures, whether of Fields, or any other Space.

3. The

3. The famous or common Measure (as 'tis called) which Land Measurers are wont to use, which in *Germany* is 16 Feet, or a Perch divided into 16 Feet, but here in *England*, according to a Statute, a Pole or Perch contains 16 Feet and an half.

4. The *Ratio* of these, which is Multiplication and Division, is Treated of before in *Civil Arithmetick*.

5. Where also we suppose enough to be said concerning the Kinds or Species arising by Multiplication : The Sum of which is,

1. Feet multiplied by Feet (for in measuring of Land they seldom descend to Inches) produce Square Feet.

2. Feet multiplied by Perches, produce Perches a Foot broad.

3. And as 16 square Feet and an half, make one Perch a Foot broad, So $16\frac{1}{2}$ such Perches a Foot broad, make a square Perch.

4. Which square Perches also are produced, if you multiply Perches by Perches ; of which 160 make an Acre of Land here in *England*, and 150 in *Germany*.

6. *Note*, In the measuring of Fields that are not accessible, that about those kind of Fields 'tis usual to constitute an Oblong Rectangle ; to be measured in the same manner, as if this which is assumed did belong to the Field to be measured : But afterwards from the sides of this Oblong, Perpendiculars must be found to each Angle of the Field to be measured, which Excess, whereby the Circumscribed Oblong Quadrangle is greater than the Area of the Field, will be cut into lesser Triangles, or Quadrangles, the Area of each of them being added into one Sum, and that Sum being taken from the same Circumscribed Oblong Quadrangle, that which is the Remainder, will be the Area desired of the Field to be measured.

7. But if in the Circuit of a Field some Portions are Crooked, and not Right Lines, those must be cut into so many parts, until they differ but little from Right Lines.

The

The other things to be observed in the measuring of Fields, are learned more by Use than by Precepts.

O F

G E O M E T R Y.

P A R T VI.

Of the Dimension or Measuring of Solids.

T H E O R E M S.

1. **O**F Regular Plane Bodies, which are more Simple, or Plane, and alike, there are only Five: that is, 1. The Tetraedron. 2. The Octaedron. 3. The Dodecaedron. 4. The Icosaedron. 5. The Cube. See Fig. 46, 47. 48, 49, 50.

And indeed all Bodies may in some manner be said to be Regular, which consist of Superficies, being either all or some of them Uniform and Equal, to wit, in respect of Irregular, which have Superficies wholly unequal and deformed: But by *Euclid*, Regular Bodies are said to be those which are most simple, and alike, containing equal Sides and equal Angles: Equal between themselves, since all others consist of Superficies that are divers, and unlike, as, The Pyramid, The Prism, The Cylinder, The Cone, &c.

2. These are called *Platonick* Bodies, by *Plato* in *Timæus*; which are named most simple by *Aristotle*, the five Bodies of the World, that is to say, Heaven, Fire, Air, Water, Earth, which he likens or compares

pare to these five Bodies ; even as *Aristotle's* Sentence is received of *Plato* : But in what manner, from the more plausible Sentence of the *Pythagorians*, *John Kepler*, with the Applause of *Michael Maestlin*, our Master, hath built the whole World of them, as may be seen in his *Cosmographical Mystery*, and *Harmony of the World*.

And that there cannot be more than five of these kind of Regular Bodies, Geometricians demonstrate at the end of the Thirteenth Book of *Euclid*, as you may see.

2. Of *Gibbous Bodies*, the most Regular and Perfect, is the *Sphere* or *Globe*.

This Body hath the most, and those the most excellent Properties, some of which we shall briefly touch upon.

1. Therefore a *Sphere* is one, and the most Simple among all Figures, consisting of one Superficie every where alike to it self.

2. 'Tis also First or Chief, because 'tis begat of of no other Figures, neither is it begat like other Figures, and it performs the points and place in the begetting of others.

3. Since it Terminates all other Figures, it self is Terminated and defined by no other than by its own Figure returning into it self.

4. Other Figures want many things besides themselves, the *Sphere* depending only on it self, requires nothing besides it self, which is not in it.

5. In every *Sphere* are three things ; the Center, the Superficie, and the Equality of Interval or Space ; of which, one being denied, the rest fall together, and are distinct between themselves, and one is not without the other, yet all make one Spherical Figure.

6. The Center is, as it were, the Origine of the *Sphere*, from which the Superficie is understood to proceed by a number of infinite Right Lines, communicating

municating themselves to every part of the World, according to the Equality of Intervals.

7. And although the Center is invisible in it self, yet it shews every where the most Equal flowing of the Superficie, by means of the Equality of Space.

8. Therefore the Superficies is the Character and Image of the Center, and as it were the glory of it, and the way to it; and he that sees the Superficie, he also in the same manner, with it sees the Center.

9. And the Interval or Space results from the comparing of the Center with the Superficie, and it shews the Profundity or Depth of the Figure.

10. *Spheres*, as they are the most Capacious of all Figures, so are they the Receptracles of all others, but principally of the five Regular Bodies, whose Angles rest upon the Extreame Points of the *Spherical Superficies*, the other Bodies are extended by the Interval of its Amplitude, and are sustained in it.

He that attentively considers these, and such like things, may understand, not obscurely, whatsoever is the adumbration of the Mystry of the Holy Trinity in one Divine Essence; and think with himself, if such things have place in Corporeal Things, and Mundane Affairs, that the like kind which Holy Writ delivers concerning this Mystry in Spiritual Things, and purely Divine, are infinitely Sublime, and cannot be denied by Anti-Trinitarians.

3. *The five Regular Bodies approach very near to the Perfection of the Sphere.*

For, 1. As the *Sphere* is the most Simple, because 'tis inclosed by one Term, that is, of it self, and all its points are equally distant from the Center: So the five Regular Bodies have all Plane Sides and Angles, every one equal both in Kind and Magnitude, which is Simplicity: From which it necessarily follows, that the Center of all their Planes is also equally distant from the middle, forasmuch as they
— being

being inscribed in a Globe, all the Angles touch the Superficies, because they remain in it : Also since they touch the inscribed Globe by all the Centers of their Planes : Because therefore the inscribed Globe continues Immoveable, and hath the same Center with the Figure, from which things are effected.

2. The other likeness of them with the Sphere, which is from the Equality of Distances of the Planes.

4. The Figure and Number of the Planes, Sides, and Angles, such as is in the Five Regular Bodies.

| | | Its Plane. | Planes. | Sides. | Ang. |
|------------------|--------|--------------|---------|--------|------|
| The Cube. | } Hath | Quadrangled | 6. | 12. | 8. |
| The Octaedron. | | Triangled | 8. | 12. | 6. |
| The Dodecaedron. | | Quinquangled | 12. | 30. | 20. |
| The Icosaedron. | | Triangled | 20. | 30. | 12. |
| The Tetraedron. | | Triangled | 4. | 6. | 4. |

5. Among the five Regular Bodies, the Cube, the Tetraedron, the Dodecaedron, are the Chief or First Sort ; the Octaedron, and Icosaedron, are Secondary, or of the Second Sort.

For those three have the more excellent Beginning, and the most simple Angle, that is, Trilinear, and each Plane proper or peculiar : The two latter have their Beginning from the Primary or Chief, and the Angle of many Lines, and more compounded, and the Planes borrowed ; to wit, being Triangular, they are borrowed from the Tetraedron.

6. Again, of the Chief, these have the Rank of Priority between themselves : the Cube, the Tetraedron, the Dodecaedron.

For the Cube is generated only from its Base ; but the Tetraedron is divided or cut from the Cube by rejecting 4 Rectangled Pyramids : And the Dodecaedron is the Cube increased, to wit, by applying or joining 6 Pentaedrons.

7. Of the Secondary, or Second Sort, the First is the Octaedron, the Latter the Icoſaedron.

For the Octaedron, the First in its Order, arises as it were from a Cube, a Tetraedron, and a Pyramid; and from that it borrows the number of its Sides, and from this its Triangular Base.

But the Icoſaedron, the last in its Order, is begat from the Tetraedron and Dodecaedron: For again it borrows from that the Base, from this the number of its Sides.

8. The Proportion of the Semidiameter of the Orb, Circumscribing the five Regular Bodies, to the Sides of the same Planes Circumscribed, and the Semidiameters of the inscribed Orbs are thus:

| | | | | | | |
|---|---|----------------|---|-------------------------|---|------|
| Of which the Semidiameter of the Orb Circumscribing any Fig. is Parts 1000. of such parts is in — | { | The Cube | { | The Length of the Side. | { | 1155 |
| | | The Tetraedr. | | | | 1633 |
| | | The Dodecaedr. | | | | 714 |
| | | The Icoſaedr. | | | | 1051 |
| | | The Octaedr. | | | | 1414 |

| | | | | | | |
|--|---|---------------------|---|---------------------------------------|---|-----|
| The Semidiameter of the Circle circumscribed by the Planes | { | C 816 $\frac{1}{2}$ | { | The Semidiameter of the Inscribed orb | { | 577 |
| | | T 943 | | | | 333 |
| | | D 607 | | | | 795 |
| | | I 607 | | | | 795 |
| | | O 816 $\frac{1}{2}$ | | | | 577 |

9. Spheres are to one another in Triple Ratio or Proportion of their Diameters.

And Triple Ratio, or Proportion, is made from the Cubick Multiplication of the Terms of the given Ratio: If therefore we multiply the Diameters Cubically, it produces the Cubes of the Terms; and by dividing, it produces the Proportion of the Sphaeres.

Example.

Let the Diameter of some greater Sphere contain the Diameter of a lesser Globe or Sphere, times 5: so that the Proportion between them be Quintuple Sefquialter, which is as 11 to 2.

Then

Then the Cube of 2 is ————— 8.

And the Cube of 11 is ———— 1331.

Divide this by 8, it produces in the Quot. 166 $\frac{1}{2}$.

And so many times the greater Globe exceeds the lesser.

10. *The Rectangled Quadrangle contained under the Diameter of the Sphere, and the Circumference of the greatest Circle of the Sphere, is Equal to the Superficial Convex Area of the whole Sphere.*

As if the Diameter of a Sphere be ———— 7.

The Circumference of the greatest Circle is — 22.

The Area of a Quadr. made from these Sides, 154.

Which is also the Convex Superficies of the whole Sphere, whose Diameter is 7, and the Circumference of its greatest Circle 22.

11. *The Quadruple of the Area of the greatest Circle, measures the bigness of the Convex Superficie of any Sphere, As,*

If the Diameter of the greatest Circle in a }
Globe be ————— } 7.

And the Circumference of the same ———— 22.

The Area of this Circle will by *Probl. 23,* }
of *Cyclometry*, at *Pag. 171*, be ————— } 38 $\frac{1}{2}$.

And the Quadruple of this, to wit, ———— 154.

Will be the whole Convex Superficies of the same Globe.

12. *As the Circumference of the greatest Circle of a Sphere, is to its Diameter; so is the Square of the Circumference, To the Convex Superficies of the Sphere. Or, As the Diameter of any Globe, is to the Circumference; so is the Square of the Diameter, to the Convex Superficie.*

Therefore we may say,

As the Circumference 22, To the Diameter 7:

So the Square of 22, the Circumference, which is 484, To the Convexity of the Sphere, 154.

Or,

Or, As the Diameter 7, to the Circumference 22 :
So the Square of Diameter 49, to the Convexity of
the Sphere 154.

13. *The Solidity of any Sphere, is Equal to the Rectangled Solid contained under the Semidiameter of the Sphere, and a third part of the Convex Superficie of the same Sphere.*

Therefore from the Semidiameter of a Sphere given, and its Convex Superficie, the solidity of the whole Sphere may be known.

14. *Two thirds of the Number produced from the Area of the greatest Circle drawn into its Diameter, is Equal to the Solidity of the whole Sphere.*

For from this Multiplication is constituted a Cylindric Figure, whose Base is equal to the greatest Circle of the Sphere, and its Altitude equal to the same : And such a Cylinder is to a Sphere inscribed in it, in proportion Sesquialter, that is, as 3 to 2, as Archimedes hath demonstrated.

15. *The Convex Superficies of a right Cone and Cylinder, the Bases being taken away, is equal to a Circle, whose Semidiameter is a mean Proportional between the Side of the Cone or Cylinder, and the Semidiameter of the Base of the Cone ; but the Diameter of the Base of the Cylinder :*

Stereometrical Problems.

Of making the five Regular Bodies.

I.

To Form the Tetraedron. Fig. 121.

OF some Matter, make 4 equal Equilateral Triangles, which being truly folded or put together, will make the Tetraedron according to the whole Solidity.

R

II.

II.

To make the Octaedron. Fig. 122.

Make eight Equilateral Triangles, equal between themselves, or to one another, of convenient Matter, and of them may be made a solid *Octaedron*.

III.

To make the Cube. Fig. 123.

Make and dispose six Squares, equal to one another, which being truly made and fixt, or glued together, will make a *Cube*.

IV.

To make a Dodecaedron. Fig. 124.

Make of some convenient matter, as Pastboard, or thin Wainscot, or such like, twelve Equilateral Pentagons, being Equiangled; which being truly folded up, or put together, will make a *Dodecaedron*.

V.

To make an Icosaedron. Fig. 125.

Fix or glue together twenty Triangles, that are Equilateral, and Equiangled, and you will have an *Icosaedron*.

Of the Dimension or Measuring of Bodies.

N O T E.

1. Even as we measure Superficies by square Superficies; So Bodies are measured by Cubical little Bodies of one Inch, Palm, or a Foot, in Length, Breadth and Depth: Or if the capacity be sought, then by a Cubical small Vessel of a certain Measure, as well in Dry Measures, as in Liquids.

2. There-

2. Therefore when the Solidity of any Body is to be found, seek how many times such little Body determinated by a known and just Quantity, is contained in the greater.

3. The Empty is measured as the Full: If therefore we know how much matter, either of Dry or Liquid, any Cubical little Vessel of one Palm, or one Foot, or of any other Magnitude contains, and we have a Rod or Rule of the inscribed Length of such little Vessel, we may measure the Cavity of any Regular Vessel, as if it were a Solid Figure, and know the whole Capacity.

Then how to measure the outward Superficie, as well as the Solidity of any of the Bodies, we will shew in the following Problems.

VI.

To measure the Convex Superficie of any Globe or Sphere.

1. Find the Diameter.

2. The Perimeter, or the Circumference of the greatest Circle; these multiplied into each other, produce the Convex Superficie of the Sphere, by *Theorem* the 10th, at *Pag.* 240.

Or, Take the Quadruple of the Area of the greatest Circle, and you effect the same thing, by *Theorem* 11. *Pag.* 240.

Or, Make it, As the Circumference of the greatest Circle, to its Diameter: So the Square of the Circumference, to the Convex Superficie, by *Theorem* 12. *Pag.* 240.

Or, Make it, As the Diameter, to the Circumference of the greatest Circle: So the Square of the Diameter, to the Convex Superficie, by *Theor.* 12.

VII.

To find the Solidity, or the whole Grossness of any Sphere.

1. Take $\frac{1}{3}$ part of the Convex Superficie of the Sphere, by dividing it by 3.

2. Draw this $\frac{1}{3}$ part into the Semidiameter, and you will have the solidity of the Sphere, by *Theorem 13. Pag. 241.*

Or, 1. Take $\frac{2}{3}$ parts of the Area of the greatest Circle

2. Draw these into the Diameter, and you will produce the same solidity, by *Theor. 14. Pag. 241.*

Or, Make it, As 21, to 11; so the Cube of the Diameter, to the solidity of the Sphere.

For the Cube of the Diameter of the Sphere, to the solidity of the Sphere, hath the Proportion *Superdecupartient* Eleven, the same as is between 21 and 11.

VIII.

To find the Convex Superficie of the Five Regular Bodies.

1. Find the Perpendicular falling from the Center of any Plane on its side, by *Theor. the 7th. Pag. 227.*

2. Multiply it in half the side.

3. Multiply again the Product.

| | | |
|---------------------------------------|--------|-------------------|
| In the Tetraedron ———— | } By { | 12.
24.
60. |
| In the Cube and Octaedron ———— | | |
| In the Dodecaedr. and Icosaedron ———— | | |

The last product of these will shew the Convex Superficie of the Body.

IX.

To measure the Solidity of the five Regular Bodies.

In the Cube.

The Cubic Number of any side given, is the solidity of the Cube; As if the side of a Cube be six Feet,

Feet, the solidity of the same will be 216 Cubic Feet. viz. $6 \times 6 = 36$ and $36 \times 6 = 216$.

But in the others.

1. Take half the Altitude.

(And this is had, if from the upper Plane of the Body produced : or from the lower Superficie of any Plane, which shall be placed upon the upper part of the Body, a Perpendicular be let fall to the Plane of the exposed Base, and we take the half of it.)

2. Take the Convex Superficie.

3. Take $\frac{1}{3}$ part of it.

Which drawn into half the Altitude, produces the solidity of the given Body.

X.

To find the Solidity of a Rectangled Parallelipipedon.

1. Draw the Length into the Breadth.

2. Draw the product into the Depth, and you will have the Solidity of the Parallelipipedon.

XI.

To find the Solidity of an Oblique Angled Parallelipipedon, or of a Rhombus, or a Rhomboid.

1. Take the Altitude of the Parallelipipedon.

2. Draw it into the Area of the Base, and you will produce the Solidity of the Parallelipipedon.

XII.

To find the Solidity of Prisms and Cylinders.

This is done in the same manner, by drawing their Altitudes into the Area of their Bases.

XIII.

To find the Solidity of Pyramids, and of Cones.

1. Assume the Area of the Base.
2. The third part of the Altitude.

These Drawn into each other, shew what is sought.

O R,

1. Assume the Altitude; and,
2. The third part of the Base, and proceed in the same manner: You will effect the same thing if you draw the Altitude into the whole Base, and take $\frac{1}{3}$ part of the Product.

XIV.

To find the Convex Superficie of a Cone, excluding, or not taking the Base.

Make it, As the Semidiameter of the Base, to the Area of the Base:

So the Side of the Cone, to the Conical Superficie.

XV.

To find the Cylindrical Superficie, excluding the Bases.

1. Take the Circumference of the Base.
2. The Height of the Cylinder.

The Multiplication of these, solves the Question.

XVI.

To double the Cube.

Between the Altitude of the Cube, and the Double of it, seek two Mean Proportionals, the lesser of these is the side of the Doubled Cube.

NOTE.

N O T E.

If the side of the first Cube be assumed of Parts 100.

| | | |
|---------------|--|------|
| The Double | <div style="display: flex; align-items: center; justify-content: center;"> <div style="font-size: 3em; margin-right: 5px;">{</div> <div style="text-align: center;"> Of that Cube, its
 Side will be of
 the same parts </div> <div style="font-size: 3em; margin-left: 5px;">}</div> </div> | 126. |
| The Triple | | 144. |
| The Quadruple | | 159. |
| The Quintuple | | 171. |
| The Sextuple | | 182. |
| The Septuple | | 191. |
| The Octuple | | 200. |
| The Noncuple | | 208. |
| The Decuple | <div style="display: flex; align-items: center; justify-content: center;"> <div style="font-size: 3em; margin-right: 5px;">{</div> <div style="text-align: center;"> Of that Cube, its
 Side will be of
 the same parts </div> <div style="font-size: 3em; margin-left: 5px;">}</div> </div> | 215. |

Which Table may be increased as far as you please, if you Double, or Triple, or Quadruple, &c. the first Cube from the side of 100 parts given, that is 1000000, and seek the next Cubic Root of each Multiplex.

The End of Geometry.

A S T R O N O M Y.

The P R E F A C E.

Of the Definition and Parts of Astronomy.

I. **A**STRONOMY, is a Doctrine or Science prescribing the reason of measuring the Sites, the Motions, the Distances, and Magnitudes of the Superior, or Celestial Bodies, visible to this World.

These Celestial Bodies are called Stars, the Sun and Moon, the Planets; hence is the name of Syderal Science, and of Astronomy, which in old time also was called Astrology: But the Moderns attribute the name of Astrology to that Art, whereby from the Motion of the Heavenly Bodies, and their Habitude to one another, they promise the Prediction of future things: But now we call Astronomy that most noble Art, which searches and displays the Motion, Distance, and Magnitude sought, of the Sun, Moon, and Stars, and all other appearances observed in the Heavens, from some Hypotheses received, agreeing to the Nature of the thing.

For an Astronomer first of all collects the many kinds or particulars of the Heavenly Appearances, by long observation of the Stars, &c. Afterwards he assumes Hypotheses, which being supposed, he necessarily demonstrates all those things which he hath collected by Observation, and what will follow in its time: And at length he either makes an Instrument by which those things partly are taken from Heaven, also they are partly set before the Eyes; or he Explicates the same by Numbers, shewing a Method, whereby at any time, either past, or to
Configu-

come, the Apparent Face of the Heavens, and the Configuration of the Stars, and Sun and Moon, may be computed ; hence are produced *Astronomical Tables*, either of Perpetual or Annual *Ephemerides*.

2. *We constitute Three Parts of Astronomy, accomplishing each in a single Book ; of which,*

The First Explains, The Spherical Doctrine.

The Second, The Theoretical.

And the Third, The Chronological Doctrine.

For since we understand in Heaven a double Motion of the Sun, Moon and Stars : *The First*, common to all of them, which is accomplished from East to West in the space of Day and Night, or for the most part in the space of 24 Hours : *And the Second*, Proper to every one, tending from West East ; both of which we usually measure by Time : Likewise *Astronomy* is truly divided into those three Parts : from which,

3. *The Spherical Doctrine*, Delivers the aforesaid Hypotheses, which shews to us the first and common Motion of all the Heavenly Bodies, and that by setting before our Eyes the Instrument of a Material Sphere, or of an Heavenly Globe,

4. *The Theoretical Doctrine*, proposes the Appearances of the second and proper Motion of every one of the Celestial Bodies, by certain Instruments of Contemplation or Theory.

5. *And the Chronological Doctrine* explains the Ratio, or Proportion of Time, depending on both these Motions.

O F
ASTRONOMY.
B O O K I.

Of Spherical Doctrine.

C H A P. I.

Of Astronomical Hypotheses in General.

1. **A** *Astronomical Hypotheses* in general, are some Artifices assumed, from which the Celestial Demonstrations are built upon.

They are called *Hypotheses*, because an *Astronomer* says, Supposing this, or that, then this or that follows.

2. And Astronomical Hypotheses are some *Physical*, others *Geometrical*.

3. *Physical*, are those which are borrowed by the Searcher, from *Physicks*, or from *Nature*, which he either supposes as True, or also further Confirms by his Observations.

Such are Assertions concerning the World, and the parts thereof, as of Heaven and Earth, the Substance, Figure, Place, and Scituation; concerning Cælestial Bodies, the Difference, Disposition and Order, likewise the Kind and Quality of the Celestial Motions, &c.

4. *Geometrical Hypotheses*, are things drawn from the Storehouse of Geometry, only indeed ascribed to Heaven by the Imagination, but conformable to the Nature of the thing, and therefore allowed by every one; from which the Cælestial things are the better explained and declared.

Such are the Hypotheses of Circles, and Orbs, with their Parts, Diameters, Axes, Poles, &c. ascribed to the Heavens by Imagination, and for distinction sake: For because the Heaven without
Stars

Stars is uniform, and every where alike, since nothing is Real, we are forced to use the Mental Distinction of the Parts of Heaven, that we may speak distinctly concerning Distinct Things, and that we may determine more curiously the Scituation and Motion of the Stars.

CH A P. II.

Of Physical Hypotheses, and first of the World, and its Parts, Heaven and Earth.

1. **T**HIS Visible World, is a Sphære described from the Eye of the Spectator, even as from a Center, consisting of the Receptacle of all Visible things in Heaven and Earth.

(1. We call this World *Visible*, with respect to the difference, viz. of *Invisible*, concerning which, St. Paul speaks in *Colosians*, Chap. 1. which they call *Platonick*, even as this of ours; but the *Hebrews* call it the *Future World*, which is *Invisible*, and the *Visible World* they call the *Present*: And the *Invisible World* is no other than the state of the Blessed Angels and Men, fully revealed by God at the end of this World, concerning which, Philosophers treat not, but Theologians.

2. And the description given of the *Visible World*, is either *Astronomical* or *Optical*: Considering the World, even as 'tis conceived by Humane Eyes, 'tis, as it were, a Concave Sphære, described about it self, the two principal Parts whereof, according to Holy Scripture, makes the Heaven and Earth: This is the Mansion and Station of Man the Spectator, from whence he contemplates Heaven as it were an hollow Arch or Vault, richly adorned with Sun, Moon and Stars, about the measuring of which Astronomy is properly conversant, leaving many things to be discussed, as Alien from this Physical Scope.)

2. And

2. And that this World is *Spherical*, that is, 'tis terminated by a circular and round Figure, most Philosophers do agree.

For although no Human Sense can extend to the utmost Superficie of the World, nevertheless there are not wanting probable Reasons, which will perswade that it is *Spherical*.

For 1. Since the Order of the Supream Stars returns in themselves, by a Circle drawn about the Earth, and so, as it were, makes a certain hollow Vault, of a whole *Spherical* Figure, the whole Band of which comes within our View in the space of a few days ; we must consent, that the Outward and Convex Superficie of the World cannot depart from this, and be any other than *Spherical*.

2. Moreover, but that most perfect and capacious Bodies ought also to be most perfect and capacious Figures, no body will doubt : Therefore since the World also is a Body most perfect and capacious, because it contains all things by its Ambit or Periphery ; and a *Spherical* Figure is the most perfect and capacious of all others, no one can scarcely doubt of the Worlds being a *Spherical* Figure.

3. Especially since *G O D* himself is the Original of the World, and the World, as it were, the living Statue of *G O D*, to which no Figure is more like (if similitude may take place) than the *Spherical* : For as *G O D* is the Beginning of Beginnings, wanting Beginning and End, Creating and Sustaining all things : One Essence in Three Persons : So also a *Spherical* Figure, by its Nature, hath neither Beginning nor End, it gives Perfection and Place to all other Bodies, and consists of Three Parts, its Center, Superficie, and Equality of Intervals, which are indeed distinct between themselves, yet they cohere so, that one being denied, the rest are destroyed, as is also said in Geometrical things before.

3. *The*

3. The principal Parts of the World, in respect of ours, are two, viz. Heaven and Earth.

And the *Earth* is the inferiour part of the World, made or joined together of Water and Earth, properly so called Spherical, or of the Form of a Globe, allowed to Men to inhabit, that from it with the Eye, even as from a common Center we may contemplate Superior and Celestial Things.

Earth, is an Ambiguous word : For 1. It signifies the Elementary Dust. 2. The dry part of this inferiour Region distinct from the liquid. 3. This whole inferiour Region under our feet, composed of the dry and liquid part, or Water and Earth, so called in particular : which notwithstanding by *Senechdoche* from the better part, the Earth, we are wont to call it the *Terrene Globe*, the *Earth*, the *Orb of the Earth* : In which signification the word *Earth* is received almost every where in *Astronomy*.

4. And that these two Parts, to wit, the Dry and the Liquid, being joined together in one Body, contain one Superficie without doubt.

1. *Experience witnesses* ; in as much as we know that having surmounted or passed over any great Sea, we always find one or other dry places.

2. *From Celestial Observations*, of which the Proportion of Changing, according to the Distance of Places, is the same every where, whether they are beheld from the Earth, or from the Sea.

5. But that this Superficie of Earth and of Water, joined together in one Body, is *Spherical* or *Globose*, is sufficiently evinced by many *Reasons*.

1. For since, as well the Earth, so called in particular, as the Water, is round, as well according to Longitude, that is, from East to West, as according to Latitude, that is, from North to South, it must needs be, that the whole Body also consisting of Earth and Water, is Round : For whether we advance by Earth or by Sea towards the East, the Stars arise sooner to the Eastern People, and the Lunar Eclipses

Eclipses are beheld sooner than by the Western People: And the Northern Stars are more and more raised towards the North, but the Southern are depressed, until some not seen before are beheld, and on the other part, the first seen are hid: The cause of which can be no other thing but the swelling, and rotundity of the Sea and Earth. *Fig. 126.*

2. The Body made of Earth and Water casts a turned or circular shadow from it self, which is beheld in Lunar Eclipses, which shadow is not produced but from a round Body.

3. Mariners testify concerning the swelling of the Water, who can behold a Sign or Mark on the Sea shoar, sooner from the top of the Mast, than at the foot of it, the swelling of the Water hindring the sight, *Fig. 127.*

And although on the Earth there are vast Mountains, and deep Vallies, which seem to hinder the roundness of the Earth, yet because they have no Notable Proportion to the whole System of the Earth, (for the Latitude of a Mountain whose Length and Breadth contains an whole *German Mile*, which is 5000 Paces, and contains 4 *English Miles*, is in Proportion to the whole Bulk of the Earthly Globe, scarce as 1 to 2662560000.) Therefore the Mountains and Vallies cannot hinder the rotundity of the Earth, altho' they make the Superficie of it somewhat rough or rugged: And this is also evident in Eclipses of the Moon, in which the shadow of the Earth is so represented in the Moon, that nothing in any manner appears in it which is Swelling or Rough.

6. And altho' the whole Earthly Globe be vastly great in it self, yet when compared with the Superior Heavens, and the distances of the higher Stars, it hath no sensible Proportion of Magnitude, but is, as it were, like to a Point or Center.

Eor

For if it should be otherwise :

1. The Celestial Observations would differ according to the variety of Places.

2. The Eastern Stars would be seen to be much lesser to the Western inhabitants, as being more remote, than to the Eastern.

3. Two opposite Points of the Heaven would never be seen together.

4. The Diameter of the Sun would appear greater in the Mid-heaven, than when 'tis beheld to arise, to wit, being nigher by an Interval of the whole Semidiameter of the Earth.

5. Lastly, the Vertexes of Gnomons in Sun-Dials can never have the Proportion of the Center to the Sphere of the Sun, if the Earth should have sensible magnitude to the Sphere and Distance of the Sun, much less to the Distance and Spheres of the higher Planets and Stars ; all which, notwithstanding, is refuted by daily Experience.

VII. *Heaven* is to us the Superior part of the World, inclosing the Earth on every side, circumscribed by Judgment of our Eyes, by a Concave Orb, filled by Air and a Golden Sky, adorned most splendidly with Sun, Moon, and Stars.

In Holy Scripture, *Heaven is whatever incloses the Earth every where, or on every side* ; so that the Air also is therein contained, whence the Birds flying in the Air are called the Fowls of Heaven : We follow this Sentence of Holy Scripture in this place. *Aristotle*, with his followers, have placed above the Air a Fiery Sphere, and above this more Heavenly Spheres and Orbs, Concave, Clear indeed, but Solid, which encompass each other as the skins of Onions, and the Stars carry or move the inferior Orbs and Spheres, of which *Astronomers* have found out the Ten principal ones, as you may see in Fig. 128.

But as that Fire being nourished from no solid reason,

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son, the most part of Modern Philosophers have dissipated it into smook; so likewise the sagacity of the Moderns have destroyed the solid Orbs, by *Optical, Astronomical, and Meteorological* Reasons.

For, 1. If the Orbs in Heaven should be solid; to wit, Thicker than the most clear Golden Sky, then the refracted Radius's of the Stars would arrive at our Air, as the Opticks teach; therefore the Planets would appear to be irregular, and as it were in other places farther off than those which may be declared by *Astronomers*.

2. *Astronomy* also witnesses, that *Mars* sometimes is nearer the *Earth* than is the *Sun*; also *Venus* and *Mercury* is conversant one while above, another while beneath the *Sun*, which if the Orbs should be solid, cannot be done without intersection of the Orbs.

3. Moreover that those *Comets*, which *Aristotle* calls *Metcors*, pass from one Region of the Sky to another, is witnessed by most certain Observations, which the solidity of the Orbs would hinder their passage.

4. This also comes near unto the Authority of Holy Scripture, which testifies of God, in *Isai. 40.* that he hath extended the Heavens, as somewhat Thin and Rare.

From these Causes we may truly appoint or state the Celestial Region to be no other than a Golden Sky, most Liquid, Clear, more Thinner also than our most pure Air, giving way to the Moveables, not less readily than the Lights of the Sun and of the Stars, give way, that they may Dart their Lights on us.

From whence also it follows, that the Stars in this most pure Sky, like Fishes in a still Water, and Fowls in the Air, are borne and carried about, not by any other but by their own proper Nature, yet neither wandringly, but constant, and by a Regular Motion.

Yet whether or no we esteem the World flowing, and every where uncovered, or fenced by no other thing, but a most fine Golden Sky: It hinders nothing whether to believe or think, that

that the Extream Wall of the World be some Solid Sphere, keeping all things within its Ambit : To which Sphere, the Stars which we call fixt, ought to be fastned, we cannot believe that those fly about freely, and nevertheless that they do, they keep the same unchangeable space between themselves : For thus the Judgment of our Eyes seems to perswade, to which in Astronomical things we are compelled oftentimes to offer more than the Exquisite Truth.

So likewise it may be lawful to suppose Orbs or Sphaeres, not Solids, but as Circles in the same clear Sky : As also the Spherical Space to be so great, as the wandering Stars or Planets require to the finishing their Courses, concerning which we shall say more hereafter.

C H A P. III.

Of the Fixt, and Wandering Stars or Planets.

1. **S**TARS are Lucid Bodies, Globous thro' the whole Ambit of Heaven, either affixt to the extream Heaven, or suspended in the Sky ; and going about in it, that they may shine upon the Earth, and distinguish the Day and Night, and make the Changes of the other Times.

Concerning these Stars, some say many things vainly, while others suppose them to be the more thicker parts of Heaven ; Some, think them to be burning Globes ; others, Terrestrial Bodies, which are inhabited by I know not what Inhabitants : But because these things are beyond finding out by humane Sense and Reason, neither is any thing revealed to us concerning them in Holy Writ ; we shall do well, if we remain within the Bounds of Modesty, and describe the Stars, from the use (that they afford to us the Inhabitants of the Earth) assigned in Holy Scripture, which is done in our Description.

2. And of Stars some are Fixed, others are Wandering, or Planets.

3. *Fixed Stars*, are, as it were, affixt in the Concavity of the Extream Orb of Heaven, by Immutable

S

Spaces,

Spaces, surrounding the whole Visible World ; they receive all Visible things within their hollow Vault, and seem to go about the Earth, and all the Common Stars, by a daily Motion.

The multitude of them is innumerable to us Men, God himself hath made them as Signs to *Abraham*, Gen. 15. Whence *Augustin* says truly, the Stars cannot be numbred, because neither all of them are believed to be seen : For by how much any one beholds more acutely, by so much he sees the more of them ; whence also to them that behold most Eagerly, some are deservedly esteemed to be hid, *Book 16. Chap. 23. de Civitat. Dei.*

Also the Telescope hath discovered many things in these days, of which the former Ages were Ignorant : In one *Head of Orion* it hath shewn more than Twenty, and which is more, in one close place (which is called by the name of the cloudy part of *Cancer*) more than Forty. The *Galaxia*, or perpetual Milky way, and in the same Age of the World, the Heavenly Circle, which *Aristotle* esteemed rudely for a Meteor, is no other, than an heap of most small Stars shining contiguously, which hath been newly detected by the same *Dioptric Tube*, or *Telescope*.

Notwithstanding, the great number of Stars is not to be supposed Infinite ; for an Infinite Number of Stars require also an Infinite Space, which cannot be : For since the Region of the Fixed is Terminated at least on one side or other, to wit, downwards, towards our moveable World, it cannot be Infinite, since to be Infinite and to be Terminated, abolish each other mutually.

But by that which humane sight attains to, 'tis easie to find out a Number many times exceeding the number of the Stars, although the whole Area of the Heavens should be planted or set with Stars, and they should all cohere contiguously.

For assuming the Middle Circle of the ultimate Heaven,

| | |
|-------------------------------|--------------|
| ven, to be Degrees | 360 |
| Or of Second Minutes | 1296000 |
| The Diameter of this will be | 412364 |
| The Semidiameter | 206182 |
| The Half of the Circumference | 648000 |
| The Area of the Middle Circle | 133605936000 |
| The Quadruple of this | 534423744000 |

Is the Area of the whole Concavity of the Starry Heaven : But if then, no Star shall have in its Diameter more than 1", and all should be disposed contiguously into three, as the little Circles of Windows, so that they cannot be thrust more closely together, yet the number of them cannot exceed this given.

But since the Diameter of Stars of the Sixth Magnitude, which by us are not seen, hath been observed by *Tycho Brahe* to be 20", you may divide that number by 20, and it will make only 2672187200. Neither also can the number of Stars yet exceed this lesser number.

4. And from the great number of Fixed Stars, the famous Astronomers have chosen the greater, and most notable, according to fight, and have distributed those into Coasts or parts of the Heaven, by Asterisms, and certain Classes of Magnitude.

5. *The Coasts of Heaven*, are certain Parts of the same, passing by two great Circles crosswise, thro' the middle of the Earth distinctly.

Hence are made four Cardinal Coasts or Parts.

The East, wherein the Sun Rises.

The West, wherein he Sets.

The North, where the 7 Stars of the Great Bear, called *Charles's Wain*, are beheld, near the North Pole.

The South, Where the Sun is beheld by us in the Meridian, or at Noon.

(Besides these, Mariners number 28, in all 32; which are either named from these principal ones, or they denominate them from the Winds blowing from thence: Even as also, the *Latins* are wont to call the Northern course or part, *Borealis*, and *Aquilonaris*; and the Southern also, *Australis*, &c. from the Winds *Boreas*, *Aquilo*, *Auster*, blowing from hence.)

And from these Cardinal Points, some Stars are called *Oriental*, others *Occidental*, some *Northern*, others *Southern*.

6. *Asterismes*, (which are also called Stars, Sun and Moon, Constellations, Signs, &c.) are the Configurations of the more nearer and famous Stars, bearing either the Names or Images of divers things, for the sake of Memory and Distinction.

Of these Images the Antients have constituted 48, and they comprehend 1022 Stars, denominated from the certain Parts or Members of the Images; as the *Horn* of the *Ram*, the *Eye* of the *Bull*, &c. to which the Moderns have added not a few: And the Causes wherefore they assumed these kinds of Images, is likely to have been chiefly three; *The Likeness* which these Stars have in disposition with these Images; *The Efficacy* which the Ancient Astrologers superstitiously ascribed to some Constellations: Lastly, *The Immortality*, to which some were Consecrated *Heroes*, and would have their names, as it were, inscribed in Heaven.

And 'tis to be noted in this place, that oft-times the name of the whole Constellation is ascribed to any one of the brightest Stars, which with respect to *Syrus*, is called the *Greater Dog*, &c.

Asterisms of the Ancients have been included by Verses in Latin.

I. *Northern Stars.*

To the Northern parts you may behold 21 Stars.

I. *Ursa*

1. *Ursa Minor*, or the Lesser Bear. 2. *Ursa Major*, or the Greater Bear. 3. *Draco*, or the Dragon. 4. *Cepheus*. 5. *Bootes*. 6. *Corona Septentri*, or the Northern Crown. 7. *Hercules*. 8. *Lyra*, or Harp. 9. *Cygnus*, or Swan. 10. *Cassiopeia*. 11. *Perseus*. 12. *Auriga*. 13. *Serpentarius*. 14. *Serpens Ophiuchi*. 15. *Sagitta*. 16. *Aquilla*, or Eagle. 17. *Delphinus*, or Dolphin. 18. *Equiculus*, or Lesser Horse. 19. *Pegasus*, or Greater Horse. 20. *Andromeda*. 21. *Triangulum*, or the Triangle.

II. Southern Stars.

To the Southern Parts you may see 15 Stars.

1. *Cetus*, or the Whale. 2. *Orion*. 3. *Eridanus*. 4. *Lepus*, or the Hare. 5. *Canis Major*, or the Greater Dog. 6. *Canicula*, or the Little Dog. 7. *Argo Navis*, or the Ship. 8. *Hydra*, or the Water Serpent. 9. *Crater*, or Cup. 10. *Corvus*, or the Crow. 11. *Centaurus*. 12. *Lupus*, or the Wolf. 13. *Ara*, or the Altar. 14. *Corona Austrina*, or the Southern Crown. 15. *Pisces Austrina*, or the Southern Fish.

III. In the Middle, in the Zodiac Circle, are the Twelve Signs,

1. *Aries*. 2. *Taurus*. 3. *Gemini*. 4. *Cancer*. 5. *Leo*. 6. *Virgo*. 7. *Libra*. 8. *Scorpio*. 9. *Sagittarius*. 10. *Capricornus*. 11. *Aquarius*. 12. *Pisces*.

Because these Twelve Signs often Occur, for brevity in Writing they are thus noted.

| | | |
|-------------------|--------------------|------------------------|
| ♈ <i>Aries</i> . | ♌ <i>Leo</i> . | ♐ <i>Sagittarius</i> . |
| ♉ <i>Taurus</i> . | ♍ <i>Virgo</i> . | ♑ <i>Capricornus</i> . |
| ♊ <i>Gemini</i> . | ♎ <i>Libra</i> . | ♒ <i>Aquarius</i> . |
| ♋ <i>Cancer</i> . | ♏ <i>Scorpio</i> . | ♓ <i>Pisces</i> . |

But these new Afterisines added by the Moderns in the South part of Heaven, and not seen by us, are contained in these little Verses.

Phœnix, Grus, Indus, Xiphias, Pava, Anser & Hydrus :

Passer, Apus, Triquetrum, Musca, Chamaque Leon.

7. *The Magnitude of the Stars, are Constituted of six Classes.*

| | In Number, | In the North. | In the South. | In the Zodiac. |
|------------------|------------|---------------|---------------|----------------|
| Of Magnitude the | First 15 | 3 | 7 | 5 |
| | Second 45 | 18 | 18 | 9 |
| | Third 205 | 81 | 60 | 64 |
| | Fourth 477 | 177 | 167 | 133 |
| | Fifth 217 | 58 | 54 | 108 |
| | Sixth 49 | 13 | 9 | 27 |
| | Obscure 9 | 9 | 0 | 0 |
| | (Cloudy 5) | 1 | 1 | 3 |
| The Sum | 1022. | 360. | 316. | 346. |

8. The Wandring Stars, or Planets, are those which are seen under the Fixt, through the Golden Sky, not only daily and common with the Fixt ; but also every one peculiar and proper to some place ; not uncertain and wandring, but yet seem to be carried many times by a variable Motion, which change the Spaces variously, both between themselves, and also to the Fixed Stars.

Of these, Seven are observed by the Antients.

Of which,

| <i>The Order.</i> | <i>The Names.</i> | <i>The Figures.</i> |
|-------------------|-------------------|---------------------|
| 1. | Saturn. | ♄. |
| 2. | Jupiter. | ♃. |
| 3. | Mars. | ♂. |
| 4. | Sol. | ☉. |
| 5. | Venus. | ♀. |
| 6. | Mercury. | ☿. |
| 7. | Luna. | ☾. |

I. Among

1. Among these Planets ☉ and ♃ are called in Holy Writ the two great Luminaries ; not that the Moon exceeds the other Stars in bigness of Body, for 'tis manifest that 'tis the least of all ; but that by reason of its nearness, it appears greater than the rest, and casts forth a clearer light.

2. The other Planets are discerned from the Fixed Stars.

1. By the Quantity of Body, because they are according to sight, (for the most part) greater than the Fixed Stars, some few excepted.

2. By the Quality of Light, because they are also more clear, and have a more constant Light, and sparkle less (for the most part) than the Fixt.

3. By their Scituation, because they are Lower than the Fixt, and change their places variously, both between themselves, and to the Fixt, which the Fixed Stars constantly retain between themselves : Moreover the Planets remain between the Zodiack, when the Fixt Stars are dispersed every where thro' the whole Heaven.

3. And the Planets likewise are distinguish'd between themselves.

1. By Quantity of Body, for Venus, after ☉ and ♃, is the greatest of all ; next to which in Magnitude is Jupiter ; Mars is somewhat less than Jupiter ; Saturn is equal to Mars ; and Mercury is least of all.

2. By Quality of Light, Venus is the most splendid, diffused with a Golden Colour in Brightness : Jupiter is like to Venus, but less splendid, of a middle colour, between Gold and Silver : Mars sparkles with a Ruddy flame : Saturn shines less, with a Leaden colour : Lastly, Mercury is Lucid, but less White.

3. By Scituation, for *Venus* departs not from the Sun beyond 48 Degrees : *Mercury* not beyond 28° : *Venus* praeceeds the Sun in the Morning, and is called *Phosphorus*, or the *Morning Star* ; and follows the Sun in the Evening, and is called *Hesperus*, or the *Evening Star*, and is easily known by its little Golden Rays : But *Mercury* lies altogether hid, either under the too much splendor of the Sun, or if it depart from thence, yet 'tis scarce observable to young Beginners, by reason of its smallness.

And we must Note also in this place :

1. That *Galileus Galilei*, a Mathematician of *Florence*, by help of the Telescope hath likewise detected other Secondary Planets, ranging about the Primary, to wit, 4 *Jovial* about *Jupiter* ; 2 *Saturnine* about *Saturn*, which we leave in their places until more is known concerning them.

2. Moreover, there are many Modern Astronomers which think far otherwise of the Order of the Planets: And *Copernicus* and his Followers, and those Ancients, *Philolaus* and *Ecphanium*, *Pithagorians* ; *Heracledem Ponticum*, *Aristarchus Samius*, and others ; change the place of the Sun and the Earth, and place the Sun in the Center of the World, only moveable about its Center, and all the others which are in the World they suppose moving : And they receive the Earth into the place of the Sun between the Planets, moveable about of it self by a daily Motion, and the Center of the Sun by an Annual Motion, and moreover moveable by another Secular Motion : And they circumscribe the Sphere of the Moon, in which the Moon as a Secondary Planet goes about near the Earth.

Leaving this Opinion, we not considering Physics, do follow the Judgment of our Eyes with Holy Scripture, with those which place the Earth in the Center

ter of the World, especially since the Defenders themselves being also of a contrary Opinion, are forced for the most part, if not to think, yet to speak according to the Judgment, or Award of their Eyes.

3. Lastly, we must know, that there appears sometimes in the Air counterfeit shapes, running to and fro swiftly, which they call by the name of *Wasted Stars* only, and from their Species, *Falling Stars*; when notwithstanding they are not true Stars, but Exhalations gathered together in the Air, suddenly vanishing, and therefore to be remitted to the *Meteorologists*: Notwithstanding, if *Comets* and *New Stars*, more durable than others, appear (which often happens in the memory of us, and of our Fathers) it will be the Office of *Astronomers* also to search diligently into the Situation and Motion of these.

8. Lastly, That which pertains to the Figure of Stars, 'tis very likely that they are Globous.

For the Body of the *Sun*, of the *Moon*, and of other Stars, if we behold them accurately, Sense Demonstrates them to be Circular, as in Fig. 129. and they appear not only Plane Circles, but every way Round or Spherical Bodies, as appears in the Moon: For at what time and place soever of Heaven or Earth 'tis beheld, it never appears not round, which cannot be in a plane Figure, which being changed, even as 'tis by any one beholding it from a lower place, it changes its Place, and its Site or Position: And why they seem to us to be Plane Superficies, notwithstanding they are Spherical Bodies, we shall Teach in *Opticks*.

C H A P. IV.

Of the Motion of the Stars, and of the Kinds and Quality of Motions.

1. **A**LL Stars move either truly or apparently, and indeed by a Circular Motion, or returning to themselves, as the Sight of the Eyes witnesseth.

1. For all Stars which Rise and Set by a Diurnal Motion, ascend from the East to the highest part, and from thence descend by the like velocity to the West, and returning in the same manner, repeat their former course: Which since 'tis done only by Circular Motion, and being finished, may be reduced in the same manner; it follows, that the Motion of the Stars is Circular, or returning to themselves. Fig. 130.

2. Also because Stars nigh the Pole always appearing, testifies it: for these describe perfect Circles about the Poles by a Diurnal Motion, which is an evident Argument, that they are moved Circularly.

2. And the Motion of the Stars is two fold, Common and Proper.

3. Common, which also is Diurnal, or Day by Day, and is called Prime; is that whereby the whole Motion of the Stars is exercised, by force of the first Moveable, to which this Motion in the first place enters, and is turned about daily from East to West in 24 Hours, or in the Space of a Day and Night.

Therefore concerning this Motion no Body doubts, since 'tis exposed to the Eyes of every one; whence also it wants no Demonstration: But concerning the Subject of this Motion, to which it chiefly belongs, whether or no it be true, or that the fallacy is granted

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ted to the Stars only by sight ; and that the Earth indeed be turned round about on its Center (since in both ways the same wholly happens ; whether the Stars are all moved from East to West, or the Stars all Resting, the Earth be moved about its Center from West to East) Philosophers contend concerning it, which to ascribe the Motion to : Which Controversie doth not so much belong to *Astronomy* as to *Physicks* : For whether this, or that be true, in *Astronomy* also false *Hypotheses*, may be assumed while such things are, by which the Calculation of the Heavenly Motions may be truly and easily performed : Which facility, although it be offered in the *Hypotheses* of the Earths Motion, yet because it seems insolent to our Studies, we retain the old *Hypotheses* of the daily Motion of the Stars, and the resting of the Earth in the Center of the World, allowed by the Judgment of the Eyes, to which also the Style of Holy Writ agrees.

4. *The proper Motion of the Stars, which is also called the Second, is that by which the Fixed Stars altogether, but the Planets every one singly, and with respect to the former, proceed contrarily by oblique Motions from the West towards the East, according to the order of the 12 Heavenly Signs.*

1. All Stars demonstrate this Motion, which proceeds leisurely to the East ; or Observations testify that 'tis made by the Oriental, by little and little, in the Diurnal Revolution : Or that which is most noted in the Moon, *Fig. 131.* for it in the New Moon, sets together with the Sun in the Evening.

But three or four days after, 'tis beheld from Sun setting by a longer Interval towards the East : whence 'tis beheld daily to be more and more Easterly ; until Sun setting the fourteenth day, it at last appears, and shews it self above the Horizon Easternly : The same is observable as well in the Sun as in all other Stars :

Stars : The proper or second Motion therefore of the Stars, is from the West to the East, being opposite to the first Motion : But since they proceed not always in the same Parallels, and change their Declinations variously, not in divers places, but in those only through which the Zodiac with its 12 Signs marches : 'Tis also manifest that these second Motions are made according to the order of the Signs, &c.

2. And they are always joined to the first Motion, and so mixt with it, that they cannot be separated from it, unless by Imagination : Whence also in explaining the first Motion, mention also ought to be made of the second ; which ought not to be blamed, for those things which Nature hath conjoined, the Method cannot be separated.

5. *And whether or no this Motion of the Stars be equal, or unequal, is doubtful among Astronomers.*

Indeed it appears to the sight to be unequal, while some Planets proceed one while slower, another time swifter, one while higher, another while lower, sometimes they march, at other times they stand still, or also we discover them to go backward : But this is thought many times to be by reason of the many Circles and Orbs, and the diversity of Centers, which Motion can be equal only about the proper Center ; from whence the Heavenly Motions, to those that live on the Earth, and behold them from another Center, seem unequal, which are most equal every one in the Heavens about its Center, and its Poles.

But since the *Ratios* of *Physicks*, of *Astronomy*, and of *Opticks*, do not admit of real Orbs, and divers Circles ; and the diversity of Centers, and of Poles, appearing unequal in all, can scarcely be excused : There is found out among many Moderns, especially those which esteem the Sun in the Center of

of the World, for the Fountain not only of Light and Heat, but also of all Motion, which also admit of some inequality in the Motion of the Stars, and will have those Stars to be swifter that are nighett to the Sun, and to be slower by how much the more they are remote from it : Nevertheless whether true or apparent only, they think that there is inequality in the Celestial Motions.

6. *Let 'tis most certain that they are not Irregular, and without Order, but are most Regular, and in Order : Which is evident from this.*

1. That the Stars always absolve or compleat their Circuits, in fix'd and like spaces of times.

2. That after the whole Period is compleated, they run back again the same Anomaly, in the same plain manner, by a most constant Equality : Therefore the Motion of the Heavenly Bodies must needs be most Regular and Ordinate.

This is all we shall say of *Physical Hypotheses.*

C H A P. V.

Of Geometrical Hypotheses of Spherical Doctrine, to wit, of the Primary Celestial Circles.

1. **C**IRCLES indeed there are none in Heaven, since all its parts are every where most like to it self, yet by Imagination we are not hindered to ascribe certain Circles in the Heaven, as well to distinguish the Site, as to design the Motion of the Stars.

2. These Circles *Astronomers* divide into 360 Parts or Degrees, every one of which they subdivide into 60 Scruples, or First Minutes, and each of the

the First Scruples into 60 Seconds, a Second into 60 Thirds, a Third into 60 Fourths, &c.

For because Heaven is very Large, it is not sufficient to assign to Stars their Circles; but also it must be shewn what part of the Circle they are in: To the shewing of which the Sexagenary Division seems most Commodious, because of its many Aliquot Parts or Multiplicities.

3. *And of these Circles, some are Greater, others Lesser.*

4. *Greater*, are those which passing through the Center of the Earth, divide it into Two equal Parts. *Lesser*, are those whose Centers are Divers from the Center of the Earth, and therefore cut it into Two unequal Parts.

But whether they are greater or lesser Circles, yet all of them are divided in the same manner into 360 Parts, and each Part into 60 Scruples; all which parts therefore are not indeed equal, yet they are alike, or Homologous and Proportional.

5. *Astronomers in Spherical Doctrine, number Ten Primary or Chief ones, of the Greater and of the the Lesser Circles, which are,*

- | | |
|---------------------|-------------------------|
| 1. The Equinoctial. | 5. 6. The Two Colures. |
| 2. The Horizon. | 7. 8. The Two Tropicks. |
| 3. The Meridian. | 9. 10. The Two Polars. |
| 4. The Zodiac. | |

Of these the six former are great Circles, and the four latter, small or lesser Circles.

The *Meridian* and the *Horizon*, in respect of one place, are appointed Immoveable: The rest in the common Motion of the Stars are understood to be turned about together.

1. *Of the Equinoctial.*

6. *The Equinoctial, which is also called the Equator, is a Movable great Circle, described from the Center of the Earth, dividing the Heaven and the Earth in the Middle, into the Northern and Southern Parts: According to the leading of which, the whole*

whole number of Stars seem to be turned about from the East into the West.

1. The Parts or Degrees of this Circle are wont to be called by the peculiar name *Times*, because they measure Days and Hours.

2. In it are considered the *Axis* and *Poles* thereof.

Axis, is a Right Line drawn by Imagination thro' the Center of the Earth, from the South into the North, and Terminated at the Extream of Heaven on both sides.

Poles, are the two Extream Points of this *Axis*, distant 90 Degrees, or the Quadrant of a Circle from the *Equator*, (as are the Poles of all greater Circles from their Circles) the one being Northern, is called the *Arctic*, or North Pole; and the other, being Southern, is called the *Antarctic*, or South Pole; about which Poles the Immoveable Stars are seen to turn daily, see Fig. 132. Whence also that *Axis*, and these *Poles*, are accounted for the *Axis* and *Poles* of the whole World: In this Scheme the Circle *ABCD* is the *Equator*, whose *Axis* *E F* passes through the Center *G*.

2. Of the *Meridian*.

7. The *Meridian*, is an Immovable Circle drawn through the Poles of the *Equator*, and the Vertical Point of any place, Fig. 133.

Whence it appears, that there may be conceived not only one, but so many *Meridians* as there are Vertical Points, or distinct places, that happen in passing towards East or West; that is in Number: For a *Vertical Point*, is that which hangs over our Heads in any place, and in *Arabick* 'tis called *Zenith*, and its opposite Point, the *Nadir*.

3. Of the *Horizon*.

8. *Horizon* or *Finitor*, is an Immovable Circle, equidistant from the Vertical Point, and either Terminating our Sight, or dividing the whole Earth, and with it

it therefore the Visible World in the Middle, into the upper and lower Hemispheres.

Whence it appears, 1. That the *Horizon* is two-fold, one of which is *Visible*, Terminating the sight on every side : The other *Rational*, parting the Heaven and the Earth in two in the Middle, not comprehensible by Sight, but only by Reason.

2. And seeing the *Horizon* also depends on a Vertical Point, 'tis necessary likewise that there should be so many *Horizons*, as *Vertical Points*, numbred as well from the East into the West, as from the South into the North, Fig. 134.

Of the Zodiack.

9. Zodiack or Signifier, is an *Oblique Circle*, intersecting the Equator in two places, and declining from the same, by one half into the North, by the other into the South ; under which the Planets are perpetually conversant : The Sun indeed always marching under the middle of the *Ecliptic Line* ; but the rest departing also to the Sides of the same.

1. This therefore is the only Circle that hath Latitude, almost of 20 Degrees (of which its Longitude hath 360) being Ten Degrees on each side from the *Ecliptick* ; to wit, as much as the Planets march hence from the Middle Line in their greatest Longitude.

2. This Middle Line the Sun describes by its proper Motion, which for that reason is called the Way and the Orbit of the Sun ; likewise 'tis called, The *Ecliptic Line*, because the Luminaries the Sun and Moon are Eclipsed in it, either by Conjunction or Opposition.



3. And

3. And the Zodiac or Ecliptic also hath its Axis and Poles, about which the Planets finish their Courses in the Zodiac, one of them is North, and the other is South, each distant from the nearest Pole of the Equator, so much as the Zodiac by its greatest Obliquity departs from the Equator. Fig.

132. As in this Scheme, the Circle $A B C D$, is the Equator, whose Axis $E F$, passes through the Center G ; But $H B I D$ is the Zodiac, whose Axis is $K L$: And the Poles K and L are so far distant from the Poles of the Equator E and F , as the Points of the Equator and of the Zodiac A and H , or I and C are removed from one another: For $A F$ and $H L$ are Quadrants, (because the Poles are always distant from their Circles by a Quadrant) and therefore are always equal: The common Portion $A L$ being taken away, the remainder $A H$, is the distance of the Equator, and of the Ecliptic; Also $L F$, the distance of the Poles, are equal.

4. Moreover the Zodiac also is divided into 12 Parts, which are called the 12 Celestial Signs, from the 12 Constellations which have been in time past in those parts; the Name of every one of which Parts is as yet retained, although the Constellations have left those ancient or former Seats, and have removed into following Places.

5. These 12 Parts or Signs, are also the aforesaid mentioned Constellations; of which the Names and Notes are these.

| | | |
|-----------|------------|---------------|
| ♈ Aries. | ♌ Leo. | ♐ Sagittary. |
| ♉ Taurus. | ♍ Virgo. | ♑ Capricorne. |
| ♊ Gemini. | ♎ Libra. | ♒ Aquarius. |
| ♋ Cancer. | ♏ Scorpio. | ♓ Pisces. |

6. In these are noted four Cardinal Points, which are,

| | | |
|-----|-----------------|---------------|
| ○ γ | } The beginning | } Of Aries. |
| ○ ♎ | | |
| ○ ♋ | | |
| ○ ♑ | | |
| | | Of Libra. |
| | | Of Cancer. |
| | | Of Capricorn. |

The two former, the beginning of Aries and Libra, are in the Points of Intersection of the Equator and the Ecliptic D and B: And the two latter, in the Points of the greatest Obliquity, or of the declination of the Ecliptic from the Equator H and I: These, viz. ○ γ, ○ ♎, are called *Equinoctial Points*; the first indeed the *Vernal*, whence the beginning of the Zodiac is placed; the latter *Autumnal*, because the Sun being in them, which is about the Tenth or Eleventh of March, and the Thirteenth or Fourteenth of September, makes the Equinox, and the Days and Nights equal in all the Earth.

But these, to wit, ○ ♋ and ○ ♑, are called *Tropical and Solstitial Points*; the first indeed pertaining to Summer, the latter to Winter; because the Sun being Constituted in these, as it were fixes his Foot, and either begins to depart from us, or to return to us, which is about the Eleventh and Twelfth of June, and the Eleventh and Twelfth of December.

7. And these Signs are distinguished variously.

| | | |
|-------------------------------|---------------|----------|
| 1. Into four
Quadrants of- | } The Spring. | } γ δ ♏. |
| | | |
| | | |
| | | |
| | Summer. | ♋ ♌ ♍. |
| | Autumn. | ♊ ♍ ♋. |
| | Winter. | ♑ ♒ ♓. |

For the Sun being constituted in these Signs, makes these Four Seasons of the Year, the Spring, Summer, Autumn and Winter.

2. Into

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2. Into Two } Northern $\gamma \delta \Pi \ominus \Omega$
in the Middle } Southern $\imath \kappa \tau \nu \pi \times$

For these Signs pass from the Equator into the North, but these from the same into the South.

3. Into Two } Ascending. $\nu \pi \times \gamma \delta \Pi$.
in the Middle. } Descending. $\ominus \Omega \imath \kappa \tau \nu$.

For by or through these, the Sun ascends to us from the South into the North; By these he descends from us, from the North Coast into the South.

4. Into 4 Tripli-
cities or Trigrams,
distant from each
other $\frac{1}{3}$ part of a
Circle, of which,

| | | | |
|----------------|--------|---|-------------------------|
| 1. The Fiery. | } Hath | { | $\gamma \delta \tau$ |
| 2. The Earthy. | | | $\imath \kappa \nu$ |
| 3. The Airy. | | | $\Pi \pi \times$ |
| 4. The Watery. | | | $\ominus \Omega \times$ |

Fig. 135. In these Trigrams of Signs, are made the Conjunctions of the three superior Planets $\gamma \delta \tau$, which Astrologers call Great; of which we shall say more hereafter.

5. Lastly, we must Note, that the whole Heaven is cut into Twelve Parts, by Six great Circles passing through the Beginnings, and through the Poles of the Ecliptic, which also, generally speaking, are called Signs: And in this manner all Stars are referred to certain Signs, as well within, as without the Zodiac, which are comprehended under some of these Parts. Fig. 136.

5. 6. Of the Colures.

1. Colures, are Circles drawn through the Poles of the Equator, and the Cardinal Points of the Zodiac.

The former of these drawn through the Equinoctial Points is called the *Equinox Colure*; The latter, passing thro' the Solstitial Points, is called the *Solstice Colure*.

Fig. 137. They are called *Colures*, as being maimed or imperfect, and having a Tail, because in an Oblique Sphere, they have some Parts lying hid under the Horizon, as it were a diminished Tail.

7. 8. Of the Tropicks.

11. Tropicks, are two lesser Circles drawn Parallel to the Equator, through the Solstitial Points of the Zodiac.

One therefore is the Tropick of Cancer, which is drawn through the Summer Solstice: The other, the Tropick of Capricorn, which is drawn through the Winter Solstice.

9. 10. Of the Polar Circles.

12. The two Polar Circles, are lesser Circles drawn Parallel to the Equator, through the Poles of the Ecliptick.

Therefore one of the Polar Circles is the *Arctic*, which passes through the North Pole of the Ecliptick: The other the *Antarctic*, passing through the South Pole of the Ecliptick. Notwithstanding among the Antients, the Ratio of the Polar Circles hath been otherwise, for they have made them indeed Parallel to the Equator, but passing through the Intersections of the Horizon, and of the Meridian.

In Fig. 138. Is a Scheme of the Principal Parallels of the Equinoctial.

A B C, is a Meridian, or a Colure, *A C* an Oblique Horizon, or of an Oblique Sphere, *D E* the Ecliptic, *D F* and *G E*, the Tropicks of Cancer and Capricorn, equally remote from the Equator, which is placed in the Middle: *H* the North Pole, *I K* the Arctic Polar; *L* the South Pole, *M N* the Antarctic Polar: Of the Moderns *A O*, touching the Horizon

Horizon in *A*, is the Arctic; *CP*, touching the Horizon in *C*, is the Antarctic of the Antients.

CHAP. VI.

Of the Secondary, or Second Sort of Circles.

1. **A**rtificers have added some Secondary Circles to the Prime Circles rehearsed before, of which there are Circles.

- | | |
|--------------------|------------------------------|
| 1. Of Declination. | 5. Of Cælestial Houses. |
| 2. Of Latitude. | 6. Parallels of the Equator. |
| 3. Verticals. | 7. Parallels of the Horizon. |
| 4. Of Position. | |

2. Circles of Declination, are those which are understood to be drawn through the Poles of the World, and any Point of Heaven.

The Principal of these is the Meridian of any place, and from the number of them, each Colure.

3. Circles of Latitude, are those which pass thro' the Poles of the Ecliptic, and any Point of Heaven.

One of these is the Colure of the Solstices, which is drawn, not only through the Poles of the World, but also of the Ecliptic.

4. Verticals, are Circles which descend from the Vertical Point of any Place or Zenith, to the opposite Nadir, thro' each Point of the Horizon.

Again, of the Number of these is the Meridian of any place: the others are called by the *Arabians* *Azimuth*: Although they also call by this Name, the Arch of the Horizon, intercepted between these Circles, and the next Meridian, or Equator.

5. *Circles of Position, are those which pass thro the Sections of the Horizon and the Meridian, and any given Point of Heaven.*

The chief of these are the *Horizon* and the *Meridian* of any place.

6. *Circles of Cœlestial Houses, are those which pass through the Sections of the Horizon and Meridian, and each 30 Degrees of the Equator, in numbring from the Horizon; and separate the Heaven into 12 Parts, as it were Houses.*

These Circles differ nothing from Circles of Position, but that these pass through any Point of Heaven, but the Circles of the Cœlestial Houses begin from the Horizon on the Eastern Part, or as 'tis caled from the Horoscope, and hence by descending are conveyed through the lower Hemisphere, and any thirtieth Degree of the Equator.

7. *Parallels of the Equator, are Circles equidistant from the Equator on both sides; of the number of these are the two Tropicks, and the Polar Circles, and others called the Circles of Natural Days, passing through each Degree of the Ecliptic, which the Sun seems to describe daily; the Limits or Bounds of which, are the two Tropicks.*

8. *Parallels of the Horizon, are drawn through each Point of the Vertical Circles, equidistant to the Horizon.*

These Circles are called in *Arabic*, *Almicanthars*: Also Circles of Altitude, which as are also the *Parallels of the Equator*; by how much further they are removed from their Principal Circles, the *Horizon*, and the *Equator*, they become lesser until at last they end in Points; which in *Parallels of the Horizon* are the *Zenith* and *Nadir*; but in *Parallels of the Equator*, the two *Poles of the World*.

Thus far of Geometrical Hypotheses.

C H A P. VII.

Of a Material Sphere, and Heavenly Globe, the Instruments of Spherical Doctrine, and the general Use and Direction of them. Fig. 139.

1. **A** Material Sphere, or an hollow Sphere, made up only of the Circles, is any Effigies of the Mundane Sphere, made Artificially of Circles, by which, and by Astronomical, and Cælestial Hypotheses, from the first movent, are the Explanation of the common Dependency of all the Stars; and in the Explanation of Spherical Doctrine, they are made Subject to the Eye, for the sake of greater Declaration, and more evident Demonstration.

2. A Cælestial Globe, in like manner, is the Image of the Starry Heaven; besides the Circles, it shews also the Asterisms of the Stars in their Magnitude, Proportion, and Distance each to other.

For those in the Heavens remotest from us are scarcely taken; but by assuming not only Geometrical Hypotheses, but also we are help'd by meet Organs and Instruments, since they present the Hypotheses to the Sight, and as it were to the Touch.

And although divers Instruments of this sort have been Invented by Artificers, and always new ones may be found out, yet the most Noble, Catholick and Universal, are the Globe, and the Material Sphere, which we chuse to use, as being every where most obvious.

3. And the parts of both, as well of the Material Sphere, as of the Heavenly Globe, are, the Center, the Axis, the Poles, the Circles.

T 4

4. The

4. The Center, is the middle point in a Sphere, or Globe, resembling the Earth, from which, as from an Optic Center; we contemplate the Heaven.

5. The Axis, is a Right Line drawn through the Center of the Sphere, or Globe, to the Circumference; about which the Globe or Sphere is turned round, representing an imaginary Line drawn thro' the Center of the Earth, from one Extream of Heaven to the other; about which the Starry Heaven, is understood to be turned round, by the first Movement, or Motion, even as a Wheel about its Axis.

6. Poles, are the Extrems of the Axis, representing, as it were, the two Cardinal Points of the World, about which Immoveables, the whole Caelestial System seems to be turned round; one of which, being in the North, is called the Arctic; the other towards the South, is called the Antarctic Pole.

That, in Heaven, is next to the last Star in the Tail of the lesser Bear, which is called the lesser Bear Star, as also the Pole Star: But this, that is, the Antarctic Pole, is conversant in a vacant place of Heaven, opposite to the notable fixed Stars.

But on Earth the North Pole is from hence beyond the Icy Sea, which is behind Denmark, Norway, and Tartary, in an unknown place; the South Pole in the very middle of the Magellanick Earth, as yet also unknown.

7. The Circles made in a material Sphere, or Painted on a Globe, are chiefly 10, reckoned up as before.

1. The Horizon. 2. The Meridian. 3. The Equator. 4. The Zodiac. 5. 6. The Two Colures. 7. 8. The Two Tropicks. 9. 10. The Two Palsars.

These

These shew or resemble certain Tracts of Heaven, of which some of them otherwhile is divided uniformly, in parts distinct to the places.

8. And although we have said before, that there are innumerable Horizons and Meridians, according to the variety of Vertical Points; yet one Horizon, and one Meridian is sufficient to all places in a Sphere or Globe that turns, and may be moved about by ones hand, to any Vertical Point.

9. Besides those Primary Circles, there is usually added in Globes, many Circles of Latitude, and of Declination, or likewise moveable Semicircles, or Quadrants of Brass, passing about the Secondary Circles by Turns, to wit, the Vertical Circles, the Circles of Position, the Circles of the Cælestial Houses, and the Circles of Latitude and Declination.

10. Also to our North Pole, there is usually affixt an Hour Circle, divided into twice 12 Hours, having an Index on it, turning about together with the Globe.

11. Lastly, on the Convex Superficie of the Globe there is described the aforesaid Asterisms, or 48 Starry Images, observing the Proportion of Magnitude and Distance, which the Stars are discovered to have between themselves in the Heavens.

12. And in the using of this Cælestial Globe, two things are required.

1. Its Rectification.
2. Its Use.

13. The Rectification of the Globe is thus.

1. The

1. *The Globe must be fitly disposed to the Coasts of the World, which may be done by finding a Meridian Line, or by the Compass, or other ways, which we shall shew hereafter.*

2. *The Artic or North Pole must be Elevated, as each place requires.*

Which Elevation is here at London $51^{\circ} : 32'$: And for other places, the Elevation of the Pole is to be sought from *Astronomical*, or *Geographical Tables*, under the Title of the Latitude of the Place, which is the same with the Elevation of the Pole; or if these be wanting, you may find it from *Geometrical Maps*.

3. *The place of the Sun in the Ecliptic must be sought to some day given, not exactly, but also with some Latitude.*

1. *From some Diary or Journal, or by an Almanack.*

For in these, is noted the day of the Suns Entrance into each Sign of the Zodiack: Hence therefore, if for any day following, you number one Degree to the day proposed, you will arrive at the Degree of the Sign of the Zodiack, which the Sun enters on the given day.

2. *From the Circles of Months and Days of the Year, and of the Signs and Degrees of the Ecliptick.*

These Circles are seen in the Wooden Horizon of Cœlestial Globes, and elsewhere, in which the Months and Days of the Year, with the Signs and Degrees of the Zodiack are placed.

In these therefore, if you seek the given Day of the given Month (for which notwithstanding in the Leap-year, or Bissextile, after the Twenty eighth of February you must take the following day next

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to the given, because the Twenty ninth of February is usually thrust in between in the Calendars, and is omitted in these Circles of the Horizon) you will also find ascribed the Degree of the Sign of the Zodiac, in which the Sun is conversant the given Day.

4. *The Degree of the Sun being found, and noted on the Globe by a certain mark, suppose in the Meridian Circle.*

5. *To the same upper part, or the Twelfth Meridian Hour, in the Hour Circle, the Position of the Globe being not changed, turn likewise the Hour Index hence, together with the Globe, to any Hour of that Day, to be transferred either before or after Noon.*

14. *The uses of the Globe, rectified in this manner, are either General or Special.*

15. *The General are,*

1. That the Hour Index being conveyed to any Moment of the Hour Circle, the Face of the whole Heaven, and the Position of all the Fixed Stars at the same Moment of time is understood.

2. But in the Night, the Hour Index being conveyed to the Hour, or the Moment of it, in which you have decreed to Contemplate the Stars, you will have not only the whole Constellations, but also each Star (approaching by the frequent Exercise, and on every side obvious Catalogues, in which is shewn each Star of every Constellation, by diligent Inspection) may be known, with the Stars of Heaven that answer to the Fixed Stars painted on the Globe, cutting Perpendicularly, according to a Right Line.

16. *The*

16. *The Special uses of Globes, are,*
 That by the Benefit of them, the Site and Motion of the Stars are not only easily understood, but also using the Compasses we may measure them, as if we were walking among those Stars :

Of which Uses we shall say more in that which follows.

C H A P. VIII.

Of the Various Sites of the Stars, and other Points of Heaven, to the determining the Circles of Heaven, and of the Globe.

THE Stars, although they are in continual Motion, yet oftentimes the Site or Position of them is considered in some Moment of Time, with respect to a certain place, as if they were fixt there in a perpetual Seat : And there are divers kinds of these Sites.

1. *The Altitude of the Sun, or of a Star, or of any Point of Heaven, is an Arch of the Vertical Circle, intercepted between the Horizon, and the given Point of Heaven.*

It is always increasing, until the Sun, or Star, or any Point of Heaven come to the Meridian, in which it arrives to the greatest height of that Day, from which Moment it begins again to decrease

2. *The distance of the same given Point from the Vertex, is an Arch of the Vertical Circle, intercepted*

intercepted between the Point given of Heaven, and the Vertex.

And it is the Complement of the Altitude to a Quadrant ; which Altitude, if you take from 90 Degrees, you will have the distance from the Vertex.

3. *The declination of any of the Points of Heaven, from the Equator towards the North or South, is an Arch of the Circle of Declination, intercepted between the Equator, and the given Point of Heaven.*
Fig. 140.

As in this Scheme abc is the Meridian, alc the Horizon, gef an Arch of the Equator, the Poles d and k , bib an Arch of the Vertical Circle, die an Arch of the Circle of Declination, i some Star given, whose Altitude will be bi ; its distance from the Vertex ib , and the Declination ei .

4. *The Elevation of the Pole, or the Altitude of the same, is an Arch of the Meridian, intercepted between the Horizon and the Pole of the Equator, or the apparent Pole of the World.*

As in the 140 Fig. da is the Poles Elevation, being at London $51^{\circ} : 32'$, it is otherwise elsewhere.

5. *The Elevation, or Altitude of the Equator, is an Arch of the Meridian, intercepted between the Horizon, and the Equator.*

That is, the Complement of the Elevation of the Pole to a Quadrant, with us at London $38^{\circ} : 28'$: In the aforesaid Scheme, the Arch fc is equal to db , the Complement of the Altitude of the Pole da , to the Quadrant ba : For df and bc are Quadrants, described from their Poles d and b , from which if you take away the Common Arch bf , it will necessarily follow, that the Remainders db and fc are equal; seeing that, if from equal things we take away equal, or that which is common to the

time.

same, the Remainders that are left will be equal.

6. *The distance of the Vertical Circle, either from the Meridian, or from the Equinoctial, is an Arch of the Horizon, drawn between the Vertical Circle, through a certain Point of Heaven, and either the Equator, or the next Meridian.*

And this distance the *Arabians* are wont to call *Azimuth*, as aforesaid; in the former Scheme the *Azimuth* of the Star *i*, is the Arch *ah*, or *hl*, one of which is the Complement of the other to a Quadrant.

7. *The Easterly, or Oréive Amplitude, is an Arch of the Horizon intercepted between the Equator, (which cuts the Easterly part of the Horizon) and the Point of the Stars Rising, or of any Point of Heaven.*

In the precedent Scheme, the Arch *lm* is the Amplitude Oréive of the point *m*. In the Western part of the Horizon, is numbred in the same manner the Westerly Amplitude: for by how many Degrees the Sun, or a Star's Rising, is distant in either part from the Equator, by so many Degrees also, very near, it will appear descending from the West part of the Equinoctial, towards the same part of Heaven.

8. *The Longitude of a Star, is an Arch of the Ecliptick intercepted between the Point of the Vernal Equinox, and the Circle of Latitude of the Star.*

9. *The Latitude of a Star, is an Arch of the Circle of Latitude, intercepted between the Ecliptick, and the given Star.*

10. *The Right Ascension of a Star, is an Arch of the Equator, between the place of the Vernal Equinox, and the Circle of Declination of the given Star.*

'Tis

'Tis so called, because the Quantity agrees with the Ascensions above the Horizon, which the Stars have in a Right Sphere. Fig. 141.

In this Scheme $A B C$ is the Colure of the Solstices, $A D C$ half the Equator, and the Pole of it B , but $E D F$ is half the Ecliptick, and its Pole G ; the Vernal Equinox is in D , and H is the given Star; $B H I$ is the Arch of the Circle of Declination, and $G H K$ an Arch of the Circle of Latitude: Wherefore $H I$ is the Declination, $H K$ the Latitude, $D K$ the Longitude, $D I$ the Right Ascension of the Star H .

11. *The Midheaven, is an Arch of the Ecliptic, intercepted between the Point of the Vernal Equinox, and the Meridian Circle, passing through the given Point of Heaven.*

Where Note, That also an Arch of the Equinoctial, which with any Arch of the Ecliptick, passing through the Meridian, is sometimes called the *Midheaven*, or the Right Ascension of the *Midheaven*.

In the Antecedent Scheme, Fig. 141. if $A B C$ is taken for the Meridian, $D E$ will be the *Midheaven* of the Point A .

12. *And from the Section of the Horizon, and the Equator, arises the Diversity of a Right Sphere, and of an Oblique, and a Parallel Sphere.*

13. *A Right Sphere, is that, in which the Horizon and the Equator cut each other at Right Angles.* Fig. 144.

In this Position, neither of the Poles is Elevated above the Horizon, but both are in the Horizon, on which also the Axis of the Sphere leans upon.

14. *An Oblique Sphere, is that in which the Horizon and the Equator cut each other at Oblique Angles, as in Fig. 145.*

If

In this Position one of the Poles is Elevated above the Horizon, more or less ; the other is depressed beneath the same : And the Axis of the Sphere insists on the Plane of the Oblique Horizon.

15. *A Parallel Sphere, is that, in which the Horizon, and the Equator, cut not each other, but are made Parallel, as if they were United.* Fig. 146.

In this Position, the Pole of the Equator, and the Pole of the Horizon ; the *Zenith*, and the *Nadir*, are joined ; and the Axis of the Sphere insists on a Plane, perpendicular to the Horizon.

And there is but one Right Sphere, and but one Parallel ; but Innumerable Varieties of Oblique Spheres, in which one of the Poles is Elevated more or less : the Cause of which Diversity, is necessary to be known of young Beginners : For the Heaven or the Earth is not changed ever and anon, so as now to make a Right Sphere, by and by an Oblique Sphere, and at last a Parallel Sphere : But we, the Inhabitants of the Earth, behold Heaven in Divers Manners, from the place of our Habitation ; for the Sphere of the Earth is also every where Habitable, and Heaven every where above or upward.

Therefore they which Inhabit under the Circle of the Equator, which encompasses the Earth in the middle, and is distant from the Poles by a Quadrant of a Circle : To these the Poles of the Horizon rest on, and these are said to Inhabit in a Right Sphere, in which all the Stars Rise and Set Daily.

If we depart from hence, and proceed towards the North, we go under another Vertical Point, and we enter into another Horizon, distant also by a Quadrant from our Vertical Point, even as the Pole
of

of the Horizon; (for Poles are distant from their Circles by a Quadrant). Therefore now we number to the Pole of the Equator, or to the North Pole of the World; not more than a Quadrant, as before, but somewhat less than a Quadrant, and the Pole is seen above our new Horizon, a little elevated, and an Oblique Sphere is begat, in which the Stars near to the Pole do not any more Rise and Set, but make their Circuits above the Horizon; but the opposite Stars are carried beneath the Horizon, and become altogether Invisible: And this Elevation of the Pole becomes always greater, by proceeding towards the North, until we come under the Pole itself, and then we obtain a Parallel Sphere, in which, nothing more of the Fixed Stars neither Rise nor Set, but half of them are always seen above the Horizon, and half of them remain beneath the Horizon, and not to be seen: Therefore if from the Center of the Globe we conceive the Superficie of it every where upwards, even as from the Earth we contemplate the Heaven every where above us; the Pole is not elevated, which is Immovable, but the Horizon is changable; we proceeding towards the North, and proceeding on in an Immovable Globe, the mutable Horizon will come to be depressed so, that the Pole is elevated above it: But because this is contrary to our imagination, therefore we accommodate it to our understanding, and the Pole remaining in the Horizon; with it we also only elevate or depress the Globe. Concerning this thing, consider the apposited Instrument, *Fig. 142 and 143*, in which the Rotundity of the Earth, according to the Latitude; variety of Spheres, Right, Oblique, and Parallel, and all those things which we shall deliver hereafter concerning Artificial Days, may be easily understood.

CHAP. IX.

Concerning the common or first Motion, being considered in general.

Hitherto we have spoken of the Various Site or Position of the Stars, it follows, that we consider the Motion of the same, not in respect of one or other Star, but of all in general.

1. *As therefore the Material Sphere, or Heavenly Globe, with the Stars and Circles ascribed to them, may be turned about its Axis and Poles to the Horizon on the East: So likewise all the Stars, as well the Wandring as the Fixt, exercised by the first Motion, common to all, may be turned about the Center, Axis and Poles of the World, from the Eastern part of the World, through the upper part of the Meridian, to the West part of the Horizon, and hence, through the lower part of the Meridian, again to the Eastern part of the World.*

2. *The Center of this First Motion, is the Center of the Earth; or rather the Eye of the Beholder on the Superficie of the Earth.*

For because the Earth hath no Sensible Proportion, to the huge Supream Heaven of Fixed Stars, and also to the Superior Sphaeres of the Planets; so also no notable difference arises, whether we consider the Motion of the Stars, either from the Center, or from the Superficie of the Earth: Although this Diversity, about the Inferior Planets is not to be neglected, but some correction is to be used from the Doctrine of Parallaxes; for as much as the Circular Motions are not Regular, except to one and the same proper Center.

3. *Indeed*

3. *Indeed there are no real Axes and Poles in the Heaven, with respect to this Motion, yet for the sake of better understanding, we conceive a Right Line, instead of the Axis and Poles, continued through the Center of the Earth on both sides, to the Extream of the Visible World, near the Lesser Bear Star, or the last of the Stars of the Tail of the Lesser Bear, and continued to the place opposite to this : In the Extream of which Axis we conceive the Poles of this first Motion, one being the Arctic, or North Pole, the other the Antarctic, or South Pole.*

4. *The Stars going round about this Axis and Poles, describe parallel and equidistant Circles, those which are nearest the Poles being the lesser ; and those which are further off are greater Circles ; of which the middlemost and greatest is the Equator, being the Rule and Measure of this whole Motion, Fig. 147.*

Although these Circles are imaginary, yet they do not wholly want a Foundation : For if a Mark or Sign on a Potters Wheel, be painted with divers Colours, although it be turned about most swiftly, that Sign or Mark is seen to describe an whole and perfect Circle ; therefore for the sake of Teaching, Circles of this kind are also conceived by the Imagination in this first Motion.

5. *The Terms of the first Motion in which it begins and ends, are inconstant indeed in the Horizon, but more constant or certain in the Meridian, in which Astronomers for the most part begin and end the first Motion.*

For all Stars of every place are said to be equal almost in space of time at the Meridian : In the Horizon 'tis otherwise, because 'tis varied in each Obliquity : Therefore although in a Right Sphere, the beginning and end of the first Motion, may as conveniently be taken in the Horizon, as in the Me-

ridian, since the Horizon of a Right Sphære is a Meridian in some place, and the Meridian an Horizon; therefore the *Ratio* is the same of both: Yet in an Oblique Sphære, some Stars near the Poles, and in a parallel Sphære, an half part of the Stars, never pass either under the Horizon, or rise above the same: But those Stars which are most remote from the Poles in our Oblique Sphære, on the North part, abide a longer time above, and a shorter time beneath the Horizon; but on the South part, they abide a shorter time above, and a longer time beneath the Horizon: Because of this variety, the first Motion is not commodiously Terminated in the Horizon, but 'tis needful that we assume the Meridian.

6. *The Periodical time of the first Motion, is commonly defined by the Quotidian Revolution of Diurnal Light, and of Nocturnal Darknes; either continual, if we begin from the Horizon; or cut off in the midst, if we begin from the Meridian; or by the space of a Natural Day, which we divide into 24 equal Hours.* *

For this cause 'tis usually called the Quotidian, or Diurnal Motion; and indeed with respect of the Sun's returning from the Meridian to the Meridian, this Periodical Motion of a Natural Day is exactly the space of 24 Hours; for the Revolution of the Sun from the Meridian to the Meridian, however it be, measures and constitutes the Natural Day, divided into 24 Hours: But with respect to the other Stars, as well Fixt as Wandring, this Periodical time is also to be taken at all adventure, by reason that to the first and common Motion, there is always mingled together without any Cessation of one or r'other, the proper, or second Motion of the Stars, which proceeds under the Zodiack, by a contrary way to the first Motion from the West towards

wards the East : Therefore the Stars, whose proper Motion is slower than the proper Motion of the Sun, they do return to the Meridian from whence they departed, somewhat later ; but of those which are swifter in Motion, they return somewhat sooner than in 24 Hours : But because the Difference in the Superiors is not very great, this Periodical time of the first Motion is commonly wont to be defined by 24 Hours.

7. *Therefore although the first Motion being considered in general, is common to the Exercise of all the Stars, yet with respect of one and t'other, either of Circles or of Stars, and also according to the Variety of a Right Sphere, and of an Oblique and Parallel Sphere, it admits of some Variety, of which also we must Treat.*

C H A P. X.

Of the First Motion considered in the Ecliptick.

1. **N**OT only Stars, but also Circles (the Horizon and Meridian excepted) are understood to be carried about by the first Motion ; among which, first of all the Revolution of the Equator, and of the Ecliptick, is remarkable.

2. The Revolution of the Equator is equal, even as the Measure and Rule of the first Motion in equal parts ; but of the Ecliptick unequal.

3. For as much as the Ecliptick is a Circle oblique to the Equator, it cannot be, but that the Parts or Arches of it meeting with the Equator, ascend above the Horizon, or descend beneath the same unequally.

4. For some Arches of the Ecliptic ascend Rightly, others Obliquely.

5. *They are said to ascend Rightly* (as well in a

Right, as an Oblique Sphere) with which a greater Portion of the Equator ascends, than is obtained in the Ecliptick, but those Obliquely, with which a lesser Portion of the Equator ascends.

6. This Inequality is Examined, according to the Ascensions and Descensions of the equal parts of the Equator, if we would have only a certain knowledge of those things which depend on the Ecliptick; whence the Doctrine concerning the Astronomical Rising and Setting, or of the Ascensions and Descensions of the parts of the Ecliptic, pertaining to Astronomy, is begat.

7. And the *Ascension* or *Descension* of the Parts of the *Astronomical Ecliptic*, is an Arch of the Equator, which ascends with any Arch of the Ecliptick, either above the Horizon, or descends beneath the same.

8. And it is either *Continual* or *Parted*.

9. *Continual*, is that whose beginning is taken from the Vernal Equinox: *Parted*, which is began from any other place.

10. Each is either *Right*, which is numbred in a Right Sphere: Or *Oblique*, being numbred in an Oblique Sphere.

In a Parallel Sphere, by reason of the first Motion, no portion neither of the Ecliptic, nor of the Equator can Ascend or Descend, since the Horizon, and the Equator are united, for the halves of the Ecliptick always remain, the one above, the other beneath the Horizon, and the whole Sphere is turned about even as a Milstone.

2. And we must Note here by the way also, that there is a Difference between the Right *Ascension*, of which we treat here, and that which we have defined before at Ch. 8. Num. 10. they agreeing in Quantity, whence also they have the common name, *Difference*; because the Right *Ascension*, concerning which we

Treat

Treat of in this place, is numbred only in the Horizon of a Right Sphere : But that, in the Circle of Declination is as well of a Right, as of an Oblique Sphere.

11. *Therefore concerning Right Ascensions, as also of Descensions, the following Rules are to be Noted.*

1. The Quadrants also of the Equator ascend with the Quadrants of the Ecliptick, beginning from the Cardinal Points, α , γ , ϵ and ω . Fig. 148.

2. In Fig. 149, The Parts of those Quadrants, or also whatsoever Quadrants are began elsewhere, by how much nearer they are to the Equinoctial Points, they have a lesser portion of the Equator Coascending, and by how much further off they are, they have a greater part of the same Coascending.

3. Also equal Arches of the Equator ascend with any two equal Arches of the Ecliptick, and equally distant on each part from any of the four Cardinal points

4. Opposite parts of the Ecliptick have equal Ascensions : for Example, the Signs γ & ϵ , δ & η .

5. The Ascension of any part of the Ecliptick, is equal to the Descension of the part opposite to its self.

12. *Of Oblique Ascensions, and Descensions, these are the Principal Rules.*

1. The two halves of the Ecliptick began from the Equinoctial Points, have also the Halves of the Equator Coascending.

2. The Parts of these Halves, or also of any other Halves began elsewhere, have not equal Ascensions of the Equator ; but either they have a greater, or a lesser portion of the Equator Coascending. See Fig. 150, and 151.

3. The Arches of the Ecliptick parted, by how much they are nearer to the Vernal point of the Equinox, they ascend the more Oblique ; by how

much they are nearer to the Autumnal Point of the Equinoctial, they ascend more Rightly.

4. The Oblique Ascensions of the parts of the Ecliptick, equally distant from one of the Equinoctial points, are equal.

5. The Ascension of part of the Ecliptick, is equal to the Descension of the opposite part.

13. *And from the comparing of Right and Oblique Ascensions, is begat the Ascensional Differences, by which the Right and Oblique Ascension differ between themselves.*

14. *Those in Northern Signs, being taken away from the Right Ascensions, produce the Oblique Ascensions; and they produce the same by being added thereto in Southern Signs. See Fig. 152 and 153.*

In these Schemes, *A B C* is the Horizon belonging to the East, *D E* the Equinoctial, *F G H* a portion of the Ecliptick, *K* the North Pole, *K G L* the Circle of Declination: The Right Ascension then of the portion of the Ecliptick *G H*, is *H L*, which the Circle of Declination *K G L* determines in the place of the Right Horizon: But *H B* is the oblique Ascension, and *B L* the Ascensional Difference to be taken from *H L* in the former Scheme, in which *H G* is on the North part; but to be added to it in the latter, in which *H G* is on the South part, to make the Oblique Ascension.

CHAP. XI.

Of the First Motion, considered in the Sun.

1. **T**HE First Motion of the Sun may be considered two fold, either in respect of the Horizon, or in respect of the Meridian.

2. *In*

2. *In respect of the Horizon*, by its first Motion, as a Fountain of Light, it constitutes the Day, called Artificial, and the opposite, Night : *The Day*, as long as the Center of the Sun, being above the Horizon, enlightens our Hemisphere with a clear Light : *The Night*, in which the Center of the Sun, being beneath the Horizon, Darkness approaches ; which are the two parts (with us to whom the Sun daily Rises and Sets) of which the Natural Day is compounded, which is Limited by one Revolution of the Sun, and is divided into Twenty Four Hours.

This is thought to be called a Natural Day, because 'tis produced by Nature : But the Artificial, not because it is from Art, but because 'tis accommodated to the Exercise of Artificers.

3. *In respect of the Meridian*, it constitutes a Natural Day, distinguished into two equal portions ; but so divided, that not one part gives place to Light, and the other to Darkness, but each part participates both of Light and Darkness, one of which is from Midnight to Noon, the other from Noon to Midnight.

This Reason of determining the Natural Day, even also, as aforesaid, Astronomers have chose before others, because in an Oblique Sphere the Horizon is Variable, when on the contrary, the Meridian is more constant, and is every where as a Right Horizon.

4. Nor yet doth it return to the Meridian with the same, but one time with one, and another time with another Degree of the Ecliptick.

The cause of this is the proper Motion of the Sun, or the Second Motion concurring with the First, whereby, under an Oblique Ecliptick, it creeps a long incessantly by little and little, into the following Sign, from the West into the East : Therefore seeing a Natural Day, which consists of one Revolution

tion of the Sun, cannot be finish'd before the Sun return to the Meridian, 'tis manifest that it returns thither with different Degrees of the Ecliptic.

5. And by reason of the obliquity of the Ecliptick, this daily Motion of the Sun doth, as it were, constitute Spiral Lines, not returning in themselves, but having the beginning about the end of the former, and the end about the beginning of the latter, which are esteemed for perfect Parallel Circles, by reason of the small distance of the end, and of the beginning: the middle of which is the Equator, and the Extreame are the two Tropicks.

Therefore the daily Motion of the Sun, is mixt of the First and the Second Motion, as it were a Thrid turned about to a Cylinder: But yet because the beginning and ends of these Spirals, are but little distant from each other, neither doth there arise because of them an Error worth the Noting, they are esteemed for perfect Circles; no otherwise than as if the Sun, by reason of its first Motion, should rest in the beginning of any one Degree of the Ecliptick an whole Day, and suddenly afterwards should pass, or skip to the beginning of the following Sign.

6. Also it comes to pass from this concurring proper Motion of the Sun, that its first Motion is not equal to the whole Revolution of the Equator, but somewhat longer, and some additaments happens from all the Days, to the Revolution of the Equator.

7. And this Additament is no other than a Portion of the Equator agreeing to the proper Motion of the Sun; or the Right Ascension of an Arch of the Ecliptick, which the Sun makes by its proper Motion Daily.

8. But these Additaments are not equal, but some greater, others less; yet so, that the Defect of one compensates the Excess of the other, and each Additament

ditament being collected through the Year, makes the whole Revolution of the Equator.

For as the Sun in the space of a Year, or 365 Days and $\frac{1}{4}$ almost, walks through the Ecliptick; so also these little portions of the Equator, consume the whole Equator in the space of a Year, and as well the Sun as the Additaments, are returned to the same beginning: And in this manner it comes to pass, that from 365 Days and $\frac{1}{4}$, the Revolutions of the Equator proceed to one more, viz. 366 and $\frac{1}{4}$, this last and supernumerary Day, being made not together, in one Day, but collected by parts and pieces, thro' the whole Year.

9. And the cause of the inequality of Additaments is Three fold.

1. The proper Motion of the Sun being unequal under the Ecliptick.

2. The Obliquity of the Ecliptick to the Equator.

3. The Obliquity of the Horizon in an Oblique Sphere.

1. For from the proper Motion of the Sun, being unequal, 'tis necessary also that the Arches of the Ecliptick, which the Sun daily measures, should be unequal, and therefore also the Ascensions of the Equator, ascending or descending together, are made unequal: So in these days of ours, the Sun about the Summer Solstice in Apogee, or in its farthest distance from the Earth's Center, dispatches in one Day $57' : 3''$: But about the Winter, in Perigee, or its nearest distance to the Earth's Center, it finishes $61' : 20''$, as shall be shewn hereafter.

2. And although the Arches of the diurnal Motion of the Sun, under the Ecliptick, are, or should be equal between themselves, yet by reason of the Obliquity of the Ecliptick in divers places, they will also have divers, and so unequal Ascensions.

3. Which

3. Which inequality likewise, by reason of the Obliquity of the Horizon is increased, and according to the variety of an Oblique Sphere is varied in sundry sorts.

10. And that we may measure the unequal Overplus, or Additaments, by some certain Rule, Astronomers assume, for the sake of measuring, mean, or equal Additions between the greater and the lesser; to wit, for every Day $59' : 8''$, that is, so much also as is the mean, or equal Diurnal Motion of the Sun in the Ecliptick.

And these $59' : 8''$ are produced if you divide the 360 Degrees of the Equator into 365 Days and $\frac{1}{2}$, even as if you should say by the Golden Rule of Proportion.

As 365 Days and 6 Hours, To 360 Degrees : So 1° , To $59' : 8''$.

Or the Resolution being made in the least Terms.

| | | | |
|--------|----------|--------|----------|
| Hours. | Seconds. | Hours. | Seconds. |
|--------|----------|--------|----------|

As 8766, To 129600 : So 24, To 3548.

Which divided by 60, makes $59' : 8''$.

11. And this Inequality of the Sun's Diurnal Motion, redounds also in Natural Days, which is constituted by the Diurnal Motion of the Sun, mixt with its proper Motion, which are hence also made unequal.

But concerning this, we shall say more hereafter in the Chronological Part.

CHAP. XII.

Of the First or Common Motion considered in the Fixed Stars, and the other Planets.

1. **T**HE Fixt Stars by their First Motion finish their Period sooner than a Natural Day, or

24 Hours, and return to the Meridian, whence they departed.

The reason is, because although they are seen to proceed also like as the Sun by a proper Motion, in an orderly following; yet because this Motion of theirs is most slow, neither follows the Sun in any manner; Therefore they return to the Meridian almost by so much sooner, as the Sun recedes longer in the space of a Natural Day from the former point; in which the Sun returning to the Meridian, the Natural Day at length is accomplished.

2. Hence it comes to pass, that in the Night oftentimes, some and other Stars succeed in the place of the former, until in the space of a Year, the whole Band of Stars arise above our Horizon, and are Visible to us.

For if the Sun should remain Immovable in one place of the Ecliptick, or the Stars by their proper Motion should proceed in like velocity with the Sun, we could not see the Stars but of one Hemisphere, the rest being hid by the brightness of the Sun: But for as much as the Sun by its proper Motion departs more swifter from the Stars, which proceed more slower, and comes unto others; it comes to pass that by its departure ever and anon, other and other Stars are discovered, and in the space of a Year all are brought to our sight, except those Southern Stars, which always lye hid beneath our Horizon.

3. The other Planets, by how much they are more slow than the Sun in its proper Motion, touch at their Journeys end of the first Motion, sooner than it, and by how much they are more swifter, they touch later than it, these are accomplished later than a Natural Day, those sooner.

For because the Sun, by its return to the Meridian, accomplishes a Natural Day, it necessarily follows, that

that those which run out beyond the Sun in proper Motion, come to their Journeys end later, and those which remain on this side it, sooner.

4. And they advance to us in the Northern part, or Hemisphere (our face being turned towards the East) from the Left towards the Right Hand: On the contrary, in a Southern Hemisphere, from the Right towards the Left Hand.

The reason is because the Equator, the Rule of the First Motion, is on the Right Hand to those in the Northern parts, if their Face be turned towards the East, and on the Left Hand to those who are in the Southern parts.

5. Moreover, as we are accustomed to inquire of the parts of the Ecliptick, so also we inquire the Ascensions and Descensions of any Stars, as well those that are continual from $\circ$ γ , as those which are parted from other Stars, and the Degree of the Ecliptick Rising and Setting together.

6. Lastly, As the Sun while he remains above the Horizon makes the Day, and while beneath the same, the Night; so by Analogy also, we call the abode of any Star above the Horizon, the Day of the Star; and its abode under the Horizon, the Night; although that happen in the Night, but this in the Day.

C H A P. XIII.

Of the Common Motion of the Stars, compared with the Sun.

1. **T**HE Stars, as well Fix'd as Wandring, being taken with the Sun in their common Motion, Arise and Set in divers manners, which is called

called the Poetical Rising and Setting of them.

The Reason of this Appellation is thus assigned, because the Poets in describing the times of the Year, are wont to use commonly one or another of these kinds of Rising and Setting.

2. And there are Three of these kinds of Rising and Setting.

1. *Cosmical*, } when a Star } with the Sun Rising.
2. *Acronychal*, } Rises or Sets } with the Sun Setting.

3. *Heliacal*, in which the Stars either appear from the Beams of the Sun, or are hid by them : which when it happens in the Morning, is the Morning *Heliacal* Rising, or Setting ; or if in the Evening, 'tis the Evening *Heliacal* Rising or Setting : See Fig. 154, and 155.

The two former, the *Cosmical*, and *Acronychal*, are the true *Ascensions* and *Descensions* above or beneath the *Horizon* : Whence 'tis called the true Rising and Setting : The *Heliacal* is only the Apparition or Disappearing of the Stars existing about the *Horizon* ; proceeding from the proper Annual Motion of the Sun, which sometimes by its Access and Recess to the Stars, and presence of Splendor, or absence at other times, hides or uncovers them ; hence 'tis called the Apparent Rising or Setting. See Fig. 156.

3. The *Cosmical* and *Acronychal* Rising and Setting is proper to all Stars, which Rise and Set, except *Venus* and *Mercury*.

For these two Planets cannot set *Cosmically*, and rise *Acronichally*, to wit, just over against the Sun, by reason of their shorter Digressions from the Sun, in which they never obtain the opposite place of the Sun.

4. The

4. The Morning *Heliacal* Rising, and the Evening Setting, happens to Stars, which in their Motion are slower than the Sun ; and by reason thereof, being left by the Sun in the Morning are seen again, and being overtaken by the same, in the Evening they vanish.

Such are all the Fixed Stars, and the three Superior Planets γ , μ and δ ; also Venus and Mercury, but not unless when they are Slow and Retrograde in their Motion.

5. When Stars are swifter than the Sun, they are apparent in the Evening from the Sun Beams, and they are hid in the Morning.

Such are Venus and Mercury when they are more swift in course, and the Moon : for these Stars approaching to the Sun, are in the Morning intermixt with its Beams, and departing from it in the Evening, they unfold themselves from its Beams.

6. The Terms of distance of the Sun from the Stars, by which they either Rise or Set *Heliacally*, are unequal.

For by how much they are greater and brighter, or lesser and obscure, they require also a lesser or greater distance of the Sun to their appearing, or being hid.

7. Yet Artificers have constituted these Terms, (by the observation of Stars, not far posited from the Ecliptic) nearly agreeing to the Truth.

Degrees.

| Degrees. | | In the Twilight. | |
|--|-----|---------------------------------|-------------------------|
| The Sun being
under Motion
in the Vertical
Circle under
the Horizon. | 19 | They
Appear
or
Vanish. | The smallest magnitudes |
| | 18 | | The Sixth. |
| | 17 | | The Fifth. |
| | 16 | | The Fourth. |
| | 15 | Among
the Pla-
nets | The Third. |
| | 14 | | The Second. |
| | 13 | | The First. |
| | 12 | | Mars. |
| | 11½ | | Saturn. |
| | 11 | | Jupiter. |
| | 10 | | Mercury. |
| | 9 | | Venus. |

The Moon hath not a certain Term, but appears sometimes the same Day, some times a Second, or Third, or also the Fourth Day after New Moon, according to the diversity of its Motion.

But these assigned Terms or Times of Apparition, and being hid, are numbred, not in the Ecliptick, as hath been said, but in the Vertical Circle, passing through the Sun, from the Horizon even to the Center of the Sun.

In Fig. 157. E or F are Stars in the Horizon, B C is the Sun's distance beneath the Horizon, numbred in the Vertical Circle A B C, according to the Term of the Stars appearing, or being hid.

CHAP. XIV.

Of the First Motion, considered in respect of divers Meridians and Horizons.

1. **T**HE Sun and all the Stars by their First Motion, always arise above the Horizon, are always conversant in the Meridian, and always go under the Horizon.

X

This

This is a Paradox, but 'tis true, if it be understood according to the diversity of Places : For because the Earth is round, and the Earth and Sea every where Habitable, and the Heaven is in every place upwards, and the Sun, and the other Stars in perpetual Motion : Every place hath a proper Vertical point, and consequently also a proper Horizon, and a proper Meridian : So that the Sun, the whole Day, and all the Stars, only proceed from one Horizon to another, or from one Meridian to another.

2. Therefore there are so many beginnings of the First Motion, as there are Horizons and Meridians ; that is, Innumerable.

And so also the First Motion always begins, and always is finished, and with it also the Natural Day.

3. The time of the Sun's approaching to the Meridian, although it be the same in the reason of numbring, to those People or Nations which are used to One and the same kind of Clock ; yet in its Nature, according to the diversity of Places, 'tis most Different.

For supposing after our manner, that the time of Noon and Midnight, is that in which the Sun is conversant in the Meridian, and being numbred by the Twelfth Hour is Universal, and in the whole World the Meridian be numbred the Twelfth Hour ; yet because the Meridian or Noon-time other where is divers ; therefore that which in one place is the Twelfth Hour, is in places more Western some Hour before Noon, in Eastern after Noon : For Example, At Vienna the chief City of Austria, the Twelfth Hour, is, in a manner, the Eleventh Hour before Noon at Paris, the chief City of France.

So that Eclipse which Ptolomy witnesses in Book i. Chap. 4. Of Geography, to be seen at Arbela, an Hour after Sun Set, appeared at Carthagen the Second Hour of the Night : So Pliny, Book 2. Ch. 70.

said,

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saith, That at what time Alexander the Great won that famous Victory at Arbela, the Moon was Eclipsed at the Second Hour of the Night, but the same appeared in Sicily at the beginning of the first Hour. This Eclipse happened in the Year Nabon 417, the Thirteenth Day of Epiphus of the Egyptians, which is the Second in the Olympian Year 111, in the Solstice, this is Years 351 before Christ was born, and the Twentieth of September, according to the Julian Years. See Fig. 158.

4. Therefore among so many Meridians, 'tis necessary for keeping of Order, that some one be appointed First or Chief, to which the rest are referred by the Ratio of time, and of distance, which by Ptolomy, and other Ancients, was that which passes through the *Fortunate* or *Canary Islands*, from which all the following towards the East, are numbered, according to the Series or Order of the Signs, the distance of which from the First, is called the Longitude of that place through which it passes. Fig. 133.

The Reason of this Election seems partly, that Nature it self hath made the beginning there, as well of Africa as of Europe; and partly, that places more Westernly were unknown to the Antients. The Modern Geographers begin to number the Meridians from those Islands in which the power of the Magnetick Needle turns precisely to the Pole of the World; which are called the Islands of Corvus, or the Green Cape, and are some Degrees more Westernly than the Canaries: We following Ptolomy, number the Longitude or Distance of the Meridian of Tubing, from the Fortunate Islands. $33^{\circ} : 40^{\circ}$.

So much of Spherical Doctrine.

O F
A S T R O N O M Y.
B O O K II.

Of Theoretical Doctrine.

The P R E F A C E.

I. **H**AVING finished the Spherical Part of Astronomy, which Acts concerning the First Motion ; it remains to Explain the Theoretical Part, which Explains the Ratio, or Reason of Second Motions.

2. We shall Constitute its Parts.

I. By Explaining the General Theoretical Hypotheses, which Treats of the Second Motion of the Fixed Stars and Planets.

II. We will Declare the Passions of the Planets, which attends the Motions of the same.

III. We shall propose the Judgments or Opinions of Astronomers, concerning the Magnitude and Distance of the Stars.

O F

O F
ASTRONOMY.
BOOK II.

The First Part.

Of the Second and Proper Motion of the
S T A R S.

CHAP. I.

Of the Second Motion, and the Hypotheses
of it in General.

1. **T**HE Second and Proper Motion of all the Stars, is that whereby they proceed in an Oblique March under the Zodiack, upon the same Axis and Poles; or in an orderly following, and according to the Series or Order of the Signs, from the West to the East.

*By what Signs this Motion becomes known, is shew-
 ed before at Chap. 4. Book 1.*

2. And this Motion is Twofold; one of *Longi-
 tude*, the other of *Latitude*.

3. The Motion of *Longitude*, is that in which be-
 ing moved nigh to the Zodiack, they are wont to
 march according to the Order of the Signs: Of *La-
 titude*, in which they depart from the middle Eclip-
 tick Line of the Zodiac to the Sides.

*In Truth there are not two, but only one Motion,
 in which they describe a Circle intersecting the Eclip-
 tick in two places; whence by the Motion of Longi-
 tude*

tude they are conveyed to the Sides of the Ecliptick: And because the Progression of the Stars being led according to the Zodiac, is not only considered, but also this digression or departing from the Ecliptick to the Sides; therefore for the sake of Teaching Astronomy, they have made, as it were, a double Motion, that they may speak distinctly concerning distinct things.

4. And again, each is Twofold, True and Meane.

5. The True Motion, is that, in which the Stars in the Heaven, for the most part appear to us living on Earth, to move unequally: Whence also 'tis called Apparent and Unequal. But the Mean Motion is that which they assume for the sake of Teaching Astronomy, to be the Mean and Equal, between the Swiftest and Slowest.

And it matters not whether or no, in Truth, this Motion be found in Heaven, or is only supposed by Astronomers; so that it be made fit, that from this Mean or Middle Motion, at length the True one be produced.

6. The Epocha, Root, or Beginning of place, whence the Second Motion of Longitude is wont to be numbred; is either from the first Star of Aries, or γ , or the beginning of the Sign Aries; to wit, the Point of the Vernal Equinox, notable before others, whence Astronomers usually begin the Zodiac.

For the Sun being constituted in this Point: 1. It being in the middle place, it gives the distance of its greatest Digressions from the Equator into the North and South. 2. It makes the Equinox in the whole Earth: And Thirdly, with us in our Hemisphere on the North part, making the beginning of Spring time, opens the Earth again, and Enlivens all things Decayed and Numm'd by the Winter Coldness, whence it is deservedly preferred to others.

7. And

7. And the beginning of time, whence the Heavenly Motions are at first deduced, is assumed otherwise of others.

The most famous are:

1. The Epocha of Christians numbred forward and backward, of which at this time the current Year is 1700.

2. In the beginning of the World, since the Fall of our first Father Adam, from the beginning of the Christian Epocha, distant, in our Opinion, 4010 Years, and 64 Days.

3. The Beginning of the Julian Period of Anticipation, which goes before the beginning of the World, 749 Julian Years, and 302 Days: The beginning, from the Christian Epocha, 4760 entire Years.

8. The measure of this Motion with us, are Days divided into Hours and Minutes; also Months and Julian Years, instituted by Julius Caesar.

For the Motion not only of the Intervals of Places, but also of Time is usually measured; the Ratio of which Times, since 'tis drivers with us, Astronomers also have commonly used Egyptian Years, Months and Days; we retain the Julian Time, which to us is most noted, and therefore may commodiously be used instead of measure.

And although these Heavenly Motions also are measured by these Times, (which Doctrine is constituted by us in the Third Book of Astronomy) yet nothing binders, but that also these Motions may be measured interchangeably by lesser Times; after what manner the Motions are more known to us: For so 'tis ordained in many other things, that the same in divers respects may be the measure and the measured; to wit, if that which obtains the place of Measure be also known by it self, which also happens in this of times: For who is there which knows not, that 60 Minutes make an Hour? 24 Hours a Natural Day? Who is

Ignorant how many Months are in a Year? And how many Days each Month hath? Likewise what is the Difference between Common Years and Bissextile, recurring or happening every Fourth Year; of which those have 365 Days, but this 366: And that 'tis easie to divide the 60 Minutes of an Hour into 60 Second Minutes, or Second Scruples, and if you please, of these into so many Thirds, &c. if we would find out the most small parts.

9. And the measure of the Distance of the Stars, arched indeed from a certain point of Heaven, are Circles divided into Degrees and Minutes; and that the measure of the same from the Center of the Earth, or of the World, or the Motion of any Circular Star directly removed, are either parts of a Semidiameter assumed at pleasure, or the Semidiameter of the Earthly Globe.

Concerning the Divisions and Subdivisions of a Circle into Degrees, and Minutes, or Scruples, hath been spoken of already often times, the Semidiameter of any Circle may be assumed at pleasure; and the more parts 'tis divided into, the truer will the Calculation be.

The Semidiameter of the Earth is commonly appointed of German Miles (each being determined by the Journey of two Hours) to be $859 \frac{1}{11}$, or to shun the Fraction, 860; or is of such parts $57 \frac{3}{11}$, of which the Circumference of the Earth is 360.

A German Mile contains about 4 English Miles, so that the Semidiameter of the Earth contains about 3436 English Miles, and $\frac{4}{11}$ of a Mile.

10. Lastly the measure of Planes, or of the grossness of Heavenly Bodies, is the Square or Cube of this Terrestrial Semidiameter.

For of great things, such as are Heavenly, a great measure also is convenient; then because if the Terrestrial Semidiameter be appointed of Miles 3436,

The

The Square of it will be, Miles 11806096.

And the Cube of it will be 40565745856.

English Miles.

11. The Organs or Instruments of Second Motions are of Theory or Contemplation, that is, either of a Figure, or of little Orbs Artificially made of Paper, or only of Circles described by the Compasses; in which the Motion of the Planets are peculiarly subjected to the Eyes, and the labour of Computation of the Motions either taken away, or rendered more easie.

For in things so remote and abstruse, 'tis necessary that the Mind be help'd by Sense: For which end the Theory of the Planets are found out by Artificers, that even as by a material Sphere, and Cælestial Globe, the First Motion is exposed; so by these Theories, the Second and Proper Motions of the Planets are exposed to the Eyes without tedious Calculation: Or because this Proportion and Magnitude of Observations cannot be made accurately, the Ingenious only delineate them with the Compasses, that they may also be the better comprehended by the Mind, and the Calculation may be the easier produced; for the end of Astronomy is, not only the bare knowledge of the Heavenly Motions, but also the skill of Calculating the same to any given time, which by the help of Theories is performed by Geometrical Calculation.

And although at this time, almost all Artists in Astronomy make proper and peculiar Theories to themselves, of which we cannot treat here, but refer the Studious in Astronomy to those Authors, which explain the Reasons of their Hypotheses and Theories liberally, and the manner of using them; notwithstanding all agree in some general parts, which we are willing to shew in this place,

place, leaving the more particular to their Authors.

12. The general parts then of all Theories, are either Orbs made of plain Paper, or Circles only delineated by the Compasses.

These represent, and subject to the Eyes, the Celestial Motions from their Causes, not indeed always by Physicks and Natural, but Geometrically; yet the same as Physicks perform, with which we are forced to be content, until the True and Physical Causes of all things be made known to us.

13. But of these Orbs and Circles, some are *Concentrick*, which are described from the Center of the Earth, or of the World: Others *Eccentric*, which are described from another and proper Center, peculiar to its self: Lastly, some are *Epicycles*, which being *Concentrick* or *Eccentric*, even as precious Stones Ingrafted or Inlaid in Rings, are carried about by them, and nevertheless are also moved about the proper Center.

In these are considered chiefly:

1. The *Eccentricity*.
2. The *Apogæon*.
3. The *Perigæon*.
4. The Line of True Motion.
5. The Line of Mean Motion.
6. The *Prosthaphæsis* or Equation.

14. *Eccentricity*, is the Distance of two Centers.

15. *Apogæon*, in Arabick, *Aux*, or top of the Circle, is the Point of Eccentricity, most remote from the Center of the Earth, or World: *Perigæon*, the opposite of *Aux*, or the lowest Point in the Circle next the same.

17. The

16. *The Line of True Motion of a Planet*, is a Right Line extended from the Center of the World, through the Center of the Body of the Planet to the Zodiack.

17. *The Line of Mean Motion*, is a Right Line extended from the Center of the World to the Zodiack, equidistant to that Right Line, which is drawn from the Eccentrick Center, to the Body of the Planet; or (in Theories which use the *Epicicle*) to the Center of the *Epicicle*.

18. *Prosthaphæresis*, or *Equation*, is the difference of the True and Mean Motion, sometimes to be taken from the Mean Motion, at other times to be added to the same, to produce the True Motion.

Fig. 160, In this Scheme, *a* is the Center of the World, or of the Zodiack, *chd* the Zodiack, *b* the Center of the Eccentrick, *egf* the Eccentrick, *a b* the Eccentricity, *e* or *c* the Apogæon, *f* or *d* the Perigæon; hence *d f a b e c* is called the Line of Apogæon, *a g l* the Line of Mean Motion, or of the *Epicicle* *m* in Theories which use the *Epicicle*, or of the Planet *i* when the *Epicicle* is wanting, equidistant or parallel to the Right Line *b i k*, drawn from the Center of the Eccentrick, to the Center of the Star or *Epicicle*; which lines *b i k*, *a i h*, *a g l*, fall in with one another, the Star or Center of the *Epicicle* existing in the Aux or Apogæon *e*, or its opposite *f*, but elsewhere they differ between themselves; whence the *Prosthaphæresis* or *Equation* is begat, which is an Arch of the Zodiack *h l*, or the Angle *h a l*, or *a b i a*, or *h i k*, for they are equal, which is called the *Prosthaphærical Angle*.

C H A P. II.

Of the Proper Motion of the Fixed Stars.

1. **A**LL the Fixed Stars, besides the daily Motion, in which they seem to be turned from the East into the West, have also another proper, but lingring and slow Motion, the distance of them from themselves to each other remaining unchangable, being led as the Zodiack, upon the Axis and Poles of the same, according to the Order of the Signs, they seem to move forwards from the West into the East.

1. This Motion, the Antient Astronomers were Ignorant of, by reason of the great slowness of it, which the Later Astronomers at length understood, by the changable Longitudes of the Stars from the Vernal Equinox.

2. Hence, sometime at the beginning of Astronomy, there was one Zodiack, whose 12 parts, as also the whole Circle, they named from the 12 Asterisms or Signs, which they were conversant in, as well as in those parts. But at this time after that the Asterisms had exceeded their former places, and the Constellation of *Aries* was got into the place of *Taurus*, of *Gemini* into the place of *Cancer*, the Zodiac is made, as it were, two fold: One of the remarkable Asterisms beginning from the first Star of *Aries*; the other being destitute of Stars from which it should receive a name, which begins from the Vernal Section of the Zodiack and Equator, the parts of which are indeed different from the Asterisms, but yet they retain their Pristine names taken from the Asterisms: From this cause also other ages have required other Cœlestial Globes.

3. *Nicholas Copernicus*, ascribes this Motion of the Fixed Stars, not to the Fixed Stars proceeding according

according to the order of the Signs ; but to the Equinoctial Points going backward, contrary to the order of the Signs ; and therefore he calls it the *Præcession of the Equinoxes* ; the cause of which he ascribes to the Inclination or Reflexion of the Earth's Axis.

But since the Progress of the Stars, is easier understood by *Tyraes*, or young Beginners, than the Regress of the Equinoctial Points ; we rather choose to attribute this Motion to them ; whether they are carried in the free Sky, or that which is received, and also believed more easily, to appoint them affixed to any one Solid Sphere, like to a Wall encompassing the whole World ; for 'tis certain that those change not their distances : For those which *Ptolomy* found in his Seventh Book of *Almagest*. *Hipparchus* found almost 300 Years after to be constituted in one Right Line : Also we at this Day, being Fourteen whole Ages later, behold the same unchangeably disposed in the same Line, to wit, *Capella*, the Left Foot of *Erichonius*, and *Palilicum*, or the Eye of *Taurus* : Also the same *Capella*, with the *Wagoner* in the Right Foot, and near *Orion* in the Left, and many more assigned by others.

4. Either this *Præcession of the Equinoxes*, or *Progression of the Stars*, being for the most part unequal, therefore they thought the *Prostaphæresis of the Equinoxes* needful, to find the true Motion of it : But because this inequality is as yet doubtful by the famous *John Kepler*, and if it should be certain, 'tis of no great moment, we do not think to burthen young *Astronomers* with it.

5. But altho' this Motion should be common to all the Fixed Stars, yet for the most part 'tis usually called only the Motion of the First Star of *Aries* from the Equinoctial Point ; from which all the other Stars retain their certain and unchangable distance,

stance, and are understood to move forward altogether with the $\Gamma * \gamma$.

2. And they proceed in the space of a Week scarce 1".

In an whole Year, only $\frac{1}{51}$ ".

Whence it appears, if they should pass over the whole Zodiac, it will require 25413 Julian Years, *quam proxime*.

3. But whether or no they also have the Motion of Latitude, and that they change it in succeeding Ages, is as yet doubtful: Therefore while the thing is truly found out, we may retain the Latitude for any time, the same which Tycho Brahe hath assigned to them, as may be seen in the Tables of the Longitudes and Latitudes of the Stars at the end of the Book.

CHAP. III.

Of the Proper Motion of the Sun.

1. **T**HE Motion of the Sun also is plainer than the Motion of the other Planets, and necessary to the right understanding and finding out of the same; whence 'tis deservedly preferred before the rest.

2. And the proper Motion of the Sun, is that in which, besides the daily, in which it Rises and Sets every Day, there is moreover another Motion peculiar to it, under the Ecliptick Line, the middle of the Zodiack, upon the same Axis and Poles; seeming to be moved according to the order of the Signs, in an orderly following, from the East into the West.

The Discoveries of this Second and proper Motion are, that it doth not Rise and Set always in the same place, but in its Rising ascends from the the South into the North, and again descends from the

the North into the South: Moreover in the Meridian, or Noon-time, at one time it draws nearer to our Vertex, afterwards it again departs from the same; whence it necessarily follows, that it hath another Motion besides the Diurnal and Ordinary Motion, in which it turns aside hither and thither: And since after Sun Set we always behold some or other Fixed Stars by their whole Circular compass, which before lay hid under the brightness of the Sun, and indeed going forwards according to the order of the Signs, from the West towards the East; it necessarily follows also, that the Sun proceeds the same way, to wit, according to the order of the Signs, from West to East, Circularly, by little and little: Lastly, since the mutation of the Sun's Declination spoken of but now, happens only in those places through which the Zodiack goes, 'tis manifest that this Motion is dispatched upon the Axis and Poles of the Zodiack.

2. *Artists understand the Orbit which the Sun describes by this Motion to be Eccentric, or to have a Center diverse from the Center of the World.*

The Arguments for this thing, are:

1. That the Sun in this Motion seems to go unequally, slower in the Summer, or in the Northern Semicircle of the Zodiack; swifter in the Winter, when in the Southern Semicircle: For in the Northern Semicircle he is conversant, Days 186 $\frac{1}{2}$, but in the Southern, 178 $\frac{1}{2}$; which is truly obtained from the Observations of the Days and Hours between the two Equinoxes of the Spring and Autumn.

2. Moreover the Sun also is unequally distant from the Earth; furthest in the Summer, nearest in the Winter: which inequality appears not only in the unequal apparent magnitude of the Sun's Diameter, less in Summer, and greater in Winter, observed

served by the same Instrument ; but also in the unequal Duration of Lunar Eclipses, in the same distance of the Moon, as well from the Earth as from the Ecliptick ; The cause of which is no other, than the unequal distance of the Sun from the Earth, by reason whereof the shadow of the Earth is made sometimes greater, and sometimes lesser, which we have discovered from the Opticks.

Therefore from these causes, Astronomers have made the Way or Orbit of the Sun Eccentric under the Ecliptick ; for this Eccentricity makes out, that the Motion that is equal about its proper Center, being seen from another Center, appears unequal, as the Opticks teach.

3. *In assigning the Eccentricity, or the Distance of the Centers, of the World, and of the Eccentricity of the Sun, Astronomers disagree, according to the diversity of Hypotheses.*

We in this place retain *Tychoes*, which at this time is parts 3584, of such whereof the Semidiameter of Eccentricity is 100000 : Or parts $2^{\circ} : 9'$, of which the Eccentric Semidiameter is 60° .

This Eccentricity some make to be Variable, but *Kepler* asserts it to be constant, and because he appoints the Ecliptick the Orbit of the Sun, and acknowledges some real Inequality in the Motion of the Sun, proceeding from Physical or Natural Causes : Therefore he assumes only half of the Eccentricity, of Parts 1792, or in a round Number 1800 : And so the cause of Inequality of the Sun's Motion he divides into two, between the Natural and the Optick Cause.

4. *In the Eccentricity of the Sun there is likewise to be noted.*

I. *The Apogæon*, or the Eccentric Point most remote from the Center of the Earth, in which the Sun is slowly moved in one Day $57' : 3''$.

II. *The*

II. *The Perigeon*, that is the Eccentric Point, nearest to the Center of the Earth, in which the Sun proceeds most swiftly; in one day $1^{\circ} : 1' : 20''$.

5. *The Apogæon*, is found in this our Age in the Year 1700, to be in $7^{\circ} : 25'$ of Cancer: but the Perigæon, in the opposite place of Capricorn, or ν .

6. But this Apogæon is not fix'd under one place of the Zodiac, but proceeds by little and little into the following place; the Motion, according to Kepler, being equal, but very slow; so that the Annual Motion of it is $1' : 2''$, and at length finishes its Period in 21612 Years.

7. But the Sun in its Eccentricity under the Ecliptick, from $\odot \nu$, which meets with the beginning of the Sun, performs its Circuit in Days 365, Hours 5, Minutes 48, Seconds 45.

8. Whence by making an equal distribution, its Motion under the Ecliptick is thus.

| | | | | |
|----------------------|----------------|------|------|-------|
| | s | ° | ' | '' |
| The Annual Motion is | 11 | : 29 | : 45 | : 40. |
| The Diurnal | ————— 59 : 08. | | | |
| The Hourly | ————— 02 : 28. | | | |

9. And the Motion of the Sun also is numbred from the place of the Apogæon, and then 'tis called the Annual Anomaly, the Eccentric Anomaly, the Reason of the Sun.

And this Annual Motion, is so much shorter than the Motion of Longitude from the Ecliptick, as the Apogæon proceeds yearly in orderly following, to wit, $1' : 2''$; Whence the Annual Motion of the Anomaly is $11^{\circ} : 29' : 44'' : 38''$.

C H A P. IV.

Of the Proper Motion of the Three Superior Planets $\mathfrak{h}$, $\mathfrak{v}$ and $\mathfrak{z}$.

1. **S**aturn, Jupiter, and Mars, are called Superior Planets, because they be placed above the Sun; Saturn in the highest place, Jupiter in the middle, and Mars in the lowest place, which is acknowledged by all Astronomers.

The reason of this Order is required or sought.

1. From the Quantity of Periodical Revolutions, since 'tis most like to truth, that the Planet that goeth about in more time, it also describes the greater Circle, and is higher than the others: Therefore since $\mathfrak{h}$ requires more time, $\mathfrak{v}$ less, and $\mathfrak{z}$ less than it, to perform their Periods or Circuits in, it evinces also that they are placed in this Order.

And since $\mathfrak{h}$ was seen to be hid by $\mathfrak{z}$ running under him, by the Ingenious Mathematician *Michael Maestlin*, at *Tubing*, the Ninth of *January* 1591, as he testifies in his *Epitomy of Astronomy*; but that $\mathfrak{z}$ is inferior to $\mathfrak{v}$ cannot be doubted.

3. And 'tis also known for certain, that these 3 Planets, in the Quality of their Motion, as well according to Longitude as according to Latitude, are like to one another, and disagree between themselves only in Periodical Quantity.

And for this cause they are usually conjoined for the most part by Astronomers, that the Ratio of their Motion may be discharged in one Tract.

4. Therefore how much Motion of Longitude belongs to them, they observe for the most part in these things.

1. They

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1. They do not only Rise and Set Daily, but moreover they proceed in an Oblique going leisurely from the West into the East.

2. They observe the Zodiack, and the order of the Signs.

3. Yet under which they so go, that not only their Altitude from the Earth, but also their Motion, as well in the same, as in divers places of the Zodiack is unequal, and sometimes they appear Swift, afterwards Slow, by and by Stationary, and also Retrograde.

To the Explanation and Demonstration of this Variety of Motions, some add the Epicycle to the Eccentricity, of which 'tis changed by the Circuit and Altitude, and the Motion of Eccentricity is made swifter and slower, and also seems to go contrarily; but others assume other Hypotheses, and almost every Artist a singular one, the Explanation whereof we shall supersede here, and refer the Studious to the *Rudolphine*, *Danish*, and *Lansbergius's* Tables; in which each Author explains his Hypotheses, and Demonstrates the Heavenly Motions from them; yet, lest Beginners in *Astronomy* should be Ignorant of all knowledge of the Motion of the Planets, we will assign the equal or mean Motions of each, from the *Rudolphine* Tables.

5. Therefore these Planets perform their Course under the Ecliptick from the Point of the Vernal Equinox.

In Julian Years. Days. Hours. Minutes.

♄ 29 : 155 : 03 : 12.

♃ 11 : 313 : 14 : 10.

♂ 1 : 321 : 22 : 21.

6. Therefore the Motion of the same is.

Y 2

Sig.

| | | Sig. | | | |
|---|-------------|-----------|-----------|-------|-------|
| ♂ | The Annual | — 0 | : 12 | : 13 | : 36. |
| | The Diurnal | — — — — — | 02 | : 01. | |
| | The Hourly | — — — — — | — — — — — | 05. | |
| ♂ | The Annual | — 1 | : 00 | : 20 | : 32. |
| | The Diurnal | — — — — — | 04 | : 59. | |
| | The Hourly | — — — — — | — — — — — | 12. | |
| ♂ | The Annual | — 6 | : 11 | : 17 | : 08. |
| | The Diurnal | — — — — — | 31 | : 27. | |
| | The Hourly | — — — — — | 01 | : 19. | |

7. The place of Apogæon of these three Planets at this time are in the Year 1700 compleat.

| | | Sig. | | | |
|---|---|------|------|-------|--|
| ♂ | 8 | : 26 | : 03 | : 59. | |
| ♂ | 6 | : 08 | : 10 | : 31. | |
| ♂ | 5 | : 00 | : 51 | : 31. | |

8. And they do not subsist Immovable in these places, but proceed into consequent places, but with a very slow Motion.

| | | | |
|-------------------------------|---|----|---------|
| For the Annual Motion is of — | ♂ | 1' | : 16''. |
| | ♂ | 0 | : 47 |
| | ♂ | 1 | : 07 |

9. Whence if the finishing the Period of each be required.

| | | Years. | Days. |
|----------------|---|--------|--------|
| The Apogæon of | ♂ | 17125 | : 273. |
| | ♂ | 27468 | : 031. |
| | ♂ | 11358 | : 304. |

10. The Motion of Latitude of these Planets is thence produced, because the way of them, or the continued Plane of the Orbit of them intersects the Ecliptick in two places with an unchangeable Angle.

$$\text{In } \left\{ \begin{array}{l} \text{♄} \quad 2^{\circ} : 32'. \\ \text{♅} \quad 1 : 20. \\ \text{♆} \quad 1 : 50. \end{array} \right.$$

11. The points of Intersection are called Nodes; one indeed ascending, by which a Planet in the North ascends; but the other descending, by which the same in the South descends; and the Ascendants are found at this time.

$$\begin{array}{l} \text{Sig.} \\ \text{♄} \quad 3 : 22 : 59 : 30. \\ \text{♅} \quad 3 : 05 : 32 : 12. \\ \text{♆} \quad 1 : 17 : 50 : 59. \end{array}$$

12. These Nodes are moved with respect to the Vernal Equinox, which Original or Beginning is accounted of the Ecliptick, in a following order, and they proceed in the space of a Year.

$$\text{The Nodes of } \left\{ \begin{array}{l} \text{♄} \quad 1' : 12''. \\ \text{♅} \quad 0 : 4. \\ \text{♆} \quad 0 : 40. \end{array} \right.$$

13. To the finishing the Circuit or Period of which there is required.

| | Years. | Days. |
|------|--------|--------|
| In ♄ | 18137 | : 143. |
| In ♅ | 370607 | : 2. |
| In ♆ | 32607 | : 304. |

CHAP. V.

Of the Proper Motion of Venus and Mercury.

AS those three ♄ ♅ ♆ are called Superior Planets; so these two, Venus and Mercury, are called Inferior Planets, Venus being higher than Mercury.

This Order and Place, and Periodical Quantity, which of *Venus* is less than of the rest, but greater than of *Mercury*; and that which is observed of *Mercury* under *Venus*, *Proclus* writes.

2. And besides that they Rise and Set in a daily Motion, they have this also, common with the other Planets, being led according to the Zodiack, by the Motion of Longitude from the West to the East, going according to the Order of the Signs, and they finish their Circuit under the Ecliptick.

In Days. Hours.

| | | | | | | | |
|---|-----|---|----|---|----|---|-----|
| ♀ | 224 | : | 16 | : | 40 | : | 11. |
| ♀ | 87 | : | 23 | : | 14 | : | 16. |

3. And there are also in them these unequal Motions, in like manner as of the rest; Eccentric, Swifter when they are nigh the Sun, Slower when remote from the same: And indeed most swift, when in the Morning Hours they begin to mingle themselves together under the Sun Beams, or when in the Evening Hours they depart from them.

4. But when they are either hid by the Evening, or appear in the Morning, they become Stationary; to wit, after the Evening greatest Elongations, and before the Mornings: In the Intermediate time they become Retrograde on the contrary part in Conjunction with the Sun.

5. For they run always nigh to the Sun, which by certain Spaces they sometimes go before, sometimes they follow, afterwards returning to it.

In this Course also *Venus* changes her name, and in the Morning going before the Sun, is called *Phosphorus* or *Lucifer*, the Morning Star: In the Evening, when she follows the Sun, she is called *Hesperus*, or the Evening Star.

6. *Venus* indeed departs far from the Sun, to the height of $47^{\circ} : 15'$, and makes fewer Stations, and so.

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so, many times, also is joined with the Sun: But Mercury hath shorter Excursions, and in Apogæon is restrained within 29° ; but in Perigæon beneath 18° : Also he frequently becomes Stationary, and oft times joined to the Sun, and seldom appears with it.

7. But that they also ascend on high, and again descend towards the Earth, argues in Venus the augmenting of her Body in certain places: But in Mercury the swiftness or slowness of Apparition, and being hid, which, from the augmenting of the apparent Body, it argues the descent of it to us.

8. These unequal Motions, are reduced to these mean and equal.

| | | |
|----------------|-------------|------------------------|
| | Sig. | |
| ♀ beyond the | The Annual | $7 : 14 : 47 : 36''$. |
| whole compleat | The Diurnal | $0 : 01 : 36 : 08$. |
| Circuit ——— | The Hourly | ————— $04 : 00$. |

| | | |
|---------------|-------------|----------------------|
| ♀ beyond 4 | The Annual | $1 : 23 : 43 : 15$. |
| compleat Cir- | The Diurnal | $0 : 04 : 05 : 32$. |
| cuits. ——— | The Hourly | ————— $10 : 34$. |

9. The Circles of them at the end of the Year of the Christian Epocha 1700, are,

| | | | | | |
|---|------|---|----|---|---------|
| | Sig. | | | | |
| ♀ | 10 | : | 02 | : | 24 : 24 |
| ♂ | 08 | : | 14 | : | 44 : 39 |

10. The Circles are moved in an orderly following, and proceed yearly.

| | | |
|---|-------|---------------|
| ♀ | ————— | $1' : 18''$. |
| ♂ | ————— | $1 : 45$. |

11. Whence the Revolution of them requires,

| | | |
|---|--------------|------------------|
| ♀ | Julian Years | 16605, Days 304. |
| ♂ | Years ——— | 12364, Days 120. |

12. The Nodes, in which the Eccentric Orbs of these Planets intersect the Ecliptic, and incline from it into the North and South, and makes the Motion of Latitude; are at this time:

Sig.

♂ 2 : 14 : 18 : 55.

♀ 1 : 13 : 47 : 15.

From which Limits they are distant Northern and Southern, a Quadrant of a Circle.

13. These Nodes also move from the Equinoctial in a following order Yearly.

♂ ———— 0' : 47"

♀ ———— 1 : 25.

So that ♂ requires of Julian Years 27574, Days 181.

But ♀ ———— 15203; ———— 151.

14. And the Inclination of the Planetary Orbs from the Ecliptic, is Constant and Fixt.

Of ♂ 3° : 0' : 27".

Of ♀ 6 : 0 : 54.

Yet the Latitude of Venus, by reason of the Current Optick Cause, may run out Northward, almost to 9°; of Mercury; Southern to 5°.

15. Lastly, here we must note, that which the excellent Mathematician Galilæus, a Florentine, hath detected by the Telescope, to wit, that Venus is illuminated by the Sun even as the Moon is, and also that she goes under it at all times.

For when she is direct and nigh to the Sun, she hath a round Figure, when Retrograde she is horned: Marius hath discovered the like in Mercury, by the help of the same Telescope, and found out that he is a Planet weak in Light, descending to the Earth, which is from the Sign of changing the kind of Illumination, and to diminish the Light in a corner; so that he moves less, being near to the Eyes, than

than when further off, which without this diminishing in a nook or corner, would be absurd, because other things appear greater, being near, than if they receded further. Concerning which Debate, you may see more in *Kepler's Epitomy of Astronomy.* Pag. 536.

CHAP. VI.

Of the Proper Motion of the Moon.

1. **T**hat the Moon is nearest to the Earth, and goes about it the nighest, is confest by all.

And 'tis proved from this.

1. That she performs her Circuit or Course in a shorter space of time than all the other Planets.

2. That by her coming between, she hides both the Sun, and all the other Stars near the Ecliptick, to which she is carried.

3. That she hath the most evident parallax among all the Planets, as we will shew hereafter.

2. And she is moved not only daily from East to West, by the Motion common to the rest, but moreover likewise she walks about the Earth by a proper Motion, but yet in an oblique going under the Zodiack, upon the Axis or Poles of the same, in an orderly following, or according to the Order of the Signs, from West to East.

This Motion in the Moon is very evident as hath been said at *Book 1. Chap. 4.*

3. In this Motion also she describes an Eccentric Circle, in which,

1. With respect to the Earth, she goes unequally, slower about the *Apogæon*, swifter about the *Perigæon*.

2. Also

2. Also she is unequally distant from the Earth, farther in *Apogæon*; nearer in *Perigæon*.

4. Also in those *Eccentric* places she is discovered sometimes slower, sometimes swifter, as also in other parts she is observed at one time higher, another time lower, which is *Conspicuous* as well in the *Apparent Magnitude* of the *Lunar Body*, as in the *Duration* and *Quantity* of *Eclipses*.

5. And the *Motion* of the *Moon* is deduced from divers *Principles* or *Beginnings*.

1. From $\odot$ or γ , and is called the *Motion* of *Longitude* of the *Moon*.

Understood in this Ratio.

1. The *Periodical Motion* of the *Moon*, ——— } Days. Hours.
 27 : 7 : 43 : 5.

2. The *Annual* after many *compl. Revolutions*, } Sig.
 4 : 9 : 23 : 3.

3. The *Diurnal* ———— 13 : 10 : 5.

4. The *Hourly*, ———— 32 : 56.

2. From the *Sun*, and is called the *Longitude* of the *Moon* from the *Sun*, and is thus.

Days. Hours.

The *Periodical Time* 29 : 12 : 44 : 3.

Sig.

The *Diurnal Motion* 0 : 12 : 11 : 27.

The *Hourly* ———— 30 : 28.

3. From the *Apogæon*, and is called the *Anomaly* of the *Moon's Orb*, of which,

Days. Hours.

1. The *Periodical Time* is 27 : 13 : 18 : 35.

Sig.

2. The *Diurnal Motion* is 0 : 13 : 3 : 54.

3. The *Hourly Motion* is ———— 32 : 40.

6. The *Apogæon* of the *Moon* also is *changeable*, proceeding in a following manner, by an equal *Motion*; of which,

The

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Ye. Da. Ho. Mi.

The Periodical time is 8 : 309 : 7 : 36.

The Diurnal Motion is ———— 06 : 14."

The Hourly is ———— 15.

7. And the Moon doth not remain always under the Ecliptick, which is proper to the Sun, but as the Ecliptick way of the Sun turns aside from the Equator, into the North and South, intersecting it in two places; so the Orbit of the Moon cuts the Ecliptick in two places, and turns aside from it, partly into the South, partly into the North.

The Points of this intersection are called Nodes, one indeed ascending, The Head of the Dragon, thro' which the Moon ascends from the South into the North, the Sign of which is ♉: But the other descending, The Tail of the Dragon, by which the Moon descends from the North into the South, whose Sign is ♊.

The reason of this terrible Appellation is, that the two Circles cutting each other more narrow about the Point of Section, they are referr'd by the First Astronomers to the Head and Tail of the Dragon; the Belly in the Middle being Broader.

8. Neither have these Nodes a Fixed Seat under the Zodiack, but creep along by little and little, contrary to the order of the Signs going before.

Years. Days. Hours.

In 18 : 224 : 03 : 50.

Therefore is

Sig.

The Annual Motion 0 : 19 : 19 : 43."

The Diurnal Motion ———— 03 : 11."

The Hourly ———— 8.

9. The digression of the Moon from these Nodes, is called the Motion or Anomaly of the Moon's Latitude.

And

And is no other than an Arch, composed of the Motion of the Nodes, and the mean Motion of Longitude of the Moon.

Da. Ho.

The Period or Course of this is 27 : 5 : 5 : 36.

Sig.

And the Annual Motion 4 : 28 : 42 : 46.

The Monthly Motion— 1 : 0 : 40 : 15. The 12 Sig. being a whole Circle, is omitted.

The half Monthly—— 6 : 15 : 20 : 08.

The Diurnal Motion —— 13° : 13' : 46".

The Hourly —— 00 : 33 : 05.

10. *And hence the greatest Latitude of the Moon arises to 5° ; to wit, so much as is the Angle, in which the Plane of the Lunar Orbit inclines to the Plane of the Ecliptick.*

Nevertheless from other causes, in certain places, the Ratio advances 17' also 18', concerning which Tycho and Kepler have much disputed.

THE SECOND PART OF BOOK II.

Of the Affections or Passions of the Planets.

CHAP. I.

Concerning the Kinds and Forms of Passions.

1. **T**HE Passions of the Planets, are some notable Affections or Accidents of Celestial Appearances, which may be taken notice of in any observation

tion of the Planets, while they run along their Spaces of Longitude and Latitude.

They are commonly called *Passions*, an Appellation indeed not agreeable, but yet received in use, and therefore retained.

2. *The kinds of them (for the sake of Teaching) may be appointed Four, to wit, even as they appear.*

1. From the Eccentricity of the Planetary Orbits.
2. From the Comparing of the Planets Motions to one another.

3. From comparing of the same with the Sun.

4. From the Relation of them to the Earth.

3. *Each of these Kinds, have many Particulars, or Representations under themselves, of which we shall speak briefly.*

CHAP. II.

Of the Representations, or Particulars of the First Kind of Passions.

1. **T**HE First Kind of *Passion*, hath four Particulars or Species under it self, of which the First is that in which Planets are said to be,

1. Direct.
2. Retrograde.
3. Stationary.

1. A Planet is said to be Direct, when its Body goes according to the order of the Signs.

2. Retrograde, when it goes or walks contrary to the order of the Signs.

3. Stationary, when 'tis seen to stand still for some time in the same place of the Zodiack.

2. The

2. The Causes of these Passions some impute to the Epicycles of Eccentrics, which when they are carried in the same Climate or Coast with the Eccentrics, the Planets are *Direct*; when they go more swift to the contrary Coast, they are *Retrograde*: Lastly, when they go with both, as well Eccentric as the Epicycle, in a like Pace, one while according to the order of the Signs, otherwhile contrary to them, they become *Stationary*.

161 As in Fig. 161. If a Planet be carried in the Superior Arch of the Epicycle HCF , it will be moved in a following order, and is *Direct*, describing an Arch of the Zodiack IDG ; but if a Planet appear in the lower part through FEH to A , the Center of the World, it again recedes by the measured Degrees GDI : But about F and H , in the end and beginning of Progression and Regression, when the Retrogradation of a Planet is equal to the Eccentric Progression, conveying the Epicycle, for some time it cannot be discerned that the Planet changes its place, but 'tis supposed that it wholly rests, because two opposite Motions happening mutually to it, they take away each other mutually, which Motion of the Planet they Intercept, lest it should change the place under the Zodiack.

But when the Motion is represented by the Epicycles of the same, not being in the Heavens indeed, but supposed by Human Ingenuity, but not any way, as may be Demonstrated from their Genuine Causes: Therefore some assume such Hypotheses, in which they excuse these Passions by the meer fallacy of the Sight, such as is concerning the Sun resting in the Center of the World, and the Earth going about it by an Annual Motion.

3. All the Planets are subject to these Passions, except the two great Lights, the Sun and Moon.

The

The cause of this unlikeness, the Ancients deduced from the Proportion of magnitude and swiftness of the Motion of Eccentrics and Epicycles: But the *Copernican Hypothesis* follows this exemption of the Lights, or Luminaries, further.

4. *The Second Species of the First Kind, is that in which the Planets,*

1. Are swifter, or augment their Course.
2. Are slower, or diminish their Course.
3. They are said to be indifferent.

5. *They are said to be more Swift, or to increase in Course, when their true Motion is greater or swifter than their mean.*

2. *Slower and Diminishing in Course, when their true Motion is slower or lesser than their mean Motion.*

3. *Indifferent, when the true Motion is equal to the Mean.*

The Representative Cause is, when they are Eccentric as well as Epicyclick: But the Physical and Genuine Cause some seek in *Sol*, the Fountain of all Motion, whose moving Vertue being joined to the Eccentric Planetary Orbs, is Stronger when near, and Weaker when further off: Whence the Cause is compounded from Physicks and Opticks.

6. *The Third Kind of the first sort of Passions is, in which the Planets are said,*

1. To be increased in Number.
2. To be Diminished in Number.

7. 1. *They are said to be Increased in Number, when the same Line of true Motion follows the Line of mean Motion.*

That

That is when the *Prostapharesis*, or difference of true and mean Motion, is either Added or Subtracted, to or from the mean Motion : Which difference of true and mean Motion is greater in some, and less in others ; according to the Ancient Hypotheses the *Prostapharesis* is made.

Of the Sun { In its greatest Eccentricity } $2^{\circ} : 23' : 24''$
 { In its least Eccentricity — } $1 : 50 : 41$.

Of the Moon } In Conjunctions ——— } 4 : 56 : 20.
 } In Quadratures ——— } 7 : 40 : 00.

| | | | |
|------------|----|---|-----|
| Of Saturn | 12 | : | 46. |
| Of Jupiter | 16 | : | 08. |
| Of Mars | 52 | : | 45. |
| Of Venus | 48 | : | 09. |
| Of Mercury | 26 | : | 42. |

Whence it appears, how much he mistakes, which takes the Mean Motions for the True.

8. *The Fourth Kind of the first sort of Passions, is the Ascension and Descension of the Planets.*

9. 1. They are said to Ascend when they pass from Perigæon to Apogæon, and proceed from the Center of the World higher into the Sky.

2. But to Descend, when they Abase themselves from Apogæon towards Perigæon, and draw again nearer to the Center of the World.

And this Passion, as also the next precedent, is not found out by Observations, but only by Calculations.

C H A P. III.

Of the Kinds of the Second sort of Passions.

1. **T**H E Second sort of Passions, which springs forth, from the comparing the Motions of the Planets together among themselves, being their Configuration, is usually called their Aspects.

2. And the true placing of the Planets is known, from the Earth, according to their Lines of Motion under the Zodiack.

3. And the Kinds of Aspects are indeed Innumerable; From which notwithstanding Astronomers have gather'd five of the chief to themselves by Observing of them:

1. In Conjunction or Copulation, when the Planets are conversant in the same Degree of the Zodiack.

The Note of it is $\circ$

2. In Sextile, or an Aspect that is Sexangular, when the Planets are distant from each other by a Sextant, or 60 Degrees.

The Note of it is $\ast$.

3. In a Quadrate, or in a Quadrangle, being Quadrangular, or in Quadrature, when they differ from one another by the same Quadrant of the Zodiack, or 90 Degrees.

The Note or Sign of this is $\square$.

4. Triangular, Trine or Triplicity, when the Planets are removed from each other by a Third part of the Zodiack, or 120 Degrees.

The Sign of this is $\triangle$.

Z

5. Op-

5. Opposite, or in Opposition, when the Planets are mutually distant from one another by six Signs, or 180° , and are beheld from the Earth in the Degrees of the Zodiack, opposite to the Diameter.

The Sign or Note of this is 8.

See Fig. 135.

1. To these also some add others, to wit, *Sesquiquadrate*, *Quintile*, *Sesquiquintile*, *Biquintile*, *Octangular*, *Decangular*, which we leave to their places.

2. And we must know that all these Aspects do not agree to all the Planets alike, in respect of all things: Indeed the three Superior, and the Moon, commonly pass under all these differences of Aspects, not only towards themselves by turns, but also to the other Planets: But *Venus* and *Mercury*, because of their very short Interval, *Venus* not receding from the Sun beyond 48° , nor *Mercury* beyond 29° : They cannot accomplish none of the Distances of these Aspects to the Sun: But to it *Venus* and *Mercury* recede from each other by a *Sextile* Interval, and therefore they may have a *Sextile* Aspect, and are conformable to no other besides.

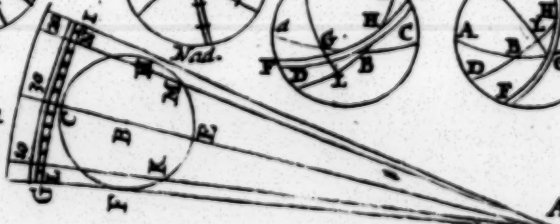
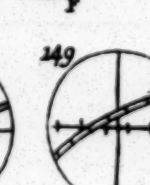
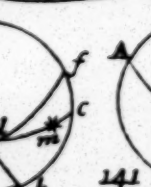
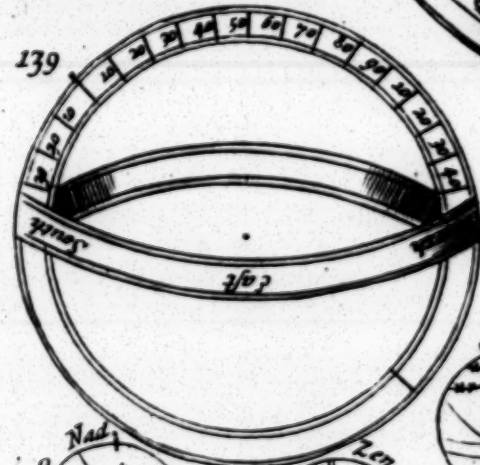
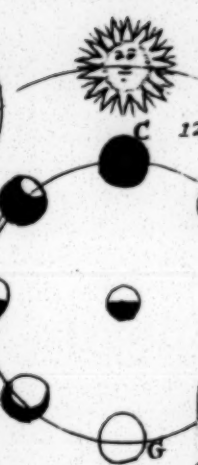
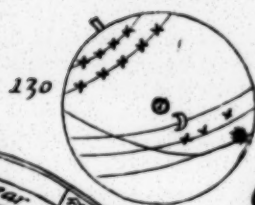
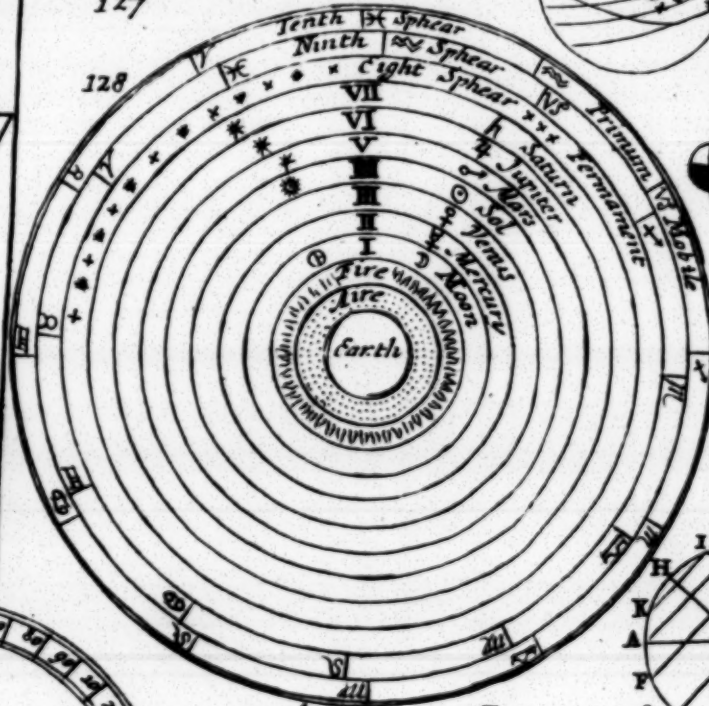
3. And these Aspects are not meer suppositions of *Astronomers*, but from the Nature of them they are brought into esteem divers ways: For,

1. According to these the Moon dispenses its Light to the Earth.

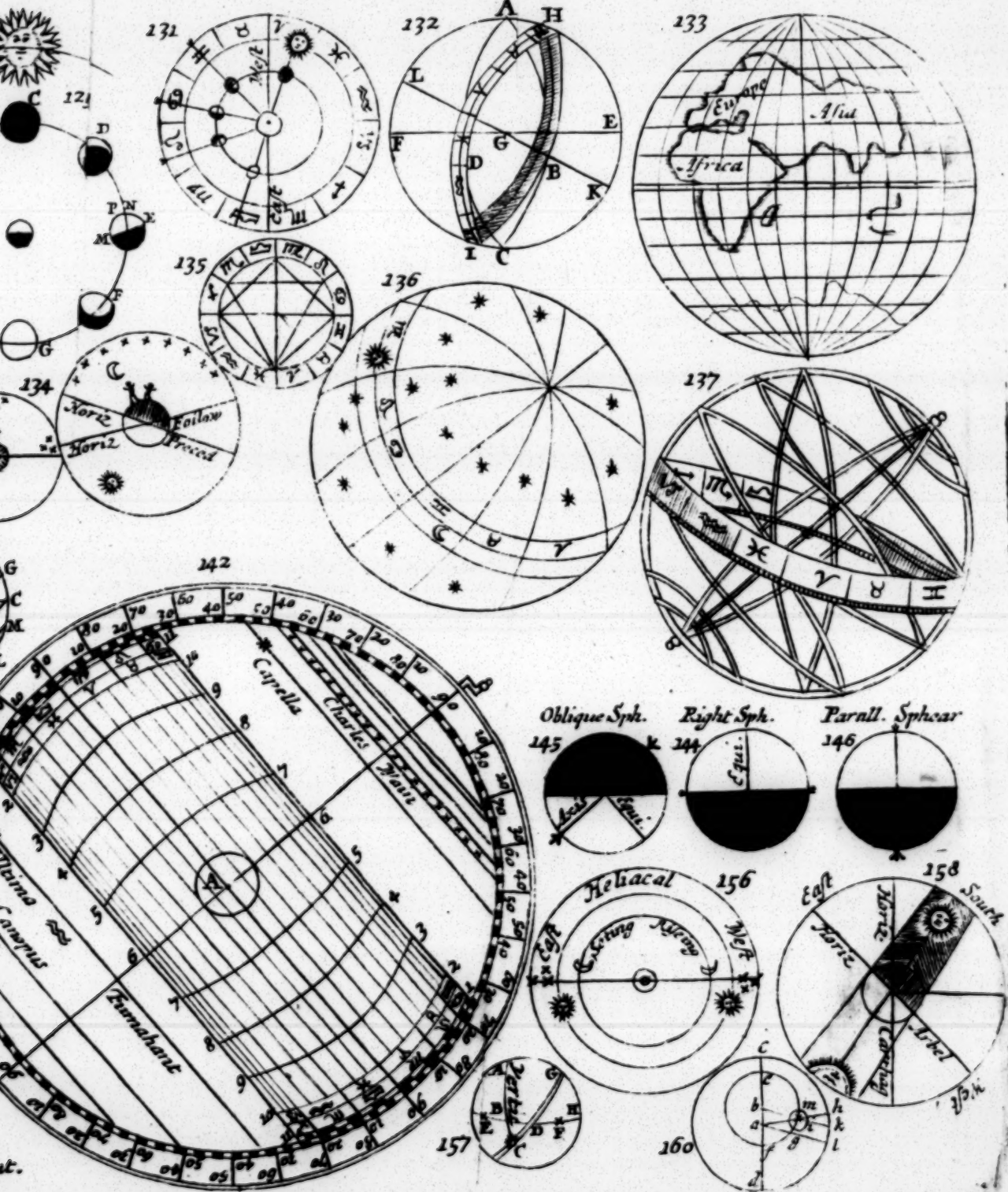
2. The Sun varies the four Seasons of the Year, and lengthens the Day, or shortens it, and makes it equal.

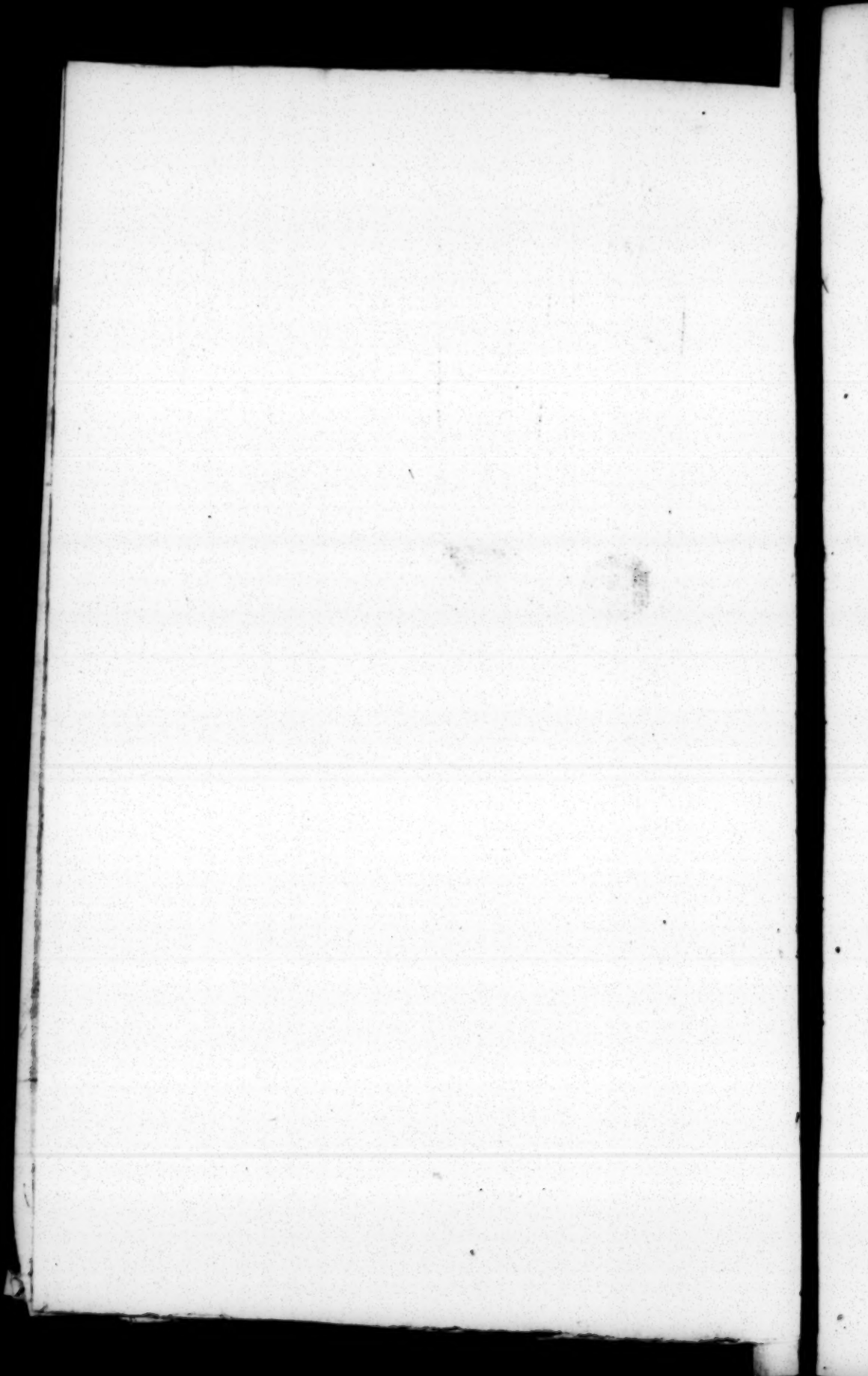
3. The Conjunctions of *Saturn* and *Jupiter*, being made about the beginning of the Zodiack, are said to be the chiefest and greatest, which follow after almost every twenty Years (to wit, always of two, *Saturn* and *Jupiter*, sometimes of three, *Saturn*, *Jupiter*, *Mars*) but distant from the former by a

Third



Place this at Page 339
or 340 to Fold all without.





Third part of a Circle, or 120° , so that every one successively goes through the Triplicities of the Zodiack; *Astrologers* call them the Fiery Triplicity in ————— $\gamma \ \Omega \ \uparrow$.

Then the Earthly in ————— $\delta \ \varpi \ \nu$.

Hence they pass into the Airy $\Pi \ \equiv \ \text{---}$.

Lastly, into the Watry ————— $\zeta \ \text{m} \ \text{---}$.

And they are conversant in one Triplicity almost twenty Years, so that to pass through the whole Zodiack and all its Triplicities, they require very near 800 Years: The more accurate Calculation is thus.

From the Hypotheses of the Ancients, between the two Conjunctions of *Saturn* and *Jupiter*, according to their mean Motions, there falls between of *Julian* Years 19, Days 314, Hours 13: In which time they pass over 8 Signs, and 3° under the Zodiack, which excess of 3° causes, that after 10 of these Conjunctions they proceed into another Triplicity; and therefore they remain in one, *Julian* Years 196, Days 216, Hours 10: And the Revolution of all the Triplicities is made in *Julian* Years 794, Days 133, Hours 16; which being run over, the great Conjunction happens again in the beginning of the Zodiack.

4. Therefore of such great Conjunctions there have only happened 8 since the Erection of the Orb, with which always happens great things to be done: to wit,

1. The Creation of the World, or of our First Father *Adam*:

2. The taking away of *Enoch*.

3. The going forth of the *Israelites* from *Egypt*.

4. The Nativity of Christ.

5. The Restauration of the *Roman* Empire; under *Charles* the VIII, which happened in the Year of the Christian Epocha 1604, also a new Star was then seen.

6. And these Aspects are either True, which are computed according to the Lines of True Motion: Or Mean, in which only the Lines of Mean Motion are considered.

They are observed in the Moon, whose Conjunctions and Oppositions *Ptolomy* calls coupling; but in Calculating the other Aspects, *Astronomers* are content with mean Motions, since the Calculation of the true Motions in this business would indeed be more laborious than useful.

CHAP. IV.

Of the Third Kind of Passions, and of the same Sort.

1. **T**HE Passions of the Third Kind, in which the Planets are compared with the Sun, are of three sorts: First in which the Planets are said,

1. To Increase in Light.
2. To Diminish in Light.
3. Hypaugee, or Combust.

1. They are said to Increase in Light when being remote from the Sun, they shine with much glittering.

2. To Diminish in Light, when being near to the Sun, they cast themselves to our sight by a dull light.

3. Combust, when they lie almost hid under the Sun-beams.

The light of the Stars indeed neither Increases nor Diminishes, but according to our Sense in a vulgar manner, we speak so, because their Light appears at one time more clear, at another time obscure, also at another time they are not plainly seen by us.

3. This

3. This Passion is proper to all the Stars, as well Fixt as Wandring, the Sun only excepted, which since 'tis the Fountain of all Light, can be hid by none more brighter.

4. And the cause of the greater or lesser light of the Stars is not one, but threefold: First indeed general, as well to the Fixt, as to the common Wandring; to wit, the greater or lesser distance of a Star from the Sun.

For the greater Light hides the lesser, according to the Axiom of Opticks: Therefore also the brightness of the Sun, by its Splendor, hides the light of the other Stars; but the Sun being departed further off, makes the same to appear more clear:

Examp. in Fig. 163. A is the Center of the World, MNO the Zodiack, GFE a Segment of the Sphere of the Sun, DCB a Segment of the Sphere of Venus, which is understood to go about in the Epicycle HIKL: The Right Lines therefore AIM and ALO drawn by the points of Contact I and L, shew in M and O the place of the greatest digression of Venus from the place of the Sun N; Venus therefore in H lyes hid, but is first beheld in P; although by reason of the Sun's nearness she is scarce apprehensible to sight; being removed from hence into I, further from the Sun, she shines with a greater light: But afterwards from I towards K, while she approaches again nearer to the Sun, she again shines less: But from Q to R she again lies hid, and, as it were, Consum'd by the Sun-beams, until having got from R, she again acquires her Splendour.

5. Another cause of the Increase and Diminution of Light, besides the other Planets, chiefly concerns the three Superiors, and is indeed the greater or lesser distance of the same from the Earth.

For the enlightned Object being nearer or further ; as the quantity of Body, so also the Clearness of the Light is increased and diminished : Therefore since also these Planets are sometimes elevated higher in the Sky, sometimes they lower themselves nearer to the Earth ; it comes to pass that by how much further they are remote from the Earth, by so much they appear lesser ; and by how much nearer they are to the Earth, by so much greater they appear, not only in quantity of Body, but also in clearness of Light.

6. *The Third cause of the increase and decrease of Light being divers, from the former, the Moon claims chiefly to her self ; and it is because the Moon having only a borrowed light from the Sun's being in Conjunction with her, the light received from the Sun is again retorted upwards to Heaven ; but the dark and shadowy whole half is presented to our sight on the Earth : And afterwards the Moon departing from the Sun, part of it is seen by little and little to turn it self to the Sun, and thereupon a greater portion than the half of it is seen, being enlightned from the same, until at length being opposite to the Sun, the whole Half is Enlightned at the Sun, and turns about the interposed Earth, and shines with a full Orb.*

i. The Moon wants proper Light, and hath only borrowed Light from Luminous Bodies, in the light of which she is conversant, no shadow interceding ; and chiefly indeed from the Sun, the Fountain of all Light in the World ; from thence we know she borrows Light, because in Eclipses sometimes the whole is suddenly lost, that not any Foot-step of it appears, when notwithstanding in the place near it, Fixed Stars may be seen of the fourth or fifth Magnitude : That the cause of this difference cannot be transferred to the thickness of Air, nor to the Shadow

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dow of the Earth, which enters in Eclipses, in which Eclipses, if she should have any proper Light, she ought to shine more splendid than without the Shadow, as in an unseasonable or dark Night, all light of the Day being absent, the light of the Moon causes such a Light, that it almost changes the Night into Day.

2. Moreover, that the same Moon shews to us always one and the same Face, we are assured of it, from the blemishes of it, which are always discerned in the same position.

3. And that 'tis not Pellucid or Shining, but is the same shady Body, is evident from this, that neither our Sight, nor the Rays of the Stars, nor the most splendid Light of the Sun, goes through it; for behind it, all the Stars to whom she comes unto, lie hid, until she be past them; also in Solar Eclipses the same so covers the Sun, that it turns the Day almost into Night.

4. Therefore since also she hath a Globous Body, and by her Monthly Course turns her self Circularly about the Sun, it follows thereupon, that the illuminated part is cast against the Sun; and altho' that half of it, which offers to the Sun, is always enlightened by the Sun: Yet our half, to wit, that which is seen by us, is sometimes turned from the Sun, that it cannot enlighten it; sometimes it is objected in part only, at other times greater, and sometimes lesser: Whence we see it illuminated only in part, and indeed by reason of the Rotundity of the Lunar Body, we see it either Horned or Bisected, or greatly Swol'n up: Lastly, sometimes she is opposite to the Sun so, that we may discern the whole, shining: In the like *Ratio*, but in an opposite manner, the Light of it again decreases: See *Fig. 164.*

In which *A* is the Center of the Sun, *B* the Center of the Earth, *C D F G* is the Orb of the Moon,

in which the Body of it goes about in ; of the two halves of which, one is always cast against the Sun, and enlightned, either the White, or that which is cut with Lines, the other is obscure, being turned from the Sun, and is depicted Black : The White part is the Illuminated half seen by us, that cut by the Lines is indeed illuminated, but shews the part turned from us : The Lines drawn from *H* and *I*, the Extreame of the Sun, to *K* and *L*, the Extreame of the Body of the Moon, determine that half of the Moon always enlightned by the Sun : But the Lines drawn from *B*, the Center of the Earth, to *M* and *N*, limit that half of the Moon *M P N* always turned towards us.

Therefore the Moon being joined to the Sun in *C*, turns *L O K*, the whole illuminated half, from us ; whence, nothing of it is conspicuous to us, (since the whole looks upwards) but the obscure or shady part looks downwards.

Without the Conjunction, such a portion of the Moon is seen to shine to us as both the halves, and hath that part common, which beholds the Sun and us ; as in *Fig. 165*, the Moon being in *D*, distant from *C* by less than a Quadrant, the common part is less than a Quadrant ; and then the portion seen, appears in a horned Curvature, and hollowed in the middle, by reason of the Curvity of the Lunar Surface, in the same manner as is seen in the present Figure : But when the Moon is in *E*, distant a Quadrant from *C*, that half of the Moon turned to us *M P N*, in *Fig. 129*, one half part is illuminated, and the other half is as yet dark or shaded ; therefore the Moon is supposed to be cut in the middle : In the same manner, if the Moon be in *F*, distant from *C*, by a space greater than a Quadrant, it also shews to us a greater part illuminated than an half, and now she begins to become swelling or gibbous. Lastly,

Lastly, the Moon being in G, opposite to the Sun, the whole half seen is illuminated by the Sun, and therefore the whole Orb shining, she imparts all the Light to the Earth, which she borrows from the Sun.

7. *And there are five divers kinds of Illumination, the Order and Appellation of which is thus.*

1. *The First happens in a Conjunction of the Sun and Moon, in which the Moon is placed between the Earth and the Sun, receiving all the Light from the Sun, she retorts or casts it upwards to the Heavens, but the Opacous or Shady Half she casts downwards to the Earth, and our Sight : Then because in such Conjunction the Moon is totally deprived of Light, she escapes our Sight, and therefore 'tis said to be more privative.*

But the time of this Conjunction is called *Interlunium*, the Moon giving no Light, for from this time she is supposed to grow up again anew, and to be enlightned : but the Moon being in Conjunction, and not shining, is usually called *New Moon*, and from hence also until she is opposite to the Sun, 'tis called the *increasing of the Moon*.

2. *The Second happens when the Moon, two or three Days after the Conjunction, goes forth from the Sun beams, and turns the part seen by us, by little and little to the Sun, and begins a new to be enlightned and appear.*

This continues to the Seventh Day, at which time the Moon proceeds from the Sun by a Sextile Aspect, even to a Quadrant : But the Moon at its first appearance, is called the *First Day*, the other Days she is said to be *Horned*, *Crooked*, by reason of its apparent Form.

3. The

3. The Third, is when the Moon at the Seventh Day, is distant from the Sun by a Quadrant of a Circle, its hollowness being filled up, shines by an half Orb.

This time is called from the Figure of the Moon, the Square or the First Quadrature, from the Position of the Moon to the Sun, but the Moon being Bifected in the middle, is usually called half Full Moon.

4. The Fourth, is, from the Seventh Day, even to the Fifteenth, and therefore after the first Quadrature even to the opposition, she is Swelling and Gibbous in the middle, and is beheld to shine by more than an half Orb.

This Moon is named, Swelling, Gibbous.

5. Lastly, the Fifth, is when the Moon at the Fifteenth Day is opposite to the Sun diametrically, being distant from it by the whole Heaven, and shines with its whole Orb.

Then the Moon is said to be Opposite, Full, shining all Night ; and at this time 'tis called Full Moon, or in Opposition.

For from this time, even to the other New Moon, the Moon begins to wax old, or decrease, and repeats all these in a Retrograde Order.

For from the Fifteenth Day to the Twenty second, she appears again Gibbous ; again the Twenty second Day she becomes halved : From hence to the Twenty seventh Day she appears almost anew again, and horned : And the last Day of appearing is called the most New Moon, afterwards on the Twenty ninth or Thirtieth Day she is joined again to the Sun, and is as it were renewed or born again.

(Therefore by reason of this, 1. In the Conjunction of the Sun and Moon, the Night is dark, and without Moon shine.

2. In

2. In the Opposition, the Moon shines all Night.

3. In the first Square numbred from the Conjunction, the former part of the Night is illuminated, and the latter part of the Night is dark, or without Moon shine.

4. In the latter Square from the Opposition, the former part of the Night she shines not, and the latter part she shines.)

8. The Second Kind of the proposed Sorts of Passions, is that in which the Planets are said to be,

1. Oriental.

2. Occidental.

3. Pernox, or remaining all Night.

9. 1. Oriental, when a Planet arises in the Morning before the Sun.

2. Occidental, when she Sets in the Evening after the going down of the Sun.

3. Pernox, when the Planet rises in the beginning of the Night, and Sets at the end of it, and so is above the Horizon all Night long.

So the Moon increasing from the Conjunction to the Opposition, is Occidental or Western ; but decreasing from the Opposition to the Conjunction, she becomes Oriental, or Eastern : *Venus* also changes Orientally and Occidentally ; for being Oriental she is called *Lucifer*, or the Morning Star, but being Occidental she is called *Vesper*, or the Evening Star.

10. And the Third Kind of the Third Sort of Passions is that in which the Planets,

1. Are compared between themselves, some Rising Heliacally swifter or sooner, others more slowly or later.

2. Each

2. Each one also appearing sometimes sooner from the Sun beams, other times tarrying longer.

II. *The causes of this thing are three.*

1. *Is the slower or swifter removing of a Planet from the Sun, or of the Sun from a Planet.*

For this Cause,

1. *Saturn* displays it self sooner from the Sun beams than *Jupiter*, and *this* sooner than *Mars*; for as much as *Saturn* pursues the *Sun* in the Diurnal Motion most slowly, *Jupiter* more swiftly, *Mars* most swiftly: one and the same of these appears or emerges sooner or swifter in *Apogæon* than in *Perigæon*, because the Motion of them is there slower, here swifter.

2. *Venus* and *Mercury* being Retrograde, depart swifter or sooner from the *Sun*, than when Direct.

3. And the *Moon* being swift in motion, shews her self sooner than if slow.

2. The Second Cause, *is the diverse Latitude of the Planets.*

For a Planet after a Conjunction Northward, appears sooner or swifter to us in the Northern part of the World, than if it were Southern: Whence also it comes to pass that some Stars by reason of their great Latitude Northward, are beheld as well in the Evening after Sun Setting, as in the Morning before the Rising of the same.

3. The Third Cause *is the unequal Ascension of the Signs of the Zodiack.*

For a Star which in Signs rightly ascending, goes before the Sun, although it be distant from it by a shorter Arch of the Zodiack, yet 'tis esteemed to be further removed from it, than if in Signs ascending obliquely, it departed from the Sun by a greater Arch of the Zodiack.

4. A Fourth Cause, is the Obliquity of the Horizon.

This alters the former cause very much, for the divers position or placing of the Horizon, changes the Angles which cuts the Ecliptic; which by how much they are lesser, they require therefore a greater distance of the Sun from the Earth for the concluding Limit of Apparition.

(According to these ennumerated Causes, first of all the Moon being deficient or renewing, hastens or suspends its Risings, or Breakings forth; for sometimes in the same Day she appears Old and New, and then 'tis called the Last and the First, to wit, if all these causes concurring they make her swifter in Course, and Northern, in Signs of greater descension; but by how much the fewer of these Causes meet together, by so much the slower, to wit, sometimes after the Second Conjunction, sometimes the Third, now and then the Fourth, she appears conspicuous by her self in the last day.)

CHAP. V.

Of the Kinds of the Fourth sort of Passions.

1. **T**HE Fourth Kind or Sort of Passions, from the Relation of the Rising of the Planets to the Earth, hath Three Kinds or Sorts.

1. The Parallaxes of the Planets.
2. The Refractions of the same, of which we shall speak in this Chapter.
3. The Eclipses of the Sun and Moon, of which we shall treat in the following Chapter.

2. The Parallax, is an Arch of the Vertical Circle, intercepted between the true place of the Star shewn from

from the Center of the Earth, and the apparent place, observed from its Surface.

For if from the Center of the Earth you draw a Right Line through the Body of any Planet even to the Zodiack : And another also from the Surface of the Earth, or from the Eye of the Spectator to the same ; 'tis manifest, that those Lines will meet in the Body of the Planet, but they will presently again part or disjoin, and that will shew to be in one place of the Zodiack, and this in another : Therefore this place which is shewn by the Right Line, drawn from the Eye of the Spectator on the Surface of the Earth, we call the Apparent place : That place which is shewn by the Right Line drawn from the Center of the Earth, we call the true place of the Planet ; and the Arch intercepted between these two places we call the Parallax, which some call the Variation, Changing, Wandring of the Sight, Unlikeness of Aspect.

(*Note, Hitherto the true and apparent place or Motion, hath been one and the same, but in this business of Parallaxes 'tis otherwise ; one is the True place, the other the Apparent ; which we thought fit to Note, lest you should be deceived by the uncertainty of the Terms.*)

2. *And this Passion agrees not to all Stars, but to those only, to whose distance from the Earth, the Semidiameter of the Earth hath Sensible Magnitude.*

Therefore from this kind of Passion are excluded the Fixed Stars, with the three Superior Planets, to whose distance the Semidiameter of the Earth hath scarcely the Ratio of a Point : The Sun either hath none, or not an whole Parallax, which is discover'd by Observations : And 'tis reckoned either from some other distance first known, or from the Ratio of the Lunar Parallaxes, or from Solar Eclipses ; and is then only used, if the most accurate Ratio of Calculation require it.

Venus

Venus and Mercury have Parallaxes without doubt, but concerning those, *Astronomers* are not Sollicitous, by reason of the Smallness : Therefore the Moon only is left, which since 'tis near to the Earth it makes a Parallax discoverable by Observations.

3. And the Parallax is considered, as well in the Zodiac, as in the Vertical Circle.

4. This is an Arch of the Vertical Circle intercepted between the True and Apparent place, and shews the difference of both.

In Fig. 166, *A* is the Center of the World, or of the Earth, *AB* the Semidiameter of the Earth, *C* the Body of the Moon; Therefore *ACD* shew that *D* is the true place of the Moon in the Zodiac, and the Right Line *BCE* shews that *E* is the seen or apparent place, in such manner as it appears from the Surface of the Earth *B*: And there is a quadrant of the Vertical Circle *FG*, between the Zenith *F*, and the Horizon *AG*, passing through the true place of the Moon *D*, and the Apparent *E*: Therefore the Arch *DE* is the Parallax considered in the Vertical, whereby the Eye in *B*, by its sight, errs from the true place *D*.

5. The Parallax considered in the Zodiac, is either of the Longitude or Latitude of a Star.

6. The Parallax of Longitude is an Arch of the Ecliptick, intercepted between two Circles passing thro' the Poles of it, of which, one is drawn thro' the true, the other thro' the Apparent place of the Star.

In the precedent Scheme, *HI* is the Ecliptic; the Arch *DH* from the true place cuts the Ecliptic in *K* at Right Angles, and consequently through the Poles : So *EL* the Arch drawn from the Visible place, cuts the same Ecliptick in *L* at Right Angles, therefore *KL* is the Arch of Parallax according to Longitude.

7. The

7. The Parallax of Latitude is an Arch of a great Circle at Right Angles to the Ecliptick, intercepted between two Parallels of the Ecliptick, one of which passes thro' the True place, the other thro' the Apparent place.

In the same Scheme MN is a Parallel of the Ecliptick drawn thro' the true place D , cutting the Arch EL in O : But PQ is a parallel passing thro' E the apparent place, and is divided by KD , continued in R : Therefore the Arch DR or OE is the Parallax of Latitude, constituting a Rectangled Parallelogram $DORE$, of which, any of the Sides are Arches of Circles, yet by reason of the small difference, they may be accounted for Right Lines: But the Diagonal of this Parallelogram DE is equal in power to the Sides DO and OE , or DR and RE .

8. The Parallax considered in the Vertical Circle, hath these Properties.

1. The Moon being in the Zenith, as happens in the Torrid Zone, the Visible or Apparent, and the True place, fall in with one another.

As in the Scheme, if it should be in F , then AF and BF make one Line,

2. The Moon being remote from the Zenith, the True place and the Apparent differ between themselves.

3. And both is in the same Vertical, but the Apparent always lower than the True place.

4. The greatest Parallax is made in the Horizon.

5. The Moon by how much remoter she is from the Earth, it produces the lesser Parallax; and the greater, by how much lower she is.

6. Refractions also increase Parallaxes, and make them greater than Just.

9. The Parallax considered in the Zodiack is thus.

1. The

1. The Moon being in 90° of the Ecliptick above the Horizon, hath no Parallax of Longitude.

2. In which the Ecliptick is Vertical, and passes through the Zenith, having no Parallax of Longitude, which can only happen in the Torrid Zone.

3. Beyond 30° of Polar Altitude to those that inhabit towards the North, the Parallaxes always happen in the South, and make the Moon more Southward than indeed she is; whence also the Northern Latitudes of it are always less than true, but the Southern greater.

4. From the East to 90° the Moon appears more Eastern than in truth she is, but from 90° to the West she appears more Western than just; there the Apparent place precedes, here it follows the True place, according to the order of the Signs.

(And 90° of the Ecliptick is that which is equally distant from the East and West: The Moon being constituted in it, is known from the following Rules.

1. If a Perpendicular being hung up, touch, as it were, both horns of the Moon, she possesses 90° .

2. If the upper Horn turn to the East, she is not as yet come to 90° : If to the West, she hath already exceeded it.)

Of Refractions.

10. *Refraction, is the Elevation or Suppression of any Star by reason of thick, and unequal transparency, to wit, the Vapors Elevating themselves from the Earthly Globe.*

By reason of these Refractions, the Stars do not always appear in that place, which indeed they are in, but appear higher from the Horizon than Astronomical Calculations admit: Likewise the Sun, and the other Stars are seen to Arise sooner, and Set later, than in truth they Rise and Set.

The Image of Sol and Luna in Vapors by Refraction, you may see in Fig. 167.

A a

The

167

7. The Parallax of Latitude is an Arch of a great Circle at Right Angles to the Ecliptick, intercepted between two Parallels of the Ecliptick, one of which passes thro' the True place, the other thro' the Apparent place.

In the same Scheme MN is a Parallel of the Ecliptick drawn thro' the true place D , cutting the Arch EL in O : But PQ is a parallel passing thro' E the apparent place, and is divided by KD , continued in R : Therefore the Arch DR or OE is the Parallax of Latitude, constituting a Rectangled Parallelogram $DORE$, of which, any of the Sides are Arches of Circles, yet by reason of the small difference, they may be accounted for Right Lines: But the Diagonal of this Parallelogram DE is equal in power to the Sides DO and OE , or DR and RE .

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1. The Moon being in 90° of the Ecliptick above the Horizon, hath no Parallax of Longitude.

2. In which the Ecliptick is Vertical, and passes through the Zenith, having no Parallax of Longitude, which can only happen in the Torrid Zone.

3. Beyond 30° of Polar Altitude to those that inhabit towards the North, the Parallaxes always happen in the South, and make the Moon more Southward than indeed she is; whence also the Northern Latitudes of it are always less than true, but the Southern greater.

4. From the East to 90° the Moon appears more Eastern than in truth she is, but from 90° to the West she appears more Western than just; there the Apparent place precedes, here it follows the True place, according to the order of the Signs.

(And 90° of the Ecliptick is that which is equally distant from the East and West: The Moon being constituted in it, is known from the following Rules.

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The Image of Sol and Luna in Vapors by Refraction, you may see in Fig. 167.

2. *The greatest Refractions are made near the Horizon: But about 45° of altitude they become insensible.*

The cause of this is, not so much that there is more Vapors about the Horizon, as that because the Beams extend to us very much Oblique: But this is not hindred not only by the diminishing of Vapors, but that in penetrating the Vapors, the light of the Sun and Stars are more direct, and arrive to us unbroken.

And by what Law Refractions are made, and how they are found from Heaven, the Ingenious Tycho Brahe hath taught in making two Tables of Refractions, one for the Sun and Moon, the other for the other Planets and Fixed Stars.

CH A P. VI.

Of ECLIPSES.

1. *A*mong Heavenly things, the Principal, and by some esteemed very strange, are the Eclipses of the Sun and Moon, or the defect and fainting of the Luminaries, as some call them.

2. *These are the Interception of the Light of the same, that it cannot be beheld by us Inhabitants on Earth.*

3. *And as the great Luminaries are two, the Sun and Moon, so also the Eclipses are twofold, Solar and Lunar.*

Of Lunar Eclipses.

4. *A Lunar Eclipse, is the Privation of the Light of the Moon received mutually from the Sun, which is made by the Earth interposing in Opposition to the Diameter of the Sun and Moon.*

1. *There-*

1. Therefore Eclipses of the Moon happen only in the Opposition of the Sun and Moon, or in Full Moon, whereupon the Moon should shine with a full Orb.

2. The cause of this is, because the Moon wants proper light, and shines only with a borrowed light from the Sun : In Opposition therefore, the Earth Intervenes between the Sun and the Moon, and not only by reason of its Solidity hinders the passage of the Sun beams to the Moon, but also together with the compassed Air, casts an obscure shadow on the part opposite to the Sun, which being covered, the Moon wants, and is deprived from light.

3. But Lunar Eclipses do not happen in every Opposition or Full Moon, but then only when the Moon is Diametrically opposite to the Sun, so that it either consists in its Nodes, or in the points of Intersection of its Circle with the Ecliptick, or not far from them : See Fig. 168.

But when in the time of Full Moon she wanders somewhat further from these Nodes to the side of the Ecliptic, then the Terrestrial shadow being weakened, she escapes free from the Eclipse or Defect.

5. *And the Lunar Eclipses are twofold : Total and Partial ; or Wholly and in Part ; in those, the whole Moon is hid ; in these, only some part of the Moon is obscured : See Fig. 169 and 170.*

6. *Both are twofold, for they are made Total, either when there is some delay in the Terrestrial Shadow, or without delay.*

In Part also, either half of the Body of the Moon is wanting, or more or less than half : See Fig. 171 and 172.

7. *The Causes of this Variety are Four.*

1. *The unequal Latitude of the Moon, or the greater or lesser distance of it from the Ecliptick.*

For a greater distance lessens Eclipses in the Ratio of Duration and Quantity, but a lesser increases the same.

2. *The unequal thickness of the Terrene Shadow.*

For the Earth which is much less than the Body of the Sun, casts from it self a Conoid shadow, which by how much higher 'tis advanced, 'tis the more diminished, until at length it decays into a point; therefore when the Moon goeth lower, she passes thro' the grosser part of the shadow: from which the Eclipse is increased; when she is higher from the Earth, she passes thro' the thinner part of the Shadow, whence Eclipses are diminished; Fig. 173.

In this Figure the Moon passes thro' *A* and *B*, the grossest part of the Shadow, but higher in *D* and *C* she passes thro' the thinner part.

2. *The unequal distance of the Sun from the Earth, which likewise alters or varies the grossness of the Terrene Shadow.*

For the Sun in *Apogæon* is remoter from the Earth, and it makes a grosser and longer shadow, but being in *Perigæon*, she makes a thinner and shorter shadow, she being nigher to the Earth.

In Fig. 174. The Sun being Higher in *C*, Terminates the shadow of the Earth *K*, but the Sun being lower in *F*, Terminates the same in *L*: Therefore the Moon by her passing through the shadow about *MN*, finds it thicker, by the Suns staying at *Apogæon* than at *Perigæon*.

4. *Lastly, the fourth Cause, is the unequal Digression of the Moon, from the true of the Sun; for the Moon being swift, shortens the time of Duration, but being slow draws it forth longer.*

8. *And Eclipses are estimated by the magnitude of Lunar Digits, of which the Diameter of the Moon is understood to have 12.*

See Fig. 171 & 172, In which you may perceive that the Defect or Eclipse of the Moon may sometimes be so great, that if it had 22 such Digits, yet all may be Eclipsed.

9. *The beginning of Lunar Eclipses is made from the Eastern part of the Moon, on which side also the light of the same is first restored.*

For

For since the Moon proceeds from the West into the East, to wit, by its proper Motion, whither the shadow enters, therefore its Eastern part is first Eclipsed, and again issues forth.

10. *But in Partial Eclipses, if the Latitude of the Moon is North, its Southern part is Eclipsed, but if the Latitude be South, she is Eclipsed in the Northern part.*

For the shadow of the Earth is always opposite to the Sun, and the Sun always remains in the Ecliptick: If therefore the Moon in Eclipse hath North Latitude, of necessity the South part of it will be Eclipsed, and the contrary.

11. *Although the Eclipse of the Moon be beheld in the same Moment of Natural Time, and in the same Quantity, by all those above whose Horizon the Moon is Risen, yet the number of this time is divers, by reason of the Variety of Meridians; by reason of which, those that are Eastern always number more than those that are Western.*

So in the Year of Christ 1497, the 18th Day of January, the beginning of a Lunar Eclipse was observed at Rome, 5 Hours and 24 Minutes after Noon; but at Norinberg, a place more West, at 4 Hours, and 52 Minutes.

Of S O L A R Eclipses.

12. A Solar Eclipse, is the Aversion or Turning away of the Sun beams from the Earth, which is by reason of the Interposition of the shadowy Body of the Moon, between the Sun and the Earth.

1. Note therefore this difference between an Eclipse of the Sun, and one of the Moon: In a Lunar Eclipse both the Moon is deprived of its borrowed light from the Sun, and we the Dwellers on Earth from the light of the Moon: But in an Eclipse of the Sun, the Sun loseth nothing of its light, but the Moon intercepts the light of the Sun from us.

2. The Eclipses of the Sun then only happen in the Conjunction of the Sun and Moon, or in New Moon.

3. And the Cause of these, is the Interposition of the Moon between the Sun and the Earth, which hinders the passage of the Sun beams to us, and the shadow being cast on the opposite part of the Sun, hides the Earth.

4. Nor yet do Solar Eclipses happen in every Conjunction or New Moon, but then only when the Moon is conversant either in the Ecliptic, or in the Nodes, or very near about the Nodes; as you may see in Fig. 129.

5. And seeing the Lunar Conjunction is threefold, 1. Mean, 2. True, (of which we have spoke before in the second kind of Passions) and 3. Visible or Apparent: When the Lines of Apparent Motion, are drawn from the sight of the Dwellers on Earth, through the Center of the Luminary, they will happen in the same Degree of Longitude of the Zodiac; knowing that Solar Eclipses are engaged only to a Visible Conjunction.

For because the Sun is not deprived of its proper light, but the Earth from the light of the Sun, by reason of the Moons being interposed, and the Sun exceeding the Earth by many parts, and the Earth the Moon in Magnitude, and therefore the shadow of the Moon cannot hide the whole Sun to all the Earth; therefore the Eclipses of the Sun are not determined to the Center of the Earth, from which the lines of true Motion are drawn, but to the sight of them, which the shadow of the Moon comprehends.

In Fig. 176, *AB* is the Semidiameter of the Earth, *B* some place given on the Earth, in which the Left Hand part resembles the East, the Right Hand part the West: Therefore when the Moon *C*, the Sun *D*, are joined in 90° of the Ecliptick, there is no difference between the visible and the true Conjunction; but between the East and 90° : The Right Line *AFG* shews the true Conjunction of the Moon *F*, and of the Sun *G*, under the point of the Zodiac *H*: And the apparent Conjunction was when the Moon

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Moon was conversant in *L*, in which place the lines *B L*, *G K*, shew the Visible Conjunction under *K*, on which side the Eclipse happens in *B*: Whence also it appears that the Visible Conjunction goes before on the Eastern part, the true following: The contrary to this is made on the Western part.

13. *And the Solar Eclipses are twofold, in like manner as the Lunar; to wit, Total and Partial, but all the Total without delay, with no stay.*

14. *The causes of this disparity, are almost the same with those which we have declared before in the Moon.*

1. The first is the greater or lesser Apparent Latitude of the Moon, or its distance from the Sun; and by how much 'tis greater, it makes both the Magnitude and the Duration of the Solar Eclipse greater, and by how much the lesser it is, it makes them the lesser.

2. The unequal distance of the Luminaries from the Earth; for the Moon being nigher to us, hides more from the Sun, than if she were more remote.

3. The unequal motion of the Moon, as well the True, as that which appears to us by changing the Parallax, makes the Eclipse of the Sun shorter, she being swifter; but longer, she being slower.

15. *Solar Eclipses are neither beheld in the same Moment of Time, nor in all Places, nor in the same Quantity.*

The cause of this, is the small true magnitude, and short removal of the Earth, and the quantity of the Terrene Globe, being greater, and the greatest quantity of the Solar Globe.

For by reason of the motion of the Moon from West to East, the Sun is first hid to those more Westernly, afterward to those more Easternly; so that while the light of the Sun is again restored to those, these expect the Eclipse to come; which happens only in those places of the Earth, which the shadow of the Moon covers, in which the Eclipses

appear Total ; round about in places not far remote, part only of the Sun seems hid, beyond them nothing of the Eclipse is seen : for the shadow of the Moon also when 'tis greatest, hides or covers not beyond seventy Miles on the Earth, and can hide the Sun no more.

17 In Fig. 177, you may perceive that the shadow of the Moon takes not away the light of the Sun from the whole Earth : For while in the place *B* the whole Sun is Eclipsed, the half part of the Sun lies on *C*, and the whole part on *D* ; wherefore if the place *D* be put to the West, and *B* to the East, the Moon by its motion tending from West, will make the Sun first Eclipsed in *D*, and after the Eclipse being ended there, the Moon being between the Sun and the place *B*, it will be Eclips'd.

16. *The Quantity of Solar Eclipses also is Estimated by Digits, as well as Lunar, so that likewise 12 Digits are attributed to the Sun.*

Notwithstanding 'tis possible that the Sun may be Eclipsed 12 Digits with 11 Parts.

And 'tis known for certain, that the Air in Total Solar Eclipses, hath been so obscure sometimes, that besides that the Stars have appeared (which also appears to have happened in the greater Partial Eclipses) they have thought it wholly Night, and there hath been need of lighted Candles or Torches ; Such an one *Mastlin* speaks of, which hapned 1563, in *August*, in which it was as dark as in the darkest Nights, neither could one see where to set his Foot, and the Stars in the Heavens did appear most clearly : And which was most wonderful, the Birds fell from the Air on the Earth, being frighted at so great darkness upon the Earth, as *Mastlin* reports. Yet there may be other causes, as well in Eclipses as without them, of these prodigious dark Days ; to wit, very thick Clouds, great force of Ashes thrown from Holes of burning Mountains : A thick consistency, either nigh about the Sun, or in the Surface of it, and other things also which may hide the Sun in some

some certain places; which may conclude the Day to be Night.

17. *The beginning of Solar Eclipses is made on the Western part of the Sun, which part also is first restored to light again.*

For the Moon being moved from West to East, this part is first hid, and first again laid open.

18. *In Partial Eclipses of the Sun, if the Latitude of the Moon is North, the Sun is Eclipsed on the North part; if South, on the South part.*

But because the Sun is always conversant under the Ecliptick, therefore according as the Moon consists, beyond, or on this side, so also the light of it is intercepted.

19. *And we are wont to Note in Eclipses.*

1. The Duration, which is the time from the beginning of obscurity, even to the end of the same.

2. The Scruples or Minutes of the Chance or Incidence, which are an Arch of the way of the Moon, which she goes thro' by her Motion of Longitude from the Sun, from the beginning of the Eclipse, in *Partial* ones to the greatest, in *Total* ones to the total obscuration.

3. The time of Incidence, is the space of time in which the Moon passes over the Scruples of Incidence.

4. In the Moon are observed also Scruples, and the time of half an Hour, and those are that Arch which the Moon passes over in her Motion of Longitude from the Sun, from the beginning of Total Obscuration, even to the middle of the Eclipse; but this the space of time it glides or passes by.

20. *To end this Discourse I shall comprise the differences of the Solar and Lunar Eclipses, as it were in a Packet, or small Bundle: Which are thus.*

1. The Sun in the Day time, the Moon only in the Night, is Eclipsed.

2. The Sun is Eclipsed and Restored leisurely, or by little and little in the New Moon, or Conjunction: The Moon is Eclipsed and Restored by Degrees leisurely in the time of Full Moon, or of Opposition.

3. The Sun is seldom, the Moon oftentimes Totally Eclipsed.

4. Yet

4. Yet in the whole, the Eclipses of the Sun are more frequent than of the Moon.

5. The magnitude of the Lunar, every where on Earth, where the Eclipse of the Moon is Conspicuous, is the same, the same time of Duration, the same Moment of Natural time, of the beginning and ending, although under divers Meridians, numbered in divers manners : But in Eclipses of the Sun at one and the same time, they are elsewhere more Partial in the North than the South, in the South than the North ; or lesser in one place than another ; beginning and ending at different Moments of time, lasting in some places shorter, in other places longer, and also in some places none at all.

6. The Moon indeed is deprived of its borrowed light : The Sun retains its Light, but that 'tis hid from us by the Moon.

7. The Moon being obscured, nevertheless sometimes she is beheld of a Reddish colour, but somewhat Brown, and drawing to a Black colour, but in the Eclipse of the Sun, the whole is hid.

8. The Eclipses of the Moon happen in the true Opposition of the Sun and Moon, but yet under no true Latitude, or very little of it : But the Eclipses of the Sun happen not in the true Conjunction of the Sun and Moon, but only in the Visible and Apparent, whose True Latitude also is not here considered, but the Apparent.

9. The Total Eclipses of the Moon are made sometimes in a long time, at other times quickly : The total Eclipses of the Sun are all without delay.

21. *Lastly instead of Corollaries, we think fit to subjoin some Questions, fit to be Noted, from the Lucubrations of the Excellent Maslin, and other Astronomers.*

1. *Whether the Luminaries are Eclipsed every Year ?*

Ans.

Answ. If we consider the certain Motion of the Sun and Moon, and make Experience the Witness; there happens not only in every Year, but every half Year, either one, or two, or also three Eclipses of the Luminaries.

2. *Whether the Sun or the Moon is oftener Eclipsed?*

Answ. It is not only the Opinion of the Vulgar, but also of some Philosophers, That the Moon is oftner Eclipsed than the Sun: But since the Terms belonging to Solar Eclipses by reason of the Parallax of the Moon are the larger Luminary Terms, it necessarily follows, that the Solar Eclipses are more frequent in general than the Lunar: Although the Solar are only seen in certain Tracts of the Earth, but the Eclipses of the Moon are Universal, which appear in like manner to all that see the Moon.

Nevertheless in particular, the Total Eclipses of the Sun are fewer than the Lunar of all sorts: But if we compare the Total of the Moon with the Total of the Sun, the Moon oftener takes away the whole Sun from the Earth, than the Earth takes away the whole Sun from the Moon: For the Body of the Earth being larger than the Body of the Moon, receives more strokes of the Moon, than the Moon from the Shadow of the Earth: Lastly, the Number of the Totals of the Sun thro' the whole Earth is the same in a manner with the Number of all kinds of the Moon.

3. *How many Eclipses may happen in one Year?*

Almost in all single Lunar Years (to wit, of 354 Days) there happens either two or three of the Sun; but of the Moon, either none, or one or two: And so in the whole, there may happen in one Year either 2 or 3 or 4 or 5 Eclipses, also sometimes 6 may happen, but not in a Lunar Year, but in a Solar, to wit, when the Eclipses happen about the beginning of the Year.

4. *Whe-*

4. *Whether or no Eclipses are more frequent in our Time, than in Time past ?*

This is proved in the Negative Opinion by this Argument, Where there is no mutation in Causes, there necessarily the Effects answer to the former Causes: But as yet there is not any mutation understood or shewn by any one in the Causes of Eclipses, either Optick or Astronomick: Therefore neither more, but frequently the same Eclipses happen now as in times past; he that would deny them to be lesser, takes on him the trouble of proving and demonstrating why, where, and when the Change is made? He that determines the contrary is deceived, from this, that in Diaries in times past Eclipses have been noted no otherwie, or seen only, as they are at this time.

5. *Whether or no the Eclipses of the Sun and Moon are Astronomick Signs of the last Days approaching?*

There are some which deny it for these Reasons.

1. Because these Eclipses proceed from Natural Causes.

2. Because the same frequently hapned in old time, as at this Day.

3. Because Christ hath said, there shall be other Signs preceeding the last Days; to wit, the Falling of the Stars, the Commotions of the Heavens, not having Natural Causes; also the Obscuration of the Sun and Moon, of which Christ speaks, proceeds from other than Natural Causes.

Divines truly affirm it from the clear Texts of Holy Scripture, in *Matth. 23. Mar. 13. and Luk. 21.* answering to the assigned Reasons: Likewise that the Rainbow was before the Flood, and had Natural Causes; and nevertheless was placed by God as a Sign that the World should not any more perish by a Flood: In like manner the Eclipses also of the Sun and Moon, altho' they have hapned from the beginning

ning of the World, which had Natural Causes, yet they may be Signs to put us in mind of the Last Day's approaching.

6. *Whether or no the Moon be Obscured by the Shadow of the Earth?*

Thus it hath been believed indeed in many Ages, which Opinion we also follow in the Doctrine of Eclipses: But the Ingenious *Kepler* is doubtful of it; who shews that the extream Beams of the Sun, which ought to Terminate the Terrene Shadow, may be broke by two Misfortunes: First, in a more thick Mean, to wit, from the Air at Perpendicular; afterwards going from thence again into a thinner Mean, to wit, from the Sky from a Perpendicular; and there coming together to sharpen and shorten the Terrene Shadow, so that it cannot reach to the Sphere of the Moon: For the Point of this refracted shadow is not longer than 43 Semidiameters of the Earth, (when nevertheless others think that it may extend it self to 268 Semidiameters) but the Moon altho' in *Perigæon*, yet is not less from us than 54 Semidiameters of the Earth: Therefore 'tis some other thing which deprives the Moon of its borrowed light, to wit, the shadow of the Air: For altho' it be a Pellucid or Clear Body, yet 'tis Opacous or Shady, because it also contains Air, especially about the Limb, as every Pellucid Globe appears more dark about its Extreames; therefore it may spread the Shadow, not indeed in it self, but in a Thinner Body, to wit, in the Sky.

7. *From whence proceeds that faint light which appears in that part of the Moon when 'tis Horned, which is turned from the Sun, being clear while beheld nigh the North West?*

Ans. It is not properly of the Moon, otherwise also it would be perpetual; when notwithstanding about the New Moon 'tis most strong in the weakest Squares,

Squares, and scarcely perceptible by sight : But secondly, 'tis from the Earth, being illustrated by the Sun, which refunds or casts out again some light into the Moon ; as also our Walls being strongly Illustrated by the Sun, dart the light into Vallies, and close dark places, and cast in the Day to them.

8. *Why the Moon in Total Eclipses sometimes is quite lost, and sometimes shines most plainly, with a Red Colour, either by the whole Body, or in part ?*

Again, the proper cause is not in the Light of the Moon, for it would be perpetual, at least in the same part of the Body ; but the Moon in that part of the Body which shines Red, falls in with the secondary Beams of the Sun, that is in our Air scatter'd about the Earth, being twice refracted, once in entring, again in going forth towards the Moon, the darkness of which discusses a little.

9. *Whether or no, and how the Moon, being much lesser than the Sun, can hide the whole great Body of the Sun ?*

Ans. Yea certainly, for by how much lesser the Diameter of the Moon is than the Diameter of the Sun, by so much she is nearer to the Earth than the Sun ; but near things are seen to have a greater Diameter than if the same were remote.

10. *Whence is that bright remaining Circle, which sometimes in Total Eclipses of the Sun is beheld round about on every side ?*

Because this Circle sometimes is beheld when the Diameter of the Moon appears greater than the Diameter of the Sun, so that the Moon ought to hide the whole Sun ; 'tis necessary that the Cause arises not Astronomically but Physically.

Kepler says it is the Pellucid Air spread about the Globe of the Moon, in which the Sun beams are broke, as well in entring from above, as in going forth downwards, and happens refracted to our sight in that

that manner in which the same Sun beams are diminished in our Air spread about the Earth; and therefore that Circle is rather of the Moon, than of the Sun, being Illuminated by the light of the Sun.

11. *Whether or no the Eclipse of the Moon may happen, both Luminaries being above the Horizon, and so without the Line of Opposition?*

That may be by reason of the Refraction of the Beams of the Sun and Moon; whence it comes to pass that the Sun, or any other Star being hid, as yet in truth beneath the Horizon, may appear above the same: Such an Eclipse *Mästlin* saw at *Tubing* in the Year 1590, the seventh of *July* in the Morning, the Scheme of which you may see in *Fig. 167.* belonging to the Doctrine of Refractions.

12. *Whether other Stars may also be Eclipsed in that manner as the Sun and Moon?*

No Star can be Darkned and Eclipsed in Light like the Moon, for *Venus* and *Mercury* are not opposite to the Sun: The Shadow of the Earth extends in no manner to the Superior Planets and Fixed Stars; But like as in the Solar Eclipse, the Moon hides the Sun, in the same manner as well the Fixed Stars as the Planets may be hid, according to the Ratio of Latitude, by all the Inferior ones.

13. *Whether there may be given some Eclipse, or such a returning again, which shall be like to the former in all things?*

To this Question *Kepler* answers in his *Epitome of Astronomy*, in this manner: By reason there are many Principles concurring to some Eclipse, there cannot be any exact order of coming back again; for the latter Series of them claims something new, which when in the first returning is not very conspicuous, yet in the second it is doubled: This returning in Lunar Eclipses, which as they are most plain, is chiefly notable, and is determined in 223 Months, that is, exactly

ally 18 Lunar Years, so that to one Decennoval Cycle, the residue is one single Solar; and it comes to pass frequently as the same, nor much different; after this time they run back and compare the Eclipses, yet from Ten Degrees of the Zodiac to the former place.

14. *Whether the Eclipses of the Luminaries are either the Causes or Præfages of Tragical Dangers? And also whether they have any thing of Advantage?*

The first, *Astrologers* testifie with great assurance, who præfage I know not what direful things from all Eclipses, and fill the minds of the vulgar People with great Terror, which ill fortune they also increase as they think, but indeed ridiculously, by the Appellations of the Head and Tail of the Dragon, about which Eclipses happen: If this be true, since every Year Eclipses happen, we are at no time safe from great Evils: And altho' it cannot be denied but that the World is perpetually troubled with great Calamities; yet for what reason this perpetual misery of the World should be joined with Eclipses, those *Astrologers* only see; but withal, unless there are Injuries in God, which some pretend, as if he had only ordained Eclipses for the destruction of Mankind; which nevertheless are, as it were, Ladders, by which we ascend into Heaven, and contemplate the works of the hand of God, to the Glory of his Name.

But what and how much benefit Eclipses afford to the knowledge of Heavenly and Earthly things, the most excellent Mathematicians *Mästlin* and *Kepler* have taught in a large Catalogue, which we will repeat as briefly as may be.

Therefore the Causes being known, the wonderfulness of these Eclipses may be found out.

1. The Course of the Moon, whose true place, without Eclipses is not observable, by reason of the Parallax of the Moon.

2. The

2. The Sun is both higher and greater than the Moon.
3. Both these Luminaries are Spherical, unequally distant from the Earth, and therefore each describes an Eccentrick Orbit by its Motion.
4. The Earth is less than the Sun, and greater than the Moon, situate in the middle of the Zodiack, making together, with the Water, one Round and Spherical Body, somewhat uneven indeed, because of the Mountains, yet those are incomparable to the whole Body of the Earth.
5. The Sun is the Fountain of all Light in the World ; but the Moon delights not in its own, but in the borrowed Light of the Sun ; and is of it self a dark Body, Opacous, Thick, Uneven, full of Gaps, most like to our Earth ; for which reason she is called by Aristotle, and other Ancient Philosophers, *The Ætherial Earth* ; The Parts of Similitude, are assigned thus by the excellent *Mestlin*, in his *Disputations of the Planets Passions*, Printed at Tubing in the Year 1606.
 1. Both the Body of the Moon and the Earth is Round and Opacous, wanting proper light ; being enlightned by the Sun, in whole opposite shew or Arch they cast a Shadow.
 2. As in the Earth, beheld from on high, there appears Spots, by reason of the Terrestrial places, Sea Coasts, Campain and Mountainous Fields : So also the like Spots are beheld in the Moon.
 3. Since the Moon on one part is illuminated by the Sun, but the part turned from it is dark, and in a certain space of time the light of the Sun passes about the whole Moon : in it there will be, as also in the Earth, the Vicissitude or Change of Day and Night ; but that of the Earth is in 24 Hours, and that of the Moon is in Days-29 $\frac{1}{2}$.
 4. As the Earth intercepts the Light of the Sun from the Moon, so the Moon intercepts the light of the Sun from the Earth.
 5. As the Terrestrial Night is illuminated by the Moon, so the Lunar Night by the Earth.
 6. At what time the Earth hath New Moon, in the same time the Moon hath Full Earth.
 7. In Squares, as on the Earth the Moon is seen

by an Half Visage, so in the Moon the Earth is only seen by an Half Visage.

8. As a thin Air encompasses the Earth, receiving the Image of the Sun by Refractions ; so the same happens in a thin Body about the Moon : But that we may return again to the advantage of Eclipses ;

6. They are for the most part a singular help to the Enquiry of the Differences of Meridians, or the Longitude of Places : And therefore the observation of Eclipses in divers places is Useful, especially in Hydography, and Naval Affairs, and in all Geography.

7. Lastly, from the Eclipses of the Sun and Moon, a Reason is found of Measuring the Distances of the Sun and Moon from the Earth, as also of the Magnitude and Proportion, that these three Bodies of the Sun, Moon, and Earth, have between themselves, which otherwise may seem impossible.

Therefore by these Wings the Mind of Man flies up into the Heavenly Theatre ; and by these Charms (as the Poet saith) draws down the Sun and Moon on the Earth ; with which words the excellent *Mæstlin* concludes this Discourse of Eclipses.

THE
THIRD PART
OF
BOOK II.

Of the Distance and Magnitude of the Stars.

The PREFACE.

1. **T**HE Wandring Stars or Planets, altho' they are carried about by no real Spheres or Orbits, which the Ancients have introduced without necessity, or sufficient reason, but flying about in the pure
and

and free Sky, even as Birds in the Air ; not indeed wandringly, but by an appointed and ordained Motion : Yet because by their Periodical Motions, they are sometimes more remote from the Center of the World, at other times more near ; therefore so much Ætherial space is only attributed to them, as the Variety of Motion, which every Planet requires ; which space it wont also to be called by the Ancient Astronomers, the Sphere or Orbit of a Planet.

2. Therefore oftentimes indeed mention is made of the Proportion and Dimension of the Heavenly Orbs ; but by these we understand no other thing than the Greatest, Least, and Mean Dimensions and Proportions of Distances, which the Planets obtain by running through their assigned Spaces.

3. And even as other Astronomers assume other Hypotheses of the Planets Motions ; so also they ascribe different Distances and Magnitudes to the Stars.

4. Whence we gather indeed, That these Magnitudes and Distances of them are not so certain, nor yet also to be far from Truth ; or else those Hypotheses represent not the Heavenly Motions, which for the most part rely on such Distances ; which Motions notwithstanding exhibited by the Hypotheses, agree exactly enough with the Heavens, not only to the Eye of the Artificer, but are testified likewise in some measure by the Vulgar People.

5. We in this place, notwithstanding we have premised the Opinions of the Ancients, shall follow the Rudolphin Tables, restored by Tycho Brahe, the Phoenix of Astronomers, and published by John Kepler ; Which although he place the Sun in the Center of the World, and numbers the Distances of the Stars from it ; and places the Earth (which others believe to be the Center of the World) in the place of the Sun ; Nevertheless if any one like it not, he may easily change the place of the Sun and of the Earth again ;

which may be done without Detriment in any Case: For as the Interval of the Journey is the same, whether you set out from London to York, or from York to London; so the Distance is the same, whether you number it from the Sun to the Earth, or from the Earth to the Sun.

CHAP. I.

Of the Distance of the Stars, according to the Opinion of the Ancients.

1. **T**HE Ancients who thought the Stars could not move of themselves, without Orbs carrying them about, have feigned many Heavens, so that one rests upon another, even as the Peels or Skins of Onions, the Inner one sustaining the Outer; and they appoint the Semidiameter of the Superior Sphere so great, as to contain a known Quantity of the Inferior Sphere.

2. And they use instead of a Measuring Rod, the Semidiameter of the Terrene Globe, which commonly is appointed of *German Miles* 868, or of *English Miles* (accounting one *German Mile* to contain four *English*) 3472.

3. Therefore since *Ptolomy* hath in times past set down the Distance of the Sphere of the Sun, and of the Moon; *Alphraganus*, *Albategnius*, and others, have accommodated other Spheres to this:

And they number Semidiameters of the Earth, from the Center of the Earth, to the Concave Sphere of

| | | | |
|--|---|------|-------|
| of the Earth, to the Concave Sphere of | ☉ | 33° | : 33' |
| Hence to the Concave of | ☽ | 30 | : 37 |
| Thence to the Concave of | ♂ | 102 | : 50 |
| To the Concave of | ☿ | 953 | : |
| To the Concave of | ♂ | 100 | : |
| To the Concave of | ♂ | 7656 | : |
| To the Concave of | ♂ | 5529 | : |
| To the Concave of the Sphere of the | } | 5705 | : |
| Fixed Stars | | | |



The Sum from the Center of the Earth to the Fixed Stars 20110.

4. From this Ratio there are made of Semidiameters of the Earth.

| | | |
|--|---|-----------------------|
| From the Center
of the Earth to the
Convexity of — | { | ☉ ————— 64° : 10° |
| | | ☽ ————— 167 : |
| | | ♀ ————— 1120 : |
| | | ☉ ————— 1220 : |
| | | ♂ ————— 8876 : |
| | | ♂ ————— 14405 : |
| | | ☿ or the Fixt 20110 : |

This last Distance being Multiplied by 3440, the English Miles contained in the Semidiameter of the Earth, produces the Altitude of the World, from the Center of the Earth, to the Fixed Stars, to be 69178400 English Miles.

This Space of the World, although it be very large, scarce any one of the Astronomers hath lessened it by solid Reasons: But the Rudolphine Tables increase it very much, whose Opinion also we will hear.

CHAP. II.

Of the Dimension of the Great or Solar Orb, according to the Rudolphine Tables.

1. **W**Hich Orb, the *Rudolphine* Tables, according to the Opinion of *Nicholas Copernicus* attributes to the Earth, and by reason of the excellent use which it performs, in the more easie dispatching Astronomic Calculations, they call it the great Orb: By us 'tis assigned the Orb of the Sun, from the diverse Position or Placing of which to the Earth, 'tis also known; which is counted to be Eccentric from the inequality as well of the Motion of the Sun, as of the Distance of it from the Earth. :

B b 3

2. And

2. And indeed, of such parts as the Semidiameter of the Great Orb is assumed — } 100000

Of the same parts, the Eccentricity of this Orb hath — } 1800

3. Whence the greatest Distance of the Sun and of the Earth, is of the same parts — } 101800

The Least Distance is — 98200

The Mean Distance therefore is — 100000

4. And because this division of the Semidiameter of the great Orb into 100000 parts, is Arbitrary, and assumed for this end only, that the proportion of the greatest and least Distance of the Sun, and of the Earth, and the Quantity of the Eccentricity may be expressed: Therefore if we would know the true Distance of the Sun and of the Earth, 'tis requisite that this Arbitrary Measure should be reduced (as we commonly say) to another Noted or Renowned Measure, which is, as aforesaid, the Semidiameter of the Earth, of *English* Miles 3440, or of *German* 860.

5. Of these Semidiameters then, the Author of the *Rudolphine* Tables assumes, for the greatest Distance of the Sun, and of the Earth — 3469.

From which he makes the Least — 3346.

And the Mean, or the Semidiameter of the Great Orb — 3407.

The Foundation from whence this Calculation arises, you may see in *Copernicus* his *Epitome of Astronomy*, Book 4. Pag. 474. and the following.

CHAP. III.

Of the Dimension of the Sphere of the Moon.

1. **I**N the Dimension of the Sphere of the Moon, almost all Astronomers agree: For of such parts as the Semidiameter of the Orb of ☾ is 100000.

Of

Of the same, they assume the Eccentricity—4362.

Whence the *Apogæon* is Distant—104262.

And the *Perigæon*—95638.

2. *Tycho* and *Kepler* make these parts :

Of the greatest Distance, Semidiameters }
of the Earth ————— } 59.

Of the Least ————— 54.

Which make the Mean ————— 56½.

CHAP. IV.

Of the Dimension of the Spheres of,

♄ ♃ ♅ ♄ ♅.

And of the Fixed Stars.

1. **O**F the same Parts as the Semidiameter of the great Orb is 100000. Of such parts the Distances are discovered as follows.

Of ♄.

The Greatest Distance is — 1005140.

The Least ————— 896860.

The Mean ————— 951000.

Of ♃.

The Greatest Distance is — 545074.

The Least ————— 520000.

The Mean ————— 532537.

Of ♅.

The Greatest Distance is — 165465.

The Least ————— 138235.

The Mean ————— 152350.

Of ♄.

The Greatest Distance is — 72914.

The Least ————— 71912.

The Mean ————— 72413.

Of ☿.

The Greatest Distance is ——— 38806.

The Least ——— 36065.

The Mean ——— 37435 $\frac{1}{2}$.

2. For in measuring the Intervals of these Planets from the Center of the World, instead of a Measuring Rod, we use the Semidiameter of the Great Orb, the measure of which we have assigned in Semidiameters of the Earth afore, according to the Opinion of *John Kepler*.

If you draw these into the Assigned Numbers of Distances, and divide the Product by 100000, it will produce the Planetary Distances from the Center of the World, in Semidiameters of the Earth, which if you multiply by 3440, you will have the same in English Miles: But if by 860, you will have them in German Miles: Which Calculation we leave to the curiosity of Astronomers.

3. And from the aforefaid, it appears, That the Distance of *Saturn* is somewhat less than Decuple of the *Earth* from the *Sun*: Of *Jupiter* Quintuple: Of *Mars* Sesquiple: Of *Venus* Subsesquitertia: Of *Mercury* about Subtriple.

4. That which pertains to the Distances of the Fixed Stars; *Kepler* assumes the Semidiameter of the Sphere of the Fixed Stars, to be to the Semidiameter of the Sphere of *Saturn*, as this is to the Semidiameter of the Body of the *Sun*.

And he hath found by Mathematical Instruments, that the Diameter of ☉ is ——— 30'

Whence the Semidiameter is ——— 15

The Right Sine of which is ——— 4363.3093

The Sine of its Complement is 999970.4807

Which divided by ——— 436

Produces ——— 229

Semidiameters of the *Sun*, by which the *Sun* is distant

stant from the Earth: Which Distance, since it is the Semidiameter of the great Orb; and this, as appears from what's said before, is a little greater than a Tenth part of the Semidiameter of the Sphere of γ . It follows, that the Sphere of Saturn hath in its Semidiameter, almost Ten times more Semidiameters of the Sun, that is about Two Thousand.

But if the Semidiameter of the Sphere of the Fixed Stars, contains in it Two Thousand also of the Semidiameters of the Sphere of γ , it will make of the Semidiameter of the Solar Body, about 4000000.

And of Semidiameters of the Earth about 60000000.

For the Semidiameter of the Sun, contains Semidiameters of the Earth 15.

CHAP. V.

Of the Causes of these Distances.

1. **T**hat the World is Sphærical in its outer Surface, is owned by all Philosophers; and they give this Reason also among the rest for the cause of it: That the World is according to the similitude of the Architect, that is, of God the Creator, who is an Infinite Being, wanting Beginning and End: Therefore since the express Image of this Infinity is in a Sphærical Figure, in which, according to Nature also, there is neither beginning nor end: Therefore that the World is Sphærical is not to be doubted.

2. Since it is so, and God hath disposed all things in a sure Proportion, as the Wise Man witnesses, when he says, *That he hath made all things in Number, Weight, and Measure.* It was disputed by the Ingenious Mathematician *John Kepler*, with the Applause

plause of his Master *Mich. Maſſlin*, That the greater Interior Bodies of the World, viz. The Planets, are disposed by their Intervals, according to the Rule of the Five Regular Bodies, the *Cube*, the *Tetraedron*, the *Dodecaedron*, the *Icoſaedron*, and the *Oſaedron*; which, after the Sphere, are the Chief, the most Noble, the most Simple of all Bodies; being Equilateral, and Equiangular, Inſcripſible, and Circumſcripſible of the Sphere, ſo that they touch the Inſcribed Sphere by all their Sides, and the Circumſcribed by all their Angles; of which, among an infinite multitude of Bodies, there is not, but only theſe five, as is Demonſtrated by Mathematicians.

| | | | |
|--|-----------------------------|---|-------------------------------------|
| 3. Therefore, what Proportion is between the Sphere Circumſcribing and Inſcribing. | 1. The <i>Cube</i> . | The ſame very near is between the Orbs of — | 1. <i>Saturn & Jupiter.</i> |
| | 2. The <i>Tetraedron</i> . | | 2. <i>Jupiter & Mars.</i> |
| | 3. The <i>Dodecaedron</i> . | | 3. <i>Mars & the Great Orb.</i> |
| | 4. The <i>Icoſaedron</i> . | | 4. The <i>Gr. Orb & Venus.</i> |
| | 5. The <i>Oſaedron</i> . | | 5. Of <i>Venus & Mercury.</i> |

4. But that there may want ſomewhat in the Leſt, the cauſe of it, the Inventor aſſigns thus, That the Orbs were not only fitted by their Diſtances to the Rule of the Five Regular Bodies, but alſo, to the Harmonick Proportions of Motion, to which even as to the Idea of Heavenly Muſick, they were made meet: Hence it hath been neceſſary, that thoſe Figurative Intervals, which are exhibited by Uniform Figures, without Variation, ſome little is taken away, and left to the Liberty of Harmoniſts, to repreſent the Harmonies of Motion.

5. Theſe Harmonies of Motions of the Slower and Swifter Planets being beheld from the Center of

of the World, exprefs not Sounds, but Mufical Numbers : The ſame Author *John Kepler* hath expoſed many things in his Books of the Harmony of the World ; of which, that young *Aſtronomers* may have ſome taſte, I will diſcover one Example.

| | | | | | | | |
|--|---|-----|---|-----|--|-----|---------------------------------|
| The Motions of the Planets Slowest and Swiftest, beheld even as from the Sun, are theſe, |  | 1 | : | 46. | Between which the Harmonick Proportion is, | 4. | 5. Tertia Major. |
| |  | 2 | : | 15. | | | g. h. |
| |  | 4 | : | 30. | | 5. | 6. Tertia Minor. |
| |  | 5 | : | 30. | | | g. b. |
| |  | 26 | : | 14. | | 2. | 3. Diapente. |
| |  | 38 | : | 1. | | | g. d. |
| |  | 57 | : | 3. | | 15. | 16. Semitonium. |
| |  | 61 | : | 18. | | | e. f. |
| |  | 94 | : | 50. | | 24. | 25. Diesis. |
| |  | 97 | : | 37. | | | b. h. |
| |  | 164 | : | 0. | | 5. | 12. Diapaſon cum Tertia minore. |
| |  | 384 | : | 0. | | | |

CHAP. VI.

Of the Magnitude and Proportion of the Planets, and Fixed Stars mutually.

I. Of Planets.

1. **T**O the finding theſe Magnitudes and Proportions of the Stars, their Diſtances are required, and the Apparent Magnitudes of their Diameters : Therefore even as theſe, by reaſon of di- verſity of Hypotheſes, are aſſigned divers ways by ſeveral Men ; ſo alſo the Opinions concerning their Magnitude and Proportion, are Various.

2. The

2. *The Opinion of Alphragan, and other Ancients.*

I. ♄.

It contains in its Diameter, Diameters of the Earth $4\frac{1}{2}$: And the Proportion of Diameters is as 9 to 2, which being multiplied Cubically, produces the Proportion of the Bodies of ♄ and the Earth, to be as 729 to 8: The greater Number being divided by the lesser, makes $91\frac{1}{8}$, the whole that ♄ is greater than the Terrene Globe, and so many times the Body of *Saturn* contains in it self the Terrestrial Body.

2. ♃.

Contains Diameters of the Earth ———— $4\frac{2}{3}$.
 Hence the Proportion of them, is as — 32 to 7.
 But the Proportion of their Bodies, is, as ———— ———— } 32768 to 343.
 And ♃ is greater than the Terrene Globe $91\frac{11}{16}$.

3. ♂.

Hath in its Diameter, Diameters of the Earth $1\frac{1}{2}$.
 And the Proportion of them is, as ———— 7 to 6.
 But of their Bodies, as ———— ———— 343 to 216.
 Whence ♂ is greater than the Earth ———— $1\frac{11}{16}$.

4. ☉.

Contains Diameters of the Earth ———— ———— $5\frac{1}{2}$.
 The Proportion of these is, as ———— ———— 11 to 2.
 But of their Bodies, the Proportion is, as 1331 to 8.
 Therefore the Sun is greater than the Earth $166\frac{1}{4}$.

5. ♀

5. 8.

Venus, Mercury, and the Moon, are less than the Earth : Therefore the Diameter of the Earth contains,

Diameters of Venus ————— $3 \frac{1}{4}$.

The Proportion of which is, as ————— 10 *to* 3 .

But of their Bodies, as ————— 1000 *to* 27 .

Therefore the Earth is greater than Venus $37 \frac{1}{4}$.

6. 8.

The Proportion of the Diameters, of Mercury and of the Earth is assigned by some to be, as 28 to 1.

So the Proportion of their Bodies } 21952 *to* 1 .
will be, as —————

7. 2.

The Diameter of the Earth, contains Diameters of the Moon ————— $3 \frac{1}{4}$.

The Proportion of which is, as ————— 7 *to* 2 .

But of their Bodies, as ————— 343 *to* 8 .

Therefore the Earth contains the Moon $42 \frac{1}{2}$ *times.*

And the Moon is equal to One three and fortieth part of the Earth, less by $\frac{1}{4}$ part.

☉ and ☾.

The Proportion of their Diameters is as 187 to 10; therefore of their Bodies, as 6539203 to 1000 : Whence the Sun is greater than the Moon 6539 times.

II. Of the Fixed Stars.

3. *The True Magnitudes of the Fixed Stars we cannot know accurately ; because also the Distances of these is not so certain, and the observation of the appearance of Diameters unconstant : Since because every*

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every ones Eye is singular, and the Estimation is most different in so small a Quantity: And also because by Optick Tubes they appear lesser by it, by how much it is better and greater.

4. But from the Opinion of Alphragan, and other Ancients, and from the Distance of the Stars 20110.

Stars of the First Magnitude.

Contain Diameters of the Earth ———— 4 $\frac{1}{2}$.
 Therefore the Proportion of Diameters, } 19 to 4.
 is, as ————— }
 And of Bodies, as ————— 6859 to 64.
 Therefore they are greater than the } 107 $\frac{1}{4}$.
 Earth, times, ————— }

Stars of the Second Magnitude.

Contain Diameters of the Earth ———— 4 $\frac{1}{2}$.
 The Proportion of Diameters is, as — 260 to 60.
 Of Bodies, as ———— 19465109 to 216000.
 The Excess of them above the Earth 90 $\frac{1}{11} \frac{1}{10} \frac{1}{10}$.

Stars of the Third Magnitude.

Have Diameters of the Earth ———— 4 $\frac{1}{2}$.
 The Proportion of them, as ———— 25 to 6.
 The Proportion of Bodies, as ———— 15625 to 216.
 The Excess above the Earth ———— 72 $\frac{1}{11} \frac{1}{10}$.

Stars of the Fourth Magnitude.

Contain Diameters of the Earth ———— 3 $\frac{4}{5}$.
 The Proportion of them is, as ———— 19 to 5.
 The Proportion of Bodies, as ———— 6859 to 125.
 They exceed the Earth ———— 54 $\frac{1}{11} \frac{1}{10}$.

Stars

Stars of the Fifth Magnitude

Include Diameters of the Earth — — — — — $3 \frac{1}{4}$.
 And the Proportion of them is, as — 119 to 36.
 The Proportion of Bodies, as 1685159 to 46656.
 The Excess of them above the Earth — $36 \frac{1}{4} \frac{1}{4} \frac{1}{4}$.

Stars of the Sixth Magnitude.

Have Diameters of the Earth — — — — — 2 $\frac{1}{2}$.
 The Proportion of which is, as — — — — — 21 to 8.
 The Proportion of Bodies, as — — — — — 9261 to 512.
 The Excess above the Earth — — — — — 18 $\frac{1}{2}$.

And this is the Opinion of the Ancients concerning the Mutual Proportion of the Stars.

5. To John Kepler it seems agreeable to Nature, and to Astronomick Observations, that there should be the same order of Magnitude between the Planets, which is also of Spheres: So that among the Planets, the greatest is the Globe of Saturn; Jupiter follows this; Mars follows Jupiter: Not the Sun after Mars, but according to the Opinion of Copernicus, the Earth: And less than this is Venus, and Mercury least of all, because lowest.

6. And he thinks that the Bodies of the Planets have Proportion of Intervals; so that by how much higher or lower any one is from the Earth, so it is also greater or lesser.

7. In this Ratio, or Proportion, Saturn in greatness of Body, will be somewhat less than Ten Times greater than the Earth: Jupiter more than Five Times: But Mars Sesquiplus or $1 \frac{1}{2}$: And Venus somewhat lesser than $\frac{1}{2}$: Mercury somewhat greater than a Third part of the same, and $\frac{1}{3}$.

8. And the Proportion between the Semidiameter of the Earth, and the Interval or Space of the Earth

Earth from the Sun: And likewise the Equality between the Bodies of the Earth and Sun he assumes; so that how many times the Semidiameter of the Earth is contained in the Interval of the Centers of the Sun and the Earth, so many times also the Body of the Earth is contained in the Body of the Sun.

9. Therefore since the greatest Interval or Distance of the Sun from the Earth, as aforesaid, is made of Semidiameters of the Earth 3469, with a Third part: Therefore so many Times also the Body of the Sun exceeds the Body of the Earth: From the Cube Root of which number it appears, that the Semidiameter of the Sun contains in it self, the Semidiameter of the Earth, a little more than 15 Times: But the Semidiameter of the Earth is of English Miles 51600, and the whole Diameter of the Sun 103200 Miles, which appears to us but about the bigness of 1 Foot, which may be divided into 12 Digits.

10. And as the Proportion of the Bodies of the Sun, and of the Earth, is to the Proportion of the Interval of the Sun and the Earth; so, according to the same Author, is the Proportion of the Interval of the Moon, and the Earth, to the Proportion of the Bodies of the same.

11. And since that Interval is of Semidiameters of the Earth very near 59; so the same number of times also the great Body of the Earth contains in it self the less and small Body of the Moon.

12. From whence also it follows, that the Terrestrial Semidiameter contains the Lunar Semidiameter somewhat less than 4 Times; whence the Diameter of the Moon is 1720 English Miles.

13. And the Proportion of the Diameter of the Solar Body, to the Diameter of the Body of the Moon, is made in the same manner as is the Proportion of the Body of the Earth to the Body of the Moon, to wit, as 59 to 1: And so the Body of the Sun contains of Bodies of the Moon

more

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more than 200000, which is produted, if you say, The Body of the Earth 1, contains Lunar Bodies 59: How many times doth the Body of the Sun, that contains the Earths 3469 times? Answ. 204671:

14: As to the Fixed Stars, in assigning the magnitude of them also, the Modern Astronomers differ much; yet so, that those also which compute most sparingly, are forced to confess, that well near all of them exceed the Body of the Earth many times,

For that is most evident, by this Popular Argument; from the Stars being taken Diametrically Opposite; such are given in Heaven in some place or other, for Example, near the Eye of Taurus, and the Heart of Scorpio.

If therefore the Earth should be greater than these Stars, it would be impossible, that one Arising, the other should Set, and their Changing by turns, those Setting, these again Arising: For the Eye elevated from the Magnitude of the Earth, must needs leave the other beneath the Horizon, which is yet wholly contradicted by Experience: Hence Tycho Brahe, who to satisfy the sinister Opinion of the Vulgar, hath, as much as he could, contracted the Stars; nevertheless the Stars of the First Magnitude he asserts to exceed the Earth 68 times, the Second 18 times, the Third 11 times, the Fourth $3\frac{1}{2}$ times, the Fifth $1\frac{1}{4}$ times, the Sixth is $\frac{1}{2}$ of the Earth.

But others increase them to a huge greatness, because they prove that they are a very great distance from the Earth; for as much as the Semidiameter of the Orb is Insensible to the Altitude of the Fixed Stars, and indeed is understood not to make one Minute in the Sphere of the Fixed Stars.

So much of the Distance and Magnitude of the Stars.

The CONCLUSION.

INstead of an Epilogue I shall annex what *Astronomers* are wont to answer to two sorts of Men, which judge awkwardly concerning such Immense Distances and Magnitudes of the Superior Bodies produced by *Astronomers*: One of which, is of them which in another manner are respecters of *Astronomy*, yea great Men among *Astronomers* by name: But they think the neatness of the most perfect Mundane Work to be overthrown, if they attribute so Immense a greatness to the Sphere of the Fixed, and to the Stars themselves, and so contemptible a smallness to the Inferior Bodies.

To these may be objected the incredible swiftness of the Fixed Stars, which they are forced to confess, who believe those in the Orb to be moved, and to finish their Course in 24 Hours: For if we assume the Semidiameter of the Sphere of the Fixed Stars not greater, than according to *Alphraganus* to be of Semidiameters of the Earth 20110, (which nevertheless among most *Astronomers* is acknowledged to be greater, and that the Diameter of the Fixed Stars is 40220 Semidiameters of the Earth) there arises so great swiftness of this Sphere, that each Star about the Equator, in the space of an Hour passes over 18118152 *English Miles*; or 4529538 *German Miles*.

For the Diameter of the Fixed Stars is of *German Miles* 40220.

Which requires a Circumference of the same Miles of 126405½.

Which drawn into the Semidiameter of the Terrene Globe 860 *German Miles*.

Produces of the same Miles ————— 108708914.

This divided by Hours ————— 24.

Exhibits the Motion of one Hour, to be Miles ————— 4529538.

This divided by ————— 60.

You will have the Motion of one Minute, of Miles ————— 75492.

If you divide the same by ————— 4000.

You will have the Motion of 1 Pulse of the Artery, Miles 1132.

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If any one abhor not this incomprehensible swiftness, why I pray should he be dismayed at that great Distance and Magnitude which some attribute to the Fixed Stars? They say that *Nature admits not this Immense Proportion*: Therefore he may say which objects this; can Nature admit of such Celerity or Swiftness, the like Example of which is not found in the World? On the contrary, it may be answered, we have in the World, and here on Earth, an Example of that Unmeasurable, or rather no Proportion: For what Proportion I pray is there between that small Animal that makes passages under the skin of the Hand, or a Mite in Cheese, and an Elephant, or that Serpent of *Pliny's*, 120 Feet long, which nevertheless greater have been seen in our Age? Likewise what Proportion is there between a Man, and I will not say the Earth, but the Diameter only, or the Semidiameter of the Earth: This contains of *English Miles* ————— 3440.

Of Paces, each containing five Feet — 3632640.

Of Feet ————— 18163200.

Divide these by 6, for so much almost is the Height or Stature of a Man, and it produces } 3027200.

That is, it will require so many Men to be joined together, or set one upon another, to reach from the Center of the Earth to the Surface thereof! If this Proportion between that said little Animal, and the Elephant: Between the Stature of a Man, and the Semidiameter of the Earth, which is as 1 to 3027200, is not absurd, which things, notwithstanding, are only of the Terrene Globe: Why therefore should the Proportion seem so absurd between the Diameter of the Sun and of the Fixed Stars, which is as 1, to 4000000; or as 15 to 60000000.

For if to the Semidiameter of the Sun, we attribute Semidiameters of the Earth ————— } 15

The Sphære of $\frac{1}{2}$ requires Semidiameters of the Earth 30000.
 And the Sphære of the Fixed Stars ————— 60000000.
 But if we make the Semidiameter of the Sun ————— 1.
 The Semidiameter of the Sphære of $\frac{1}{2}$ will be ————— 2000.
 And of the Fixed Stars ————— 4000000.

That is from that Hypotheses which we have assumed before in Chap. 4. which although the Altitude also be Stupendious, yet a less can scarcely be given, if we admit the Parallax of the Annual Orb.

The other sort of Men, from these Proportions, judge it Injurious to the Common People, who by reason of these Disagreeings and Differences, esteem the Opinions of the Learned to be foolish and vain, and deride them: 'Tis Lawful to ask of these, why they do not also renounce or deny the measures of Liquids, of Dry things, of Fields, of Distances, of Journeys; for here also is great Differences or Disagreements; neither can any one so accurately measure any thing at any time, that it shall not want or exceed at another time by many Drops, Grains, Digits, Feet or Paces; which Defects or Excesses to beware of, and to keep an equality in measuring things of all sorts, is altogether impossible: But say they, these Excesses or Defects are small, but yours very great? *Ans.* I deny them to be greater; for compare the whole World, in what manner soever the Magnitude of it is surely found out, with the Terrene Globe; and you will find so much inequality, that all the comparing and disagreement of Proportion, is not so much as is an Hundred, or a Thousand Drops compared with a Cart Load of Wine; concerning the Defect of which, if any one should contend before a Judge, and would accuse the Seller of Fraud, he would be accounted for an Impudent Man, or a Fool in Law; Lastly, there are some who are led by Religion, or rather Superstition, who call them Idle and Proud Men, who seek to measure

sure

sure the Stars, and find out their Distances and Magnitudes. : And with *Pliny* exclaim against them, *Saying, 'tis a madness to trouble any ones mind to go about such an hardy Enterprize as the measuring of the World :* But the madness is in them, who have the World for their God, which *Pliny* made it ; whose Profundity is only Scrutinized by the Spirit of God : But he that with Christians believes that the World is the Work of God, and as it were a Living Statue of him, by which he would shew to us his Wisdom, Goodness, and Power ; he will not disallow these and such like Studies and Endeavors ; unless he would esteem a Man that endeavors to know the Erection of God, beneath a Brute : Some Men creep on the Ground, but a Generous and Godly Mind also desires to mount up to the Heavens, and there also, as *Plato* saith, not to creep along, but to admire the Wisdom and Goodness of God : For neither any one of the *Astronomers* advances to it through Rashness, that what Distances, Magnitudes and Proportions he finds of the Stars, he should glory that he hath them in the Minutes or Scruples of themselves (altho' sometimes for necessity in Calculation he works to the smallest of them) but accepts all things in a larger manner, that he may understand the Works of God the Better, which hath made all things in Number, Weight, and Measure, whose Name be Blessed and Praised for evermore.

The End of Book II. Of Theoretical Doctrine.

O F
A S T R O N O M Y.
B O O K III.

Of Chronological Doctrine.

The P R E F A C E.

1. **T**HE Stars, besides many other uses, are also designed by God for this, that from thence we the Dwellers on Earth should seek the Distinction and Numbring of Times, of Days, of Months, and of Years.

2. Whence the Astronomical Science ought not to be excluded from this Chronological, or Doctrine of Times; which others intermix with Spherical and Theoretical Doctrine, but we are pleased to deliver it separately, and to divide it into Three Parts.

3. Of these it Aets,

I. Of Astronomical Time.

II. Of Political Time.

III. Of Ecclesiastical Time.

O F

O F
ASTRONOMY.
 B O O K III.

The First Part.
Of Astronomical Time.

C H A P. I.

Of the Definition and Division of Chronology.

1. **C** *Chronology* is a Doctrine delivering the Reason of distinguishing and computing of Time.
2. And it is either *Astronomical* or *Historical*.
3. *Astronomical*, is that which considers time as the measure of Duration, sought from the Heavenly Motions, and Explains the Variety of it.
4. *Historical*, which accommodates this measure to things done in the World.

Concerning this we shall add a few things towards the end: The Astronomical we shall now briefly pursue.

C H A P. II.

Of the Diversity of Times.

1. **T** *IME* then in this Place is the measure of Duration of any thing, taken from the Heavenly Motions.

(For the word *Time* is Ambiguous: Sometimes it signifies the Duration of Things absolutely considered; sometimes a Duration determined by a certain Measure: Lastly, sometimes the Measure

it self of Duration, when 'tis sought from any other Durations, but then especially when from Heavenly Motions.)

2. *And this Time is either Astronomical, or Political, or Ecclesiastical.*

3. *Astronomical*, is that which is constituted as accurately as may be to the Rule of Cælestial Motion.

4. And 'tis either *True*, which is computed to true Motions; or *Mean*, which is computed to mean or middle Motions.

That is Unequal; but this Equal, and chiefly regarded; for the Measure is equal to it self, and agrees constantly.

5. *Both kinds, are Days, Months, and Years.*

CHAP. III.

Of Days, and the Kinds and Parts of Days.

1. **T**HE least Interval of Time Instituted by God himself in the First Creation of things, is a Day.

2. *Which is wont to be distinguished into Natural and Artificial.*

(Concerning both, so far as it agrees most closely with the Motion of the Sun; and some foreknowledge of them, is necessary to the right understanding the Cælestial Motions, although we have touched upon some things before in Book I. *Of Astronomy*, Chap. 8. But we have reserved the further Delivery concerning Natural and Artificial Days to this Third Book.

And although 'tis less fit for distinction to divide the Day into Natural and Artificial, for Art hath Instituted neither, but both by Nature it self; yea rather are they Instituted by God the Author of Nature; yet because 'tis entertained by the Ancients, we also retain it, which since 'tis agreed of concerning Things, we ought to be easie in Words.

3. *A Natural Day, is the Space of Time, continuing either by a continued or interrupted Revolution together of Solar Light, and of succeeding Darkeness.*

Or speaking Astronomically.

'Tis the Space of Time in which the whole Equator Circle,

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Circle, together with the Additament answering to the proper Motion of the Sun, is turned back again, or makes its Revolution.

1. The maker of the Day, yea the Day it self is Light, or the Sun: For the Light God himself hath called the Day, not plainly in what manner 'tis Light, but in what manner 'tis present, and enlightens by action; the absence of which involves all things in Darkness, by a perpetual Change, the Period of which is Evening and Morning as the Scripture says, or Light and Darkness; also God gave it the name of *Day*, which is this our Natural *Day*: The Law and Rule of which, since it is the Equator, or the Equinoctial Circle, therefore also from the Revolution of this, a Day may be defined.

2. And because the Sun the Fountain of Light, besides its daily Revolution, proceeds also by its proper Motion under the Ecliptick slowly, there is added daily to dispatch the Natural Day, such a Portion of one Revolution of the Equator, as the Sun requires in its Progression under the Ecliptick.

3. And these Portions are called *Additaments*, or *Overplus's* to be added to the Equator, and are no other than the Ascensions of the Sun's Diurnal Motion, or Portions of the Equator, competent to the Diurnal Motion of the Sun.

4. And since this Diurnal Motion of the Sun is unequal, these Additaments are also made unequal, which inequality also redounds in the Natural Days.

5. Lastly, because we ought necessarily to assume some mean or equal, between unequal, as the measure of inequality: Hence therefore,

4. *Natural Days are twofold,*

1. True and Unequal, which are also called *Apparent*, to which besides the whole Revolution of the Equator, true Additaments and Apparent in the Heaven, but unequal, are to be added.

2. Mean

2. Mean or Equal, to which are added mean and equal Additaments, more than the whole Revolution of the Equator, being assumed by Astronomers for the Subject of teaching.

1. Those True Days are in daily use: and indeed are commonly thought equal, because each one is divided into 24 equal Hours according to Sense, and the Hour 12 in Sun-Dials always agrees to the Meridian line or shadow; nevertheless the cause of which agreement is govern'd not by the Equality of the Day, but by the Diligence of a Pendulum Clock, and its Hour of 12, daily prolonging or withdrawing, and directs it to the arrival of the Sun to the Meridian Line: Which, if truly a Clock or Watch be made, which for some certain Days wants not Human Direction, at last the difference between the Meridian and the Hour of 12 will be discovered, and therefore the inequality of the true Natural Days, and of the Apparent in Heaven.

2. And the Mean or Equal, are used by *Astronomers*, which they frame to the Tables of the Cœlestial Motions, and they invent the same to this end, that they may measure that Inequality by these Equals: Whence also the swift Motions taken out from the Tables, agree not with the Heaven, before Reduction be made of equal time to unequal, or the contrary.

3. And altho' the inequality of Pairs of Days next to one another be not observable; also one only of the shortest being compared to one of the longest, hath not very much difference, yet some many Days in order in some one part of the Year, being joined to one another, being compared with the same number of Days in another part of the Year, a difference evident enough appears to arise beyond those, and makes them Shorter and Longer.

4. And the Additaments which *Astronomers* constitute

stitute between the greatest and least, being Mean and Equal, are so much, as the Sun's mean Diurnal Motion is, to wit, $59' : 8'' : 20'''$, from thence an equal Natural Day consists of $360^\circ : 59' : 8'' : 20'''$ of the Equator, for which nevertheless, for the most part we assume only 360° .

5. *Indeed the beginning of a Natural Day ought every where to be taken from the Horizon of the places, even as from the time of Light and of Darkness; but in an Oblique Sphere it pleases Astronomers, by cutting off in the midst, as it were, the Vicissitude of Light, and of Darkness, beginning it from the Meridian Circle, as most constant, or from Noon or Midnight.*

1. In a Right Sphere the Ratio or Reason, is the same of the Horizon and of the Meridian: And the Horizon of one place is the Meridian of another, and the Meridian the Horizon: From this cause, the Ratio is all one, whether we begin from the Meridian, or from the Horizon, for the Natural Day.

2. But in an Oblique Sphere the Ratio is unlike, for the Horizon is so varied according to the variety of the Oblique Sphere, that 'tis not only different, but also it acquires another Obliquity at the Equator.

3. Moreover the Sun in some places of the Earth, many Days, neither Rises above the Horizon, nor Descends beneath it; as among those who inhabit under the Poles in a Parallel Sphere.

Therefore we cannot take in it always one beginning of the Natural Day, but instead of it we must choose a Meridian, which indeed is also various, according to the variety of places, but is in it self a right Horizon of places every where.

Nor yet also is the Horizon in an Oblique Sphere altogether so to be rejected, that when necessity requires to begin the Natural Day in it, that it cannot be done in no manner: For,

6. The

6. *The parts of a Natural Day are the Artificial Night and Day.*

The beginning and end of these must necessarily be taken from the Horizon.

7. *For the Night is the space of time, in which the Sun lies hid beneath our Horizon.*

8. *But the Artificial Day is the space of time, in which the Sun tarries above our Horizon.*

The reason of this Appellation some assign to be this, that these Artificial Days are accommodated to Exercise.

9. *Moreover as the Natural, so also the Artificial Days and Nights are very unequal in an Oblique Sphere.*

The inequality of Natural Days, as well in a Right as an Oblique Sphere, proceeds from the Inequality of the Sun's Motion, and the Obliquity of the Ecliptick: But these in an Oblique Sphere depends on the greater or lesser Obliquity of the Horizon and the Equator; concerning which Note what follows.

I. *In a Right Sphere there is a perpetual Equinox.* See Fig. 178.

That is, the Days to the Days, and Nights to the Nights, and all the Days are equal to the Nights.

II. *In an Oblique Sphere there are only two Equinoxes, to wit, in which the Days next to the Nights, and the contrary, are equal.*

This comes to pass from the Sun's being in $\circ \gamma$, and $\circ \alpha$, to wit, in the Spring time about the Tenth or Eleventh of March, and in the Autumn about the Thirteenth or Fourteenth of September.

III. *In the North part of our Oblique Sphere, the Days increase from the Vernal or Spring Equinox, and are made longer than their adjoining Nights, even to the Summer Solstice.*

See

See Fig. 179. That is, In which ☉ enters into ♋, about the Eleventh or Twelfth of June, and makes the longest Day and the shortest Night.

IV. From the Summer Solstice the Days decrease again in a Retrograde Order, and the Nights increase, yet so that they are shorter than the Days, even to the Autumnal Equinox.

V. From this Autumnal Equinox, the Nights increase, and become longer than the Days, even to the Winter Solstice.

That is, in which the Sun entring ♏ w about the Eleventh or Twelfth of December, makes the longest Night, and the shortest Day.

VI. From the Winter Solstice the Nights decrease again in a Retrograde Order, and the Days increase, yet so, that they are shorter than the Nights even to the returning of the Vernal Equinox.

VII. Then from the Sun's being in places of the Ecliptick, equally remote from the same Solstitial Point, the Days are equal to the Days, and the Nights to the Nights.

VIII. And from the Sun's being in places of the Ecliptick, equally remote from the same Equinoctial Point, or also in opposite places, the Day of the one is equal to the Night of the other.

And by how much greater the Obliquity of the Sphere is, or the Elevation of the Pole, by so much the greater also are the increase of the longest and shortest Day and Night; yea, and of all the rest, as well increasing as decreasing, observing a due Proportion: See Fig. 180.

IX. In the South part of our Oblique Sphere, all these things are in the same Proportion between themselves, but in a contrary manner, and in opposite Signs,

For

For they have the longest Day and the shortest Night, the Sun being in $\circ \text{ } \gamma$; on the contrary, the longest Night and the shortest Day; the Sun being in the beginning of ♋ : Hence, when with us the Days increase, by the Sun's ascending from $\circ \text{ } \gamma$ to us, there they decrease, by the Sun's departure: And when with us they decrease, by the Sun's departing from us, from $\circ \text{ } \text{♋}$: There they increase by the Sun approaching to them.

X. The Cause of all these is:

I. The Section in divers manners of the Horizon, and of the Circles of the Sun, or of the Parallels of the Equator.

For in a Right Sphere, the Equator, and all its Parallels are cut by the Horizon into two equal parts, and as always the Semicircle of the Ecliptick Arises with the Sun in the East, and Sets in the West; so also the Semicircle of the Equator, as well as the Parallels of it, ascend and descend with the same.

Therefore since the Sun abiding above the Horizon, it makes the Day, and being beneath the same it makes the Night, 'tis manifest from hence that the Days and Nights in a Right Sphere, are equal, as in Fig. 178.

But in an Oblique Sphere it happens contrary: For from the Pole, being elevated above the Horizon; the Ecliptic and the Equator are never cut into two equal parts, unless when the Sun is in the Sections of the Equator and of the Ecliptic, and therefore being conversant in the Equinoctial points $\circ \text{ } \gamma$ and ♈ , it cuts both the Ecliptick and the Equator into two equal parts, and with the Semicircle of the Ecliptic, the Semicircle also of the Equator ascends and descends: If the Sun be conversant in all the other places of the Ecliptick, the Parallels of the Sun are unequally cut by the Horizon; so that the Sun running thro' the Northern Signs, with us the greater part of
every

every Parallel is above the Horizon, and the lesser beneath; the contrary of which happens if the Sun be passing thro' Southern Signs: From which the inequality of Nights and Days, in an Oblique Sphere, appears plainly. Fig. 179.

2. *Another Cause is, the inequality of Ascensions and Descensions.*

For altho' half of the Ecliptick Rises and Sets every Day and Night, yet not also half of the Equator ascends or descends together: But in Northern Signs more than half ascends and less descends: In Southern, less than half ascends, and more than half descends: Therefore since the Portions of the Equator ascending and descending together, measure the Day and Night, again the cause of inequality is manifest.

II. *In a parallel Sphere the whole Year is divided into one Artificial Day, and one Night; yet the Day, in a Northern Parallel, exceeding the Night in Magnitude, the contrary of which is made in the Southern part.* Fig. 181.

1. The cause of this is, because in this Position of the Sphere, half of the Ecliptick being always elevated above the Horizon, all the Parallels of the Equator which arrive with it, it shews above the same: Therefore how long the Sun is conversant in this half, so long is the Day, without any Night intervening: On the contrary, the other half of the Ecliptick, with the Parallels of the Equator, always lie hid beneath the Horizon, and makes no Day, but continual Night; which delay in both, extends it self to half a Year, or six Months.

2. Hence therefore from the Analogy of our Artificial Days it may be said, that by only one Annual Revolution of the Sun, only one Natural Day is made; the Artificial Days of which compleat half a Year, and the Nights in like manner half a Year.

And

And because the Annual Orbit of the Sun is Eccentric, the *Apogæon* of which continues in the Northern half of the Ecliptick, in which the Sun is moved slower than in the half on the South part, and therefore abides longer in the Northern half, than in the Southern: Whence in the Northern part the Day is made a little longer than the Night; on the contrary, in a Parallel Sphere, in the Southern part, the Night is longer than the Day.

XI. *And the length of an Artificial Day or Night may be known in an Oblique Sphere, by the Dimension of the Diurnal or Nocturnal Arch.*

XII. *And a Diurnal Arch is an Arch of the Equator, from the Rising of the Sun to the Setting of the same, or the springing forth of Day.*

Like to this, is the Parallels of the Natural Day, in which the Sun is conversant, shewing the Segment above the Horizon.

XIII. *A Nocturnal Arch is an Arch of the Equator, which from the Setting of the Sun, even to its Rising, or in the Night, descends beneath the Horizon.*

Equal to this, is that which from the Setting of the Sun even to its Rising, ascends over against it; likewise the inferior Segment of the Parallel of the Sun.

(But how the Length of the Day and of the Night, may be found from these Arches, shall be shewn in the Problems.)

CHAP. IV.

Of Hours and Minutes.

1. **T**HE Days, as well Natural as Artificial, are divided also into Hours and Minutes, or Scruples of Hours.

I. This distinction was not used by the most Ancient, who generally signified whatsoever time, either by the Four Seasons of the Year, *Spring, Summer, Autumn, and Winter*: And the Day they divided into Four Vigils, the first of which was from
our

our Hour of 8 even to 9; the other, or second, from 9 to 12; the third from 12 to 3; the fourth from 3 to 6.

2. The day also, as well Natural as Artificial, they divided or distinguished into time before Noon and after Noon, numbering from Midnight with respect to the Natural Day; but from the rising of the Sun with respect to the Artificial Day.

3. Also there have been in old time, many other parts of the Natural Day, to wit, Midnight, Cock Crowing, the Morning Twilight, Day break, the Morning, Noon, the Evening, the setting of the Sun, Candle tinding, the Evening Twilight, unseasonable Night, the dead time of the Night; but we at this time distinguish the day more concisely into Hours and Minutes.

2. An Hour is either the twenty fourth part of a Natural Day, or the twelfth part of an Artificial Day and Night.

3. And 'tis either equal or unequal.

4. An equal Hour is the twenty fourth part of a Natural Day.

1. 'Tis called also an Equinoctial Hour.

And is a space of time, in which $15^{\circ} : 2 : 28''$ of the Equator ascends above the Horizon, yet for which, for the sake of roundness, for the most part we number 15° of the Equator for one Hour, and back again one Hour for 15° of the Equator.

2. But since the Hours of a Natural Day are said to be equal, that is to be understood, not to be true; for because the Natural Days are unequal, which are measured by Hours, also the Hours themselves must needs be unequal; not indeed the Hours of one day, in respect of which they are truly said to be equal, but in the Ratio of divers days considered, which inequality, notwithstanding because it is small, is not heeded, but is accounted for equality.

5. *An unequal Hour is the twelfth part of an Artificial Night or Day.*

1. These Hours are called Temporal, perhaps for the conveniency of doing of things, likewise Judaical or Jewish, because they were used by the Jews, as appears from Holy Writ.

2. And again we must note, that these Hours are called unequal, not with respect to one Artificial Day or Night, for so they would be all equal, but with respect of divers Nights or Days, for truly the Hours of these are unequal, more long of the longer days, more short of the shorter.

3. Therefore according to the diverse Consideration, all Hours (except of equal days, the Hours always by Astronomers are feigned equal) are also equal and unequal: The equal are all the Hours of one and the same day, the unequal, the Hours of divers days: Of Natural Days, because of the unequal Additaments, and the obliquity of the Ecliptick: Of Artificial, since for this reason, as also because of the obliquity of the Horizon: But because that inequality is small and unestimable, therefore the Hours of it are accounted equal; but this, because great and notable, therefore the Hours of it are esteemed unequal.

4. Others distinguish the unequal Hours also into Jewish and Planetary; and these they number and distribute in another manner than the Jewish, altho' some confound them, and account them for the same: But because the vanity of Astrologers have feigned these Planetary Hours, as also the days of the Week are denominated by them; therefore to speak concerning them, we think it not worth our while.

6. *The Sixtieth part of an equal Hour is called a Minute or first Scruple, the Sixtieth part of this a second Minute or Scruple; and so on, even to the smallest Division.*

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For 'tis worthy of the most precious Treasure of time, that we should have the Ratio or Proportion of all, yea, of the most Minute Particles of it; neither should we suffer any time to slip or pass away without some Fruit, or to spend it vainly; for all things are other Mens, Time only is ours, as *Seneca* saith; which indeed neither can suffice for necessary things, altho' it should be preserved most diligently.

And we must note, that the Method of converting of Degrees and Scruples of the Equator into Hours and Minutes; and back again, Hours and Minutes into Degrees and Scruples of the Equator, may be thus,

1. The turning of Degrees and Scruples of the Equator into Hours and Minutes.

Assume for any of the Equator { For one Degree } 4' { Of an Hour
 { A first Scruple } 4"
 { A second Scruple } 4"
 Or

Multiply as well the Degrees as the adhering Scruples by 4, and it produces from Horary Degrees first Minutes, from Scruples of the Equator Horary second Minutes, &c. which afterwards, if they increase beyond 60, they must be reduced by Sexagenary Division to the former Species.

Example.

Degrees of the Equator 31° and Scruples 43' How many Hours and Minutes do they constitute? Answer, from multiplying by 4 comes forth 124' and 172" Horary Minutes, that is, 2°: 6': 52".

2. The turning Hours and Minutes into Degrees and Scruples of the Equator.

Assume { For one Hour } 15° { Of the Equator
 { For one first Minute } 15'
 { For one second Minute } 15"
 Or

Multiply as well the Hours as the Scruples by 15, and it produces from the Hours, Degrees; from the first

first Minutes is produced first Scruples of the Equation; from second Minutes is produced second Scruples of the Equation, &c. to be reduced to the former Species or Rank, if there be need.

Example.

Hours 2, Minutes 6, Scruples or second Minutes 52. How many Degrees and Scruples of the Equation do they make? Answer, from the Multiplication is produced $30^{\circ} : 90' : 780''$, that is $31^{\circ} : 43' : 13''$ of the Equation.

CHAP. V.

Of Astronomical Years, and in particular of the Solar.

1. **A**N Astronomical Year in general, is the Revolution of any Star, made together by its proper, and mean or equal motion.

Hence then they are.

| | Of Julian Years. | Days. | Hours. |
|---------------------------|------------------|-------|--------|
| 1. The great Syderal Year | 25411. | 215. | 0. |
| 2. The Saturnine | 29. | 155. | 3. |
| 3. The Jovial | 11. | 313. | 14. |
| 4. The Martial | 1. | 321. | 22. |
| 5. The Venereal | | 224. | 17. |
| 6. The Mercurial | | 87. | 23. |

2. But in particular, an Astronomical Year is either Solar or Lunar.

For although the Revolution of any Star measure time, and may constitute a Year, yet these two Luminaries are those before the others; and according to the Course of them the Annual times are determined.

3. A Solar Year, is that space of time in which the Sun, departing from one point of Heaven, returns to the same.

And

4. And

4. And it is twofold.
1. Sydereal, and
 2. Tropical, or turning about.
 5. A Sydereal Solar Year, is that space of time in which the Sun departing from the 1st of Aries, returns again to the same.

For altho' any fixed Star may be appointed for the beginning of this Year, yet always we make choice of some Illustrious or bright Star: So the Egyptians began their Year from the rising of the Dog Star, which hapned in the Summer; the Grecians from the rising of the 7 Stars; we from the 1 Star of γ near to the Equinox: But because the length of a Sydereal Year is of more subtil observation, hence it comes to pass that Artificers do not agree in Minutes.

| | D. | H. | M. | S. |
|----------------------------|-----|-----|-----|------|
| By Copernicus this Year is | 365 | : 6 | : 9 | : 39 |
| By Tycho | 365 | : 6 | : 9 | : 26 |

6. The Tropical Solar Year, is that space of time in which the Sun under the Ecliptick departs from the point of the Vernal Equinox, and returns again to the same.

In which the precession of the Equinoxes, or the motion of the fixed Stars is unequal; they also have this unequal Year, one while greater, another while lesser; between which the mean or equal is:

| | D. | H. | M. | S. |
|--------------------|-----|-----|------|------|
| According to Tycho | 365 | : 5 | : 48 | : 45 |

The beginning of this Year, which may be taken in every place, Astronomers place in α or the Vernal Equinox, in which most notable point also they begin and end the Zodiack.

7. And this Year is distinguished into 4 parts, which are, Spring, Summer, Autumn and Winter.

Commonly 'tis distinguished into two only, viz. Summer and Winter; but better into 4 parts, which are called the 4 Quarters of the Year, or the 4 Sea-

sons of the Year, in which is made the most evident mutation of the Air.

8. The Spring, *with us is the first part of the Year, from the Vernal Equinox to the Summer Solstice.*

In this time the Sun ascends to us thro' γ , δ and η , whence these Signs are called Vernal.

9. The Summer, *is the second part of the Year, from the Summer Solstice to the Autumnal Equinox.*

In this time the Sun passes thro' the 3 Summer Signs, ϵ , ζ , η , thro' which it departs from us, and descends towards the South.

10. Autumn, *is the third part of the Year, from the Autumnal Equinox, even to the Winter Solstice.*

In this time the Sun descends further thro' the 3 Autumnal Signs θ , ι , κ , until it hath departed the furthest from us into the Solstice.

11. Winter *is the fourth and last part of the Year, from the Winter Solstice to the Vernal Equinox.*

In this time the Sun passes thro' the 3 Winter Signs λ , μ , ν , thro' which it ascends again from the South, and returns to us again.

CHAP. VI.

Of the Lunar Year and Astronomical Months.

1. **A**N Astronomical Lunar Year, *is that which is taken from the motion of the Moon.*

2. *And 'tis twofold.*

1. Simple, persevering by one revolution of the Moon.

2. Compounded of many Revolutions of the same.

3. A Simple Lunar Year, otherwise called by a Name more in use, from *Months*, and 'tis twofold, Periodical and Synodical.

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4. A Periodical Month, is that in which the Moon under the Ecliptick is turned about from one to the same point.

D. H.

This is done in 27 : 7 : 43 : 5

5. A Synodical Month, is that in which the Moon departs from the Sun, and having compleated all the diversities of her appearances, returns to the same.

D. H.

This Month consists of 29 : 12 : 3 : 44

1. And this is understood of the mean or equal motion of the Moon; whence also the mean or equal Months are made; but the true Months are very unequal; of the Periodical, by reason of the inequality of the Moons motion; of the Synodical, by reason of the inequality both of the Stars and of the Sun and Moon.

For in the Summer, the Sun being in *Apogæon*, marching slowly, the Months grow shorter, because the Moon being swift, soon overtakes the Sun, being the slower: In the Winter, the Sun being about the *Perigæon*, marching swiftly, the Months are longer, because the Moon is slow, and overtakes the Sun, being more swift, but slowly.

Moreover, since the Moon is slow in her *Apogæon*, swift in *Perigæon*, therefore slower there than here, all other Circumstances being the same, it finishes the residue which remains to it, to compleat the period, by its motion to the Sun.

Therefore the causes being compounded into one, when the Sun is in *Apogæon*, the Moon in *Perigæon*, the Synodical Month hath Days 29, Hours 6, Minutes 42; but when the Sun is in *Perigæon*, the Moon in *Apogæon*, the Month is almost 30 intire days, for they want only Hours 4, and Minutes 23; because of this inequality Astronomers neglect the true Months, and are content with mean, being equal.

D 1 4

2. Also

2. Also we must know, that to these two kinds of Months a third also is wont to be added, which they call the *Month of Illumination*; and 'tis the interval or space from the first day of beholding the Moon, after the Assembly of both the Luminaries, even to the last day of the Moons vanishing: These Months also are very unequal, for that cause, that the Moon in falling and renewing, some times promoting her appearance, other times suspending it, for the causes declared in the Theory of the Moon.

3. Lastly, we must note, altho' the Months, properly so called, are made by the course of the Moon, yet Analogically also they are constituted Solar Months by some Astronomers, which are the space of time, in which the Sun views one Sign of the Zodiac, or a twelfth part of it; which, since 'tis made by an unequal motion, therefore they are also unequal; but if from the true, being unequal, we assume the mean, being equal, it will make the Solar Month of D. H.

30 : 10 : 29 : 6 : 19.

6. A Compounded Lunar Year, is the space of time, in which the Moon, after some Assemblings or Conjunctions with the Sun, comes to it again, and is united with it not far from the former place.

7. And this Year also is twofold, 1. Common, which comprises 12 Synodical Changes, and 2. the Leap Year, which hath 13 Synodical Changes.

For because the Moon after 12 compleat Changes, doth not overtake the Sun, but after 13 Changes out goes the same, therefore the Lunar days that are wanting, are reserved until they constitute an whole Month in the second or third Year, which they intermingle, that the Conjunctions of ☉ and ☾ may continue; whence the Year runs up to 13 Months.

8. Again, both are twofold.

1. Mean, which is the Rule of the rest.

2. True,

2. True,

True, being irregular from the mean, more or less.

9. The Mean is the Common.

Days. Hours.

Of 354 : 8 : 48 : 38 : 10.

And the Leap Year.

Of 383 : 21 : 32 : 41 : 21.

Thus far of Astronomical Time.

PART. II.

BOOK III.

Of Politick or Civil Time.

CHAP. I.

Of Politick or Civil Days.

1. Politick or Civil Time, is that which any People or Nation, from rude and popular Observation of Celestial Motions, constitute to themselves for the use of the Citizens.

Therefore the Rule and Measure of this is Astronomical Time; to which, by how much the nearer the Politick approaches, it is accounted the more perfect.

2. The Parts of this, like as also of Astronomick, are.

Days, Years, Months.

3. Politick Days, differ not otherwise from true and unequal Astronomical, than that some begin them

in another place, and distribute them into some and other Hours, or number them otherwise.

For that we may speak only of some.

1. The *Hebrews* by Divine Authority, begin the day from the setting of the Sun; considering the order of the Creation of the World, for from the World, being just begun upon the Face of the Abyss, or that great heap of Water, made in the first day, which was as it were, the matter of the future Edifice, they were as yet in Darkness, to which Light succeeded, even as the form of all things; and so there was made from the Evening and the Morning the first day, as the Scripture speaks *Genesis*, 1.

2. These are imitated by the Ancient *Athenians*, but at this day *Silesians*, *Bohemians*, *Marcomans*, *Italians*, &c. the Clocks of these founding the 24 Hours from Sun setting; and the end of the Civil day they also make the beginning of the other.

3. The *Chaldeans* or *Babylonians*, and at this time the *Norinbergs*, begin the day from Sun rising.

4. The *Imbrians*, the *Egyptians*, the *Arabians*, they place the beginning of the Civil day in the Meridian or Noon time, which also Astronomers do; but the *Romans* in the Midnight.

5. We at this day follow both, and begin the Civil day either from Midnight or from Noon; for from both places the number of 12 Hours is turned about to its beginning.

4. And as the Civil day among others, hath another beginning, so they distribute the same also divers ways into Hours.

5. For altho' 'tis almost every where received, that 24 Hours constitute a Natural Day, yet some make them equal, others unequal.

Understand equal in respect of one Natural day; for if we consider divers Days, since they are unequal,

qual, the Hours also must of necessity become unequal.

6. They which have the Hours of one day equal, some of them number them intirely, from 1 to 24; but some bisect them in the middle of Day and Night: Lastly, others number the Hours of the Day and of the Night asunder, and either Night or Day being ended, they return to Unity.

And these, if they Inhabit in an Oblique Sphear on the North part, in the Summer time they have the Artificial Days longer by many Hours than the Nights, in the Winter time the Nights are longer than the Days.

7. They which use unequal Hours, divide the Artificial Day into 12, and the Night into the same number of Hours.

Then since the Days in an Oblique Sphear being compared between themselves, and to the Neighbouring Nights, are in no manner equal; also the Diurnal Hours of divers Days being compared between themselves, and with the Nocturnal Hours next to them, must needs be made unequal.

CHAP. II.

Of Politick Years.

1. **A** Politick or Civil Year, is a certain number of whole Days, collected either from other causes and reasons, or from the Sun or Moon, or from the Periods of both Motions; and neglecting the Astronomical, these being instituted by the People of any Nation, they are received.

From this Cause they need the Intercalation, as they call it, or the inserting of Days or of Months, lest the defect, though small in the beginning, in
success

success of time increase too much, and the Civil Year be changed unmeasurably from the Astronomick.

2. And of these Politick Years, also some are Solar, which are accommodated to the motion of the Sun; some Lunar, which are estimated chiefly, according to the course of the Moon.

3. Of the Solar there are many Kinds, three of which will be sufficient for us in this place.

The Julian, The Gregorian, The Egyptian.

4. The Julian Year is, that which Cæsar Julius Cæsar either constituted, or used in the Roman Empire, being ordained by a publick Edict.

5. And 'tis twofold.

1. The Common, of Days 365.

2. The Leap Year, Bissextile or Intercalation, recurring every fourth Year, of Days 366.

The cause of this diversity is this.

1. Julius Cæsar would not neglect the Hours in the Astronomick Solar Year, or the Yearly Hours, adhering to the Circuit of the Sun, but thought to intermix them in his Civil Year, which he would have to be Solar, that it might exactly agree with the Heaven.

2. But when he considered that the Sun finished its Course in Days 365, and Hours 6, therefore he appointed, that even as the Astronomick, so also his Civil Year should have so many Days and Hours.

3. But because these 6 Hours being observed every Year, was not fit for Civil use, (for since 'tis convenient that the Years and the Days be conveyed from one and the same beginning; these 6 Hours, every Year at the beginning of the Year he withdrew) therefore the 3 former Years he intirely neglected them, and made every one of these Years commonly of 365 Days.

4. But the fourth Year, in which these 6 Hours 4 times taken, make 24 Hours, and therefore an whole

whole Natural Day; this day he ordered to be planted between; and therefore every fourth Year he made to consist of 366 Days: Whence every fourth Year he called *Embolimæus*, but we *Bissextile* or *Leap Year*, because that day of Intercalation in old time was placed to the Sixth day before the Calends of *March*, either before or after, and the Sixth of the Calends of *March* twice numbred; but now at this time, that day of Intercalation, (or which is to be added) is joined to the last day of *February*, which therefore obtains 29 Days, when otherwise it hath only 28.

5. And we must know, that this *Julian Year* of Days 365 and Hours 6, is longer than just; and the Astronomick Year of Days 365, Hours 5, Minutes 49, is exceeded by it by about 11 Minutes, or a little more: Whence in succeeding times arose some inconveniency; and it came to pass that the Ingress of the Sun into the first Scruple of *Aries*, or the point of the Vernal Equinox, which *Cæsar* would have affixt to a certain day of the Month of *March*, remained not there, but every Year anticipated, or took up before that time almost 11' of an Hour; which from the time of *Julius Cæsar*, to the Year 1653, being collected Yearly, increased beyond 14 days, by which the Equinox crept forward towards the beginning of the Year.

Hence is arisen

6. The *Gregorian Year*, which *Gregory* the XIII. Pope of *Rome*, about the Year of the Christian Epoch 1580, Instituted among them.

This Pope to correct the Error of *Julius Cæsar*, and being willing to reduce the Equinox to the days which was deserted, and to take care of this Anticipation for the future, he introduced this Year, not received by all, but only by his Romans.

Then 1. that he might Retract the Equinoxes to their former places, from which they had crept forward

ward from the former days, he concealed or took no notice of 10 days in the *Julian Year*, and commanded them to be over-past, or not accounted in numbering.

2. And that they should not again be removed from their Seats, which may happen by reason of too much Intercalation, and assuming the Year longer than just; he commanded that in the following Years of the Christian Epochs 1700, 1800, 1900, &c. which otherwise ought to be Intercaled, that the Intercalation should be omitted.

And this is the reason that those which use the Pontifick *Gregorian Kalender*, always antecede us 10 Days; which Anticipation, by reason of the neglect of Intercalation in the Year of the Christian Epochs 1700, being this present Year, in which I am Writing this Book in *English*, it will be 11 days, and in the Year 1800 12 days, in the Year 1900 13 days, if the World so long remain.

7. The *Egyptian Year*, is the space of time, always containing 365 Natural Days.

We call this the *Solar Year*, not because it depends on the Sun's Motion, but that it approaches nigh to the *Solar Year*, neglecting only 5 Hours and 49 Minutes, which adhere to the Circuit of the Sun, Astronomically considered.

But because these Hours and these Minutes every fourth Year constitute almost an whole day, it comes to pass, that from the neglect of them the beginning of every fourth Year is Anticipated by one Day; and therefore in 1460 Years an whole Year is wanted away: Example, from the 1 of January to the 31 of December, from this to the 30 of the same, &c. And thus the *Julian* and the *Egyptian Year* in 1460 Years agree again at last; and then the Egyptians number the Year 1461.

CH A P. III.

Of the Civil Lunar Years.

1. **L**unar Politick Years, are also either *Simple*, which consists of one Revolution of the Moon.

Or *Compound*, which consists of more.

2. *Simple Lunar Years*, (which as we have also admonished before, are called more rightly and more usefully Months) are by turns, of Days 30 and 29, made in imitation of a Synodical Month, (viz. a Conjunction of the Moon) in which she being departed from the Sun, returns again to the same.

For the Moon from a Conjunction with the Sun, returns again to the same by mean or middle Motion, in Days 29, Hours 12, Minutes 44. And lest the 12 Hours that are annex'd should cause trouble, for them in the other Month also an whole Day is assumed; whereby it comes to pass, that what is withdrawn from one Month, advances again to the other; and therefore some equality of Months is preserved.

3. A *Compound Lunar Year*, also is twofold, *Loosed* and *Bound*.

4. *Loosed*, is that which is exempted from the Laws of the Suns Course, being made in some measure from many changes of the Moon, or from the meetings of the Sun and Moon.

5. This again is twofold.

1. *Common*, of 12 Months, an half part of which is of 29 Days, the other half of 30, whence the whole Year is made of 354 Days; for 29 drawn into 6, makes 174, and 30 drawn into 6, makes 180, both being added, are equal to 354 Days, or a common Lunar Year.

2. *Leap Year*, in which one Day is added over and above the 12 Changes; so that the Year is made of 355 Days.

The Turks from the Discipline of the Arabians, use such Years, and of the Leap Year or Intercalation, this is the Ratio or Reason; to wit, because one change of the Moon contains besides 29 Days and 12 Hours, also 44 Minutes, which constitute 11 Days in the space of 30 Years: That therefore the Ratio of a Lunar Year should stand firm accurately, 11 Days have been Intercalated exactly in 30 Years; which Intercalation, least it should be made together, and at once, they distribute it

it into the 30 Years; so that 19 Years are Common, of Days 354, but 11 Years, to wit, 8, 11, 13, 16, 19, 21, 24, 27, 30, are Leap Years of 355 Days.

Hence 'tis evident, that one Period or Circuit is of 10631 Days, which make 360 Changes, every one of which is.

D. 29. H. 12. : 44.

which agrees sufficiently to Astronomick Truth; therefore the beginning of the Turks Year also is constant, and runs out, from Spring to Winter, from Winter to Autumn, hence to Summer, until in the Thirtieth Year it passes again into the Spring time: And 30 of our Years are among the Turks very near 31.

6. A Politick Lunar Year Bound, is that which so observes the course of the Moon, that some of the Years being elapsed in the Period, they make the Lunar equal to the Solar, and they agree exactly.

This Agreement is sought by many; it brings forth various Cycles, in which they would include it.

For hence it is:

1. Of Eight Years, by Cleostratus, Tenedius, Harpalus, Eudoxius, which since it hath been found vitious in success of time, there hath succeeded.

2. The Nine Years of Metonius, in which also since it hath been found vitious, there is received.

3. The Period of Calippus of Seventy Six Years, from which divers forms of Solar and Lunar Years have sprung up among divers People, the Reasons of which many Men, (among whom Joseph Scaliger is chief) have sought with much Labour; but by reason of the obscurity of the same, and the forgetfulness of Ages and of Authors, which have remembered the Differences of times, nevertheless they have left the Truth in doubt.

It

It is not less uncertain what form of Years the Fathers used before and after the Flood, also the Hebrews before and after the Babylonick Captivity, concerning which there are divers Opinions of divers Men: Indeed the Jews at this time use the Bound Lunar Year, as exact as may be; which form they boast to have received from Moses himself, but they are deceived; for that form after the Destruction of the Second Temple, at last was introduced, as hath been demonstrated by many; but the form of Years of the Ancient Hebrews lyes hid in the dark: Therefore he that shall labour earnestly in reducing those Years to our Julian Form, will undertake much Labour, with little or rather no Benefit.

Therefore 'tis sufficient for us at all adventure, to know the beginning of the Years which were used by the Ancients, as well Jews as Gentiles, or how many Months and Days they included in their Year, for from hence, if not exactly, yet we may institute such comparings with our Years, which will not be far from Truth: We will shew hereafter the measure in Years of the Hebrew Bibles, to the Studious in Theology.

CHAP. IV. Of Politick Months.

1. **M**onths, have their Name from Measuring, if we believe Cicero, because they measure the length of the Year, and divide it into parts.

2. Therefore the Reason and Denomination of Politick Months is divers, according to the diversity of Nations, and of People.

3. Our Julian Year is divided into 12 Months, and they are in a Common Year,

| | Days | Sum | | Days | Sum |
|----------|------|------|-----------|------|------|
| January | 31. | 31. | July | 31. | 212. |
| February | 28. | 59. | August | 31. | 243. |
| March | 31. | 90. | September | 30. | 273. |
| April | 30. | 120. | October | 31. | 304. |
| May | 31. | 151. | November | 30. | 334. |
| June | 30. | 181. | December | 31. | 365. |

Ec

Notte

Note, 1. In the Bissextile or Leap Year, February hath one Day more, to wit, 29; whence in such a Year, the Year it self, and the Sum of the rest, is increased by Unity, or one Day.

2. We number the days of these Months in a Natural Order; but in what manner they were numbred among the Romans, appears from the following Verses.

*Prima dies Mensis cujusque est dicta Calenda,
Sex Nonae Maius, Octobet, Julius & Mars.
Quatuor at reliqui, dabit Idus quilibet octo,
Inde dies reliquos omnes dic esse Calendas.*

Quas retro numerans, dices à Mense sequente.

3. Likewise the number of days of our Months are included in the following Verses.

*Thirty days have September,
April, June, and November.
And all the rest have Thirty and One,
Excepting February alone.
Which commonly hath Twenty and Eight,
But in Leap Year one more by Fate.*

4. But what is the reason of the Appellation of Nones, of Ides, and of Calends, may be sought of Grammarians; with which Derivations we will not load this Chronology.

5. The beginning of the first Month, and of the whole Year, happens almost in the middle of Winter, because of the Winter Solstice, the Cardinal point of the Zodiack, (in one of which Nature seems to have placed the beginning of the Year) and also the New Moon) which Caesar would not neglect; but 'tis like) in Old Time, when he constituted the Year from hence, the beginning of our Year was not far from the Calends of January, which nevertheless anticipates the Solstice at this time, because the Year is longer than just, by some days.

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4. The Egyptian Year, hath 12 Months, each of 30 days; the Names of which have been,

- | | | |
|-------------|---------------|-------------|
| 1. Thot. | 5. Tybi. | 9. Pachon. |
| 2. Phaophi. | 6. Mechir. | 10. Payni. |
| 3. Athyr. | 7. Phamenoth. | 11. Epiphi. |
| 4. Chaac. | 8. Pharmuthi. | 12. Mesori. |

In the end of the Month Mesori, they have annexed 5 days to be intercalated or added, to make the Year of 365 Days: And altho' Thot hath always been the first Month of the Year, yet the beginning of it wanders thro' the whole Year, going backwards every 4 Years one Day, not unless after 1460 Years, (which Period is called the great Cynic Year) returning to the Pristine Seat; and 'tis called Cynic, because the beginning of the Year returns to the rising of the Little Dog, a Celestial Star, from whence it first departed.

5. The Loose Lunar Year of the Turks, also hath 12 Months, to which they attribute 30 and 29 Days one after another.

And they are,

- | | | | | | |
|-----------|-----|-------------|-----|-----------|-----|
| Maharram | 30. | Giumadi I. | 30. | Ramadhan | 30. |
| Sepbar. | 29. | Giumadi II. | 29. | Schewal | 29. |
| Rabie I. | 30. | Regeb | 30. | Dhilkabde | 30. |
| Rabie II. | 29. | Sababin | 29. | Dilbaga | 29. |

In the Year of Intercalation this last Month hath 30 Days.

6. These have been the Months of the Hebrews.

- | | | | | | |
|------------|-----|---------------|-----|-------|-----|
| Tisri | 30. | Schebach | 30. | Siwon | 30. |
| Marcheswan | 29. | Adar | 29. | Tamuz | 29. |
| Casten | 30. | Nisan or Abib | 30. | Ab | 30. |
| Thebeth | 29. | Iiar or Ziph | 29. | Elul | 29. |

These 12 Months contain 354 Days, but when they use the Intercalation, they add one Month of 30 days, Ve-adar, as if it were said Adar repeated; and the Year is made of 384 Days.

The Hebrews have also a twofold beginning of their Year; the Ecclesiastick from the Month Nisan, insti-

puted by Moses, the Politick from the Month Tifri, from the Custom of the first Patriarchs, of which that happens in the Spring Time, this in Autumn; and indeed, as 'tis supposed, about both the Equinoxes; and the New Moon next either Equinox is estimated for the First Moon.

7. The Months of the Ancient Athenians and Macedonians have been.

The Attics begin their Year from the Summer Solstice, or from the New Moon next to it.

The Macedonian Months in the beginning were Lunar, computed to the motion of the Moon, of altern Days 30 and 29; but after the Macedonians came to be in the Power of the Romans, and received the Julian Kalendar, they indeed retained the Names of their Months: But as to the beginning and number of Days, they accommodated the same intirely to the Julian Form; and Audinaus is made January, which the rest follow in their Order.

CHAP. V.

Of divers Epochas.

1. *Of the Julian Period.*

1. **T**IME is deduced from some certain and remarkable beginning, which the *Latins* call *Radix*, others *Æra*, (a Word of an unknown or uncertain Derivation.

2. Since there are many of these which are used in divers Forms of Years, and they begin from divers times of the Year; 'tis most useful, that if we would have the sure beginning of each, before all the known, we must have some *Universal Epocha*, by its Years containing all others in it self, and so cited from a great while since, and continued to many Ages; and having a known Form of Year, and running from a sure and stabl beginning of a Year.

3. Since no such can be, (to wit, which also of it self includes the begining of the World) it is law-
ful

ful to devise one, which the Ingenious *Joseph Scaliger* hath done, by introducing the *Julian Period*.

4. It proceeds not from some real beginning, but is *Proleptick* and *Assumptive*, long before the beginning of the Creation of the World; before which, notwithstanding we imagine Years of this kind to have been; that also we may assign in them, to the World it self some sure beginning.

5. And it consists of *Julian Years*, beginning from the *Calends* of *January*, and so is most noted, and most fit to design the beginning of other *Epochas*.

6. 'Tis composed from 3 *Cycles*, (which shall be explained hereafter) of the *Sun* 28 Years, of the *Moon* 19, of *Indiction* 15 Years, multiplyed in themselves, and so contains 7980 Years.

7. And it flows from that imagined or feigned time, in which those 3 *Cycles* of Years begin together; whence it appears, that this *Cycle*, as it began before the Creation of the World, so it will not be compleated before the end of the World.

CHAP. VI.

Of other Epochas, in what Year and Day they begin of the Julian Period.

From the beginning of the Julian Period there are
Years of Julian

| | Period. | Days. |
|--|---------|-------|
| 1. To the Fall of our first Father }
<i>Adam</i> . ————— } | 749. | 301. |
| 2. To the <i>Judaic Æra</i> , which they }
think falsely to be the <i>Epocha</i> }
of the World. ————— } | 952. | 279. |
| 3. To the beginning of the Uni- }
versal Flood. ————— } | 2405. | 156. |
| 4. To the <i>Æra</i> of the <i>Babylonians</i> . } | 2480. | 0. |
| 5. To the Year of the calling of }
<i>Abraham</i> , and of the Divine }
Promise ————— } | 2773½. | 0. |

E e 3

6. To

| | Julian Years | Days |
|--|--------------|------|
| 6. To the going forth of the <i>Israelites</i> from <i>Egypt</i> . ———— } | 3202. | 112. |
| 7. To the Building of <i>Solomon's</i> Temple. ———— } | 3681. | 113. |
| 8. To the first Olympiad of <i>Iphitus</i> . ———— } | 3937. | 203. |
| 9. To the Building of <i>Rome</i> by <i>Cato</i> . ———— } | 3961. | 111. |
| 10. To the beginning of the <i>Æra</i> of <i>Nabonassar</i> , of which the <i>Egyptian</i> Years are from the beginning of wandering. ———— } | 3966. | 56. |
| 11. To the first Destruction of <i>Jerusalem</i> , and the <i>Babylonic</i> Captivity. ———— } | 4108½. | |
| 12. To the beginning of <i>Cyrus</i> . ———— } | 4152½. | |
| 13. To the loosing the <i>Babylonic</i> Captivity. ———— } | 4174½. | |
| 14. To the beginning of the <i>Peloponnesiack</i> War. ———— } | 4282½. | |
| 15. To the Second Year of the base, Born <i>Darius</i> , and the Edict of renewing the <i>Israelites</i> . ———— } | 4293. | |
| 16. To the taking <i>Babylon</i> by <i>Alexander</i> the Great. ———— } | 4383½. | |
| 17. To the Years of the <i>Greek</i> <i>Maccabites</i> . ———— } | 4404½. | |
| 18. To the beginning of the <i>Julian</i> Years, constituted by <i>Julius</i> <i>Cæsar</i> . ———— } | 4668. | |
| 19. To the true day of <i>Christ's</i> Nativity. ———— } | 4706. | 358. |
| 20. To the Epocha of the <i>Christians</i> . ———— } | 4713. | |
| 21. To the 5 Year of <i>Tiberius</i> <i>Cæsar</i> , and the beginning of <i>John</i> <i>Baptists</i> Preaching. ———— } | 4740. | 232. |
| 22. To the Passion of <i>Christ</i> . ———— } | 4746. | 113. |
| 23. To the Second Destruction of <i>Jerusalem</i> . ———— } | 4783. | 243. |
| 24. To the Year of <i>Christian</i> Epocha 1700 compleat. ———— } | 6413. | 0. |

Note.

1. From each of these Epocha's, if you take away 749 Years and 301 Days, you will have the distance of

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of the same from the beginning of the World.
Example.

The Passion of Christ happened in the Year of the Julian Period ————— 4746.

In the compleat Day ————— 113.

How many Julian Years and Days did that come to pass from the beginning of the World? Answer, Take away Years 749, Days 301; and there is left, Years 3996, Days 177, for the distance of Christs Passion from the Creation of the World.

2. If you desire to know the distance of two Epocha's, take away or Subtract the lesser from the greater, the residue shews what you seek?

Example.

The present Year of the Julian Period is 6413.

How much is it distant from the true Birth Day of Christ? Answer, Take away from the given

Years 6412, Days 366.

The Years of Christs Nativity 4706, 358.

There is left ————— 1706, 008.

P A R T III.

of B O O K III.

Concerning Ecclesiastical Time.

C H A P. I.

Of the Definition and Distinction of Ecclesiastical Time.

1. **E**ccllesiastical Time, is that which is observed in the Church, or by the Congregation of the
E c 4 People

People of God, for the publick Exercise of Divine Service.

The Reason of the Invention and Disposition of these times, was called by the Ancients, The Ecclesiastical Accompt.

2 And Ecclesiastical Time may be distinguished into *Judaic* or *Jewish*, used among the *Jews* of old time; and into *Christian*, used at this very day among Christians.

For the whole Race of Mankind, in Holy Scripture is divided into Jews and Gentiles; The Jews from the Hebrews and Patriarchs, Abraham, Isaac and Jacob, or of the Off-spring of Israel; in Old Time they were the Congregation and Elect People of God: But after that they stubbornly rejected Christ, the Saviour of Mankind, they themselves also were rejected by God; and in their place or stead the Gentiles were received, who embraced the Doctrine of Christ, which are now called Christians from Christ.

3: And Ecclesiastical Times, as well *Jewish* as *Christian*, are *Feasts* instituted in both Churches; in which, daily Business being omitted, Divine Worship was publickly Exercised.

CHAP. II.

Of the Weekly Feasts of the Jews.

1. **T**HE Feasts of the *Jews*, some were Weekly, others Monthly, some Annual or Yearly, others Annales.

The Weekly, every Week.

The Monthly, every Month.

The Annual, had recourse to each Year.

The Annales, dured the whole Year.

2. *The Weekly*, or *Septimane*, is the compass of 7 Days, instituted by God himself in the first Creation

tion of the World; with the diversity of Days of Feasts, and of Working Days.

3. *Working Days*, were the six former, in which it was lawful to do Household Affairs and publick Business; but the *Feast* was every Seventh Day, in which they forbore doing any Household Affairs and publick Business, and were employed in the Worship of God and Holy things.

4. Therefore the *Weekly Feasts* of the *Hebrews* was every Seventh Day of the Week, according to the number of the Days of the *Mundane* Creation, in remembrance as well of its Creation finished in the Sixth Day, as of Divine Quiet or Rest taken in the Seventh day; Instituted by God himself publicly in the Exercise of Divine Observance. *Genesis* 2. *Exodus* 20.

1. Hence this Weekly Feast was called of the *Hebrews Sabbath*, that is *Rest*, because in it God rested from the Work of Creation, and commanded; also that Men should rest from their Domestick and Publick Business, that they might be free for Divine Worship.

2. From this *Sabbath* also the other Days of the Week were Named, to wit, the first or next to the Sabbath was called the first of the Sabbath; the rest, the Second, Third, Fourth, Fifth and Sixth of the Sabbath.

3. From the *Hebrews*, this Divine Institution of the Week also spread among the *Gentiles*, differing from the true Church of God, and polluted by the Vanity of *Astrologers*; for the first Hour of the first Day of the Week they attribute to the Sun, as Governor and Ruler; the other remaining, they attribute to the subsequent Planets, of which they are continued in order thro' the 24 Hours of the Natural Day; the Lordship or Rule of the *Moon* falls in in the first Hour of the Second Day; the Lordship of *Mars* in the first of the Third Day, and so on: And hence it came to pass, that the Days of the Week were denominated from the 7 Planets, even as they are

are by many at this day, altho' the Government of them is Abominated as a foolish thing, yea Ungodly, not only by Christians, but also by Skilful and Godly Astrologers; from this reason many Men rather choose to call the Days of the Week with the *Jews*, the First day of the Week, the Second, the Third, &c. than the day of ☉, ☿, ♈, ♉, ♊, ♋, and ♌; or Monday, Tuesday, Wednesday, &c.

C H A P. III.

Of the Feasts of the Jews, of the Monthly, of the Annual, and of the Annals.

1. **T**HE *Monthly Feasts of the Jews*, to wit, those which had recourse to every Month, did fall in with the first day of each Month, in which God prescribed a peculiar Sacrifice to be offered to himself by the *Jews*. *Numb. 28. Verse 11.*

The Sacrifices were called Neomenia, or of New Moons, by the Latins and Greeks; by Words desumed from New Moon, which nevertheless by the Hebrews are not denominated from the Moon; but they simply use the Word Kodeich, which signifies any Innovation.

2. *The Annual Feasts*, were 3, Universal, to which the whole Nation of the *Israelites* ought to gather together into the Holy City, or to the Temple of the Lord; to wit.

1. *The Passover, or the Feast of Easter.* 2. *The Feast of Pentecost, or Whitsuntide; and 3. The Feast of Tabernacles.*

3. *The Passover*, befell on the 14 and 15 of the Month *Nisan*; *Pentecost* was the Fiftieth day after the Passover, and always fell on the 6 day of the Month *Siwan*: *The Feast of Tabernacles* began with the 15 day of *Tisri*.

1. To these Solemn Feasts did adhere other Holy Days, partly going before, partly following after;

con.

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concerning which, as also concerning Sacrifices and other Ceremonies observed in these days, you may consult Holy Writ, with the Commentaries of Divines.

2. And the *Jews* not being content with these Feasts instituted by God, added some other Feasts and Fasting Solemnities, to wit.

1. The days of *Purim*, or of Lots, in which *Haman* sought a fit day for the extinguishing of the *Jews*, being Celebrated the 14 and 15 of the Month *Adar*. *Esther*, Chap. 9.

2. The Feast of gathering together of Wood, Celebrated in the Month *Ab*, of which *Josephus* mentions Book 2. Chap. 17. of the *Jewish Wars*.

3. The Feast of Renovation of the Temple, instituted by *Juda* to the *Maccabires*, on the 25 day of the Month *Casleu*. 1 *Machab*. Chap. 4.

4. The Feast of Satisfaction of the Ark of the *Israelites*, instituted by *Simon Asmon*, on the 23 day of the Month *Isar*. 1 *Maccab*. 13.

And of the Solemnities of Fasting, *Zacharias* speaks Chap. 7. and 8. which were.

1. The Fast of the Besieging of *Jerusalem* by *Nebuchadnezzar*, Celebrated the 10 day of *Thebet*.

2. The Fast of occupying the City, Celebrated the 9 of *Tamuz*.

3. The Fast of the Desolate City, on the 9 of *Abib*.

4. The Fast of the Murder of *Gedaliah*, on the 3 day of *Tyri*; in which, under the *Babylonick* Captivity, came the Solemnity of *Hesters* Fasting, which was on the 13 of the Month *Adar*. *Esth*. Chap. 4. &c.

5. The Annal Feasts of the *Jews* were every Seventh Year, a Sabbathic; and 7 times 7, that is, every Forty Ninth Year a Jubilee.

Concerning these, see *Levit*. 25. *Deut*. 15 and 31. where is Treated of the Solemnities, Priviledges, and Immunities of these Years.

Some number them in the *Civil Years*, which nevertheless are either unknown or uncertain what kind of Year the *Hebrews* used before and after *Moses*; which

which ignorance or uncertainty makes these Years also ambiguous; therefore some fly here for help to the Mystical Years, which they deduce from the Week of Days of the Creation, as a *Radix* in Arithmetical and Geometrical Progression, in this manner: As God from 7 Days made the Week of Days, so they from 7 such Weeks, or 49 Days make the Month, and from 7 Months or 343 Days the Mystical Year; of which every Seventh is *Sabbathick*, and every Nine and Fortieth is to them a *Jubilee*.

CHAP. IV.

Of the Feasts of Christians.

1. **T**HE Feasts of Christians are twofold, *Weekly* and *Anniversary*.

Those have recourse to every Week; these to every Year.

2. *The Weekly Feasts*, are the first day of every Week, which is called the Lords Day, for that 'tis as it were a day Consecrated to our Lord God.

This Custom issues from Divine Institution, in which one Day of every Week was designed to Divine Worship; And as among the Jews the Seventh Day of the Week was a Feast, and Holy; so among the Christians, from the Authority of the Apostles of Christ, the First Day of the Week was elected; and so the Sabbath was changed from the Seventh Day of the Week into the First.

3. *Anniversary Feasts*, some are *Immovable*, which always retain the same day of the Month in the *Julian Year*, altho' they happen on different Days of the Week; and others *Movable*, which happen in other Years, on other Days of *Julian Months*, altho' they retain constantly the day of the Week.

The

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The immovable Feasts among us are, And are Celebrated.

1. The Feast of Christs Circumcision. January 1.
2. Epiphany, or Twelfth Day. January 6.
3. The Conversion of Paul. January 25.
4. The Purification of Mary. February 2.
5. Of Matthias the Apostle. February 25.
6. Of the Annunciation of Mary. March 25.
7. Of St. George. April 23.
8. Of Philip and James, Apostles. May 1.
9. Of John Baptist. June 24.
10. Of the Apostles Peter and Paul. June 29.
11. Of James the Apostle. July 25.
12. Of St. Lawrence. August 10.
13. Of Bartholomew the Apostle. August 24.
14. Of Mathew the Apostle. Septemb. 21.
15. Of Michael the Archangel. Septemb. 29.
16. Of St. Gallus. October 16.
17. Of Luke the Evangelist. October 18.
18. Of Simon and Jude the Apostles. October 28.
19. Of St. Martin. Novemb. 11.
20. Of Andrew the Apostle. Novemb. 30.
21. Of St. Luke. Decemb. 13.
22. Of Thomas the Apostle. Decemb. 21.
23. The Nativity of Christ. Decemb. 25.
24. Of Stephen the first Martyr. Decemb. 26.
25. Of John the Evangelist. Decemb. 27.
26. Of Innocent Children. Decemb. 28.

5. The Movable Feasts are:

1. The Passover of Christians, or the Feast of Christs Resurrection; on which depends.
2. The Feast of Ascension, the 40 day after the Passover.
3. The Feast of Pentecost, the 50 day after the Passover.

4. The

4. The Feast of the Holy Trinity, the 8 day after the Celebrating of Pentecost.

5. The differences of all the Lords Days after Trinity, designed by their Numbers 1, 2, &c.

(And for the finding these Feasts, there are devised some Cycles of Years or Periods; concerning which also we shall now Treat.)

CHAP. V.

Of the Cycles of Julian Years.

1. **C**ycle, in this place is the Periodical Collection of many *Julian* Years, devised for this end, that Ecclesiastical Times and others from a certain time, returning in the Orb, may more exactly be designed.

2. Of these Cycles, the 4 Principal are,

- | | |
|----------------------------------|-----------------------------------|
| 1. <i>The Cycle of the Sun.</i> | 3. <i>The Cycle of Indiction.</i> |
| 2. <i>The Cycle of the Moon.</i> | 4. <i>The Period of Julian.</i> |

3. *The Cycle of the Sun*, is a Revolution of 28 Years, in which the Letters of the Day of the Sun, or of the Lords Day, returns by mutation in the Orb.

4. *The Dominical Letter*, is one of the 7 Letters of the Alphabet *A B C D E F G*, shewing every Year the first day of the Week, or the day of the Sun, or the Lords Day.

For 'tis wont to note which you will of the Week Days of the whole Year, with the seven first Letters of the Alphabet; so that the first day of the Year hath the Letter *A*, the Second *B*, the Third *C*, and so on, even to 8, which resumes the Letter *A*.

Therefore

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Therefore since the Common Julian Year hath 52 Weeks and 1 day, and the Bissextile or Leap Year 52 Weeks and 2 Days, and so the first day of the following Year falls in the Week of Days next from the former, and also in the Bissextile leaps over or beyond one day; for Example, If any one begin the common Year from the Lords Day or Sunday, or the first day of the Week, and it hath 52 Weeks and 1 day, the first day of the following Year will fall on the Second day of the Week, or Monday the day of the Moon; and if this be supposed to be Bissextile, since it will have 52 Weeks and 2 Days, (the following Year will begin not on Monday, but on Tuesday, or the day of Mars) Therefore also it must needs be that the *Dominical* Letter must alter, nor but after 28 Years to return to the first Order.

If no Intercalation should be made after Seven Years, the first *Dominical* Letter will repeat its place and order: But because every fourth Year, by inserting the Bissextile Day, the Series or Order is interrupted, and the intercalated day acquires neither the number nor the Letter distinct from the former; therefore the number of 7 Days must be Multiplied by the number 4 of the Bissextile, and so to the Cycle of the Sun, or to the variation of the *Dominical* Letter, is attributed 28 Years: Moreover also the *Dominical* Letter of the Bissextile Year is double, the former of which shews the Lords Day, even to the intercalated Day, or the 29 of February, and the other shews the day of the Sun, or the Lords day, thro' the remaining part of the Year.

Cycle

| | | | | | | | | | | |
|------------------------------|----|----|----|----|----|----|----|----|----|----|
| Cycle ☉ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Dom. Let. | G | E | D | C | B | G | F | E | D | B |
| | F | | | | A | | | | C | |
| Week days
or Cal. of Jan. | 2 | 4 | 5 | 6 | 7 | 2 | 3 | 4 | 5 | 7 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | |
| A | G | F | D | C | B | A | F | E | | |
| | | E | | | | G | | D | | |
| 1 | 2 | 3 | 5 | 6 | 7 | 1 | 3 | 4 | 5 | |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | | | |
| C | A | G | F | E | C | B | A | | | |
| B | | | | D | | | | | | |
| 6 | 1 | 2 | 3 | 4 | 6 | 7 | 1 | | | |

In this little Table, which begins in the Year of the Christian Epochs 1644, you have in the first place the Series of numbers of the Cycle of the Sun: Then you have subscribed the Dominical Letter of that Year. Lastly, the days of the Week, or the Calends of *January*, from which every Year begins; to which the Letter *A* is appointed.

5. The Cycle of the Moon, is the Revolution of 19 Years, in which the Ancients thought the Changes of the Moon, the Indexes of the Paschal Feast, did return to their first beginning, and was called the Cycle of 19, as also the Golden number; either from its use, in like manner dear as Gold, which in Old Time they performed in shewing New Moons; or because it was exposed to the view of all Men in Golden Letters.

Therefore because the Ancients did affix the Feast of the Passover to the Full Moon, which followed the Vernal Equinox, also because that Christ Celebrated the Passover in the Full Moon of the same Vernal Equinox, and Suffered and rose again; from this Cycle they would learn the New and Full Moon, and consequently also the Feast of the Passover: But their expectation was frustra-

ted,

ted, for the Changes of the Moon did not return precisely to the same beginning in 19 Years, but is testified by Observations to anticipate somewhat; which defect from the time of assuming of this Cycle, increased into some days, and so the Coelestial Changes were of one day, but the Changes according to the Cycle, or of the *Ecclesiasticks*, were of another; for altho' the Cycle of the Moon at this time doth not any more design the Changes, yet from the Ancient Institution, we who retain the *Julian* Calendar, take out the Feast of the Passover, or of *Easter*, from it, no otherwise than as if it did shew the Changes of the Moon exactly: And this according to the Cannons, which the Fathers of the *Nicene* Council Assembled, by the command of *Cesar Constantine*, in the Year of the Christian Epoch 325; albeit they do not agree with the Heaven, yet they have appointed it in this Sense.

1. The 21 day of March is accounted for the day of the Equinox.

2. The Full Moon happening on the 21 of March, or the next following, is accounted for the Full Moon of the Paschal Feast.

3. The Lord's Day next following this Full Moon, is the day of the Feast of the Passover.

4. If nevertheless the 14 day of the Moon, or the Full Moon, happen on that Lord's Day, the Paschal Feast must be Celebrated 8 days after, to wit, least it should agree with the Passover of the Jews.

6. The Paschal Terms, and the Epact also, belong to the Cycle of the Moon.

7. The Paschal Terms, are the number of days of March and April, with the ascribed Letters, shewing by the Golden Number and the Dominical Letter, the Fourteenth Ecclesiastical change of the Moon, and the following Lord's Day the Feast of Easter.

These Terms consist between the 21 of March and the 18 of April inclusive, in which the Full Moon never neither goes before nor follows the Paschal Feast; even as also the Feast it self neither antecedes

| | | | | | | | | | | |
|------------------------------|----|----|----|----|----|----|----|----|----|----|
| Cycle ☉ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Dom. Let. | G | E | D | C | B | G | F | E | D | B |
| | F | | | | A | | | | C | |
| Week days
or Cal. of Jan. | 2 | 4 | 5 | 6 | 7 | 2 | 3 | 4 | 5 | 7 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | |
| A | G | F | D | C | B | A | F | E | | |
| | | E | | | | G | | | D | |
| 1 | 2 | 3 | 5 | 6 | 7 | 1 | 3 | 4 | 5 | |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | | | |
| C | A | G | F | E | C | B | A | | | |
| B | | | | D | | | | | | |
| 6 | 1 | 2 | 3 | 4 | 6 | 7 | 1 | | | |

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These Terms consist between the 21 of March and the 18 of April inclusive, in which the Full Moon never neither goes before nor follows the Paschal Feast; even as also the Feast it self neither antecedes

be 22 of March, nor follows the 25 of April, according to the Verse.

*The Feast of Easter preceeds not the 22 of March ever,
Nor doth it follow the 25 of April never.*

| The Cycle D | 1. | 2. | 3. | 4. |
|--------------------|----------|----------|----------|---------|
| The Paschal Terms. | A. 5. d | M. 25. g | A. 13. e | A. 2. a |
| 5. | 6. | 7. | 8. | 9. |
| M. 22. d. | A. 10. b | M. 30. e | A. 18. c | A. 7. f |
| 11. | 12. | 13. | 14. | 15. |
| A. 15. g | A. 4. c | M. 14. f | A. 12. d | A. 1. g |
| 17. | 18. | 19. | | |
| A. 9. a | M. 29. d | A. 17. b | | |

This Table begins in the Year of the Christian *Epocha* 1653, and contains in the first place the Cycle of the Moon in its order, next the Paschal Terms in their Months, *M. March*, *A. April*; with their Letters added to the days of these Months, in which the Paschal Terms fall: Therefore the *Dominical* Letter being given, and the Golden Number, with the subscribed Paschal Term, if you number from the day of the Month, and its Letter, to the next Lords Day, you will have the Feast of *Easter*.

8. The *Epaet*, are days by which the Solar Year exceeds the Lunar.

1. For a *Solar* Year hath days 365, but the *Lunar* only days 354, therefore every Year gathers together 11 days; and when the collected or gathered number arises to 30, the Intercalation is instituted of one Month, and makes the Leap Year of 13 Months, reserving the residue to be added to the *Epaet* of the following Year.

2. And these *Epaets* are invented by the Moderns, to shew the difference between the *Solar* and *Lunar* Year.

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Year, and the New and Full Moon, and they shew the Ages of the Moon more exactly than the Golden Number; although neither these, unless Correction be used, can perform that beyond 300 Years.

3. And the Epacts proceed with the Golden Number in Old *Juban* Calendars in this manner, from beginning both from the Cycle with the Year 1653.

| | | | | | | | |
|--------------------|-----|-----|-----|-----|-----|-----|-----|
| The Golden Number. | 1. | 2. | 3. | 4. | 5. | 6. | 7. |
| The Epact. | 11. | 22. | 3. | 14. | 25. | 16. | 17. |
| 8. | 9. | 10. | 11. | 12. | 13. | 14. | 15. |
| 16. | 17. | 18. | 19. | 20. | 21. | 22. | 23. |
| 24. | 25. | 26. | 27. | 28. | 29. | 30. | 31. |

4. These Epacts also are found from the Golden Number, in this manner.

1. The Golden Number being Multiplied by 11,

2. The Product divide by 30, the residue constitutes the Epacts.

5. *The use of Epacts among others, is also this.*

1. *The Golden Number of any Year being given, the Epacts of that Golden Number Subtract from 30, the residue shews the day of the first New Moon of that Year: Example, in the Year 1652.*

The Golden Number is 19, the Epact 29, Subtract this from 30, it leaves 1; therefore the 1 of *January* this Year happens the first New Moon of this Year. Another Example, in the following Year 1653, the Golden Number is one, the Epact 11.

This Subtracted from 30, leaves 19; therefore the 19 of *January* 1653, happens the first New Moon of this Year.

2. *To this first New Moon if you add alternately 29 and 30 days, you will acquire the New Moons of the other Months, Subtracting them from the days of the antecedent Month, if the Sum exceed them.*

F f 2

Example.

Example.

To the 19 of *January* add 29, they make 48; Subtract 31, the days of *January*, there is left 17 of *February*, in which the Second *New Moon* happens.

9. *The Cycle of Indiction*, is the Revolution of 15 Years, in which also they shewed the time and the order of paying of Tribute to the *Roman Empire* of Old Time.

1. What the *Ratio* or Reason of the Cycle of Indiction is, the Opinions of Authors indeed are various; that which we shall assign, is made common, and agrees with the *German Appellation*; but whatsoever the reason of Appellation is, the thing being advanced, the Name of Cycle of Indiction continues, and the Chronological Use of it, in which the *Julian Years* may be distinguished some from others

2. And this Cycle is received instead of an *Epocha*, being restored by *Constantine the Great*, Master of the Peace of the Church, in the Year of Christ 312; and 'tis twofold, from *Cesar*, or the Emperor, and from Books containing matters of Religion: That begins the 24 of *September*, this with the 25 of *December* following.

3. The new beginning of this Cycle was made in the Year of the *Christian Epoch* 1648.

10. The *Julian Period* is the Revolution of 7980 Years, in which only three of the said Cycles of the Sun, of the Moon, and of Indiction, return to the Orb, and begin together.

Therefore in this Cycle are comprehended the 3 Superiors, which arises from drawing 28 into 19, and the Product of the Sum 512 into 15: And this Period is introduced by the Excellent *Joseph Scaliger*, being *Proleptick* indeed, and by Fiction extending it self beyond the beginning and end of the World;
and

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and hath excellent Uses, and compendiously contains what the three Superiors promise.

And to the Years of the Christian *Epocha* given, if you add the intire Years of the *Julian* Period 4713, you will have the Year of the *Julian* Period current in the given Christian *Epocha*. Example, the Year present 1700, How much is the *Julian* Period? *Answer*, 4713 being added, makes 6413, which is the Year of the *Julian* Period in this current Year passing away, of which more shall be said in the Problems.

The end of the Third Book of Chronological Astronomy.

T H E
P R A C T I C E *and* U S E
O F
A S T R O N O M Y.

SINCE the End of Astronomy is, not only to know the Motion and Site, or Position of the Stars in General, but also to design and compute the same to any given Time, and that they may be accommodated to the Practice and Use of Humane Life; it pleaseth to shew this in three parts of Astronomy, to Practice and Use, by subjoining some easie Problems, which follow.

• F f 3

I. Spherical

I. Spherical Problems.

P R O B L E M I.

To find a Meridian Line, &c.

In a Right and Oblique Sphere the Meridian Line is a right Line, passing by the Axis of the Equator, or of the World, through the Centre of the Earth, from the North Pole to the South Pole, resembled in a Plane; the understanding of which conduces very much to the observing of the Cœlestial Motions, we will shew several ways of inquiring the motion of it.

1. *By the Compass or Dial of Travellers.*

For it being turned to and fro, until the movable little Tongue or Needle, touched by magnetick vertue, answer exactly the immovable little Tongue depicted in the bottom of the Compass, then you will have what you desire; for this little Line it self, or any Parallel of it, will be the Meridian Line sought.

2. *By the Compasses and Rule, by help of an erect Gnomon.*

1. Fix a Gnomon or Style in a Place, parallel to the Horizon, erect it perpendicularly at Right Angles, and note at some time before Noon, the Sun Shining, the extream point of the Shadow, which the Gnomon casts from it self.

2. The Gnomon being taken away from the point of its Station, through the extream point of Shadow noted just before, draw a Circle.

3. Restore the Gnomon in its place, keeping precisely the same length of it and the rectitude which it had before, expect until the Shadow of the Gnomon (in the time before Noon decreasing, after Noon

Noon increasing again) touch in the described Circle again, but in another place; which point of Contract note.

4. If afterwards you bisection the Arch of the Circle between the two noted points of the Shadow, and draw a right Line from the Center thro' the point of Section, it will be the Meridian Line sought.

N O T E.

The Shadow which the Gnomon casts from it self being perpendicularly erected, or hung perpendicularly under that moment of the Hour 12 of the Meridian, shews also the Meridian Line.

1. A Meridian Line divided by another, and cut crosswise at Right Angles, that part which beholds the Sun is South, the opposite is the North; the Left Hand part of the other Line is the East, that to the Right Hand shews the West; supposing you look towards the South.

2. The Meridian Line being drawn, the same shews the Coelestial Meridian of any place.

3. The Shadow of a Style perpendicularly erected upon a Meridian Line, or of a Perpendicular suspended upon the same, agreeing exactly to the Meridian Line, it shews the point it self of the Meridian, or the moment of the Hour of Twelve.

4. The Compass being placed according to the Meridian Line, and the Sun shining, shews the hour of the day, although it should want the magnetick Needle or little Tongue.

5. A Star seen by the extrems of two Gnomons, perpendicularly erected upon a Meridian Line, is understood to consist in the Meridian it self.

P R O B L E M II.

To find the Altitude of the Sun, of the Moon, and of other Stars above the Horizon.

F f 4

1. In

1. *In the Cælestial Globe.*

1. Conform the Globe to the Position or Being of Heaven, in the given time.

2. Put down the Quadrant of the Vertical Circle annexed to the Meridian, thro' the known place of the Sun, or any Star given; the Arch of the Quadrant intercepted between the Horizon and the place of the Sun, or the Center of the Star, will shew the Altitude of the same to the time proposed.

2. *In the Heaven it self.*

For to do this, it behooves the Astronomer to be furnished with an Astronomical Quadrant, divided into Degrees, fitted with Sights and a Perpendicular: If the Center of this be placed opposite to the Sun or any Star, and so long raised or depressed while the Sun Beams pass thro' both the Sights, the third of the Perpendicular hanging freely, or any Star seen through these holes, or at least may be seen through the little extreams of the Sights, the third will shew in the Limb of the Quadrant the degree of Altitude, which the Sun or any Star obtains in that moment of Observation in the Vertical Circle.

P R O B L E M III.

To take the greatest Altitude that the Sun or any Star rising and setting, can have in the given day.

This greatest Altitude is, when the Sun or Star consists in the Meridian Circle, in which moment it begins again to decrease, as before it increased.

1. *In the Globe therefore.*

Bring under the place of the Sun or the given Star, to the Meridian of thy place; then the Arch it self
of

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of the Meridian, intercepted between the Horizon and the place of the Sun, or the Center of the Star, will shew the greatest Altitude of that day.

2. But in the Heaven.

You will have the greatest Altitude of the Sun, or of any Star mounting the Horizon, if you take it when the Sun or the Star is in the Meridian Circle it self: or if this Moment be dubious, when they consist a little beyond and on this side the Meridian, take the Altitude sometimes increasing and decreasing; which greatest Altitude you will then discover for the Meridian, and back again, you will have the Meridian for the greatest Altitude of that day.

In an Oblique Sphere the Stars that set not, do pass the Meridian twice in one Diurnal Revolution, both of this side and beyond the Pole, having there the greatest, here the least Meridian Altitude, which are taken after the same manner; and this may be done twice in one long Winter Night.

P R O B L E M IV.

To find the greatest and least Altitude which the Sun can have at any time in the whole Year in the Meridian.

This happens, the ☉ being in ♊ ♋, and ♎ ♏.

In the Globe therefore.

These places bring under the Meridian of the place, in which you may number the greatest and the least Altitude from the Horizon.

But under the Heaven.

About the middle of Summer, to wit, from the 10 to the 14 of June, likewise about mid Winter, from the 10 to the 14 of December, (between which days the Sun is highest and lowest in the Meridian) about Noon take oftentimes the Altitude of the Sun,
increasing

increasing and decreasing, and you will observe that to be the greatest which is in the end of increasing, and the least in the end of decreasing; for the greatest and least Meridian Altitude to be had of the Sun, that it can acquire in the Heaven.

In this manner you will find at London.

The greatest Altitude of the Sun $61^{\circ} : 59'$
 The least ————— $14 : 57$

P R O B L E M V.

From the Altitude of the Sun given, or of a Star, or of any point of Heaven, to know the distance of the same from our Vertex. Fig. 140.

The Altitude $h i$ being found, take it away from $h b$, or 90° : The residue $b i$ is the distance of the given point from the Vertex.

So if from 90° Or ————— $89 : 60$
 You Subtract the greatest Altitude of $\odot$ ——— $61 : 59$
 There is left ————— $28 : 01$

So much $\odot$ is distant from our Vertex, yea, in Summer, when nearest to us; which distance consists of English Miles, (allowing with Mr. Norwood, a Degree, to contain Miles $69 \frac{1}{4}$) ——— $1946 \overline{) 964}$

But in Winter the furthest distance is $14 : 57$
 Which Subtracted from 90° Or ————— $89 : 60$
 It leaves ————— $75 : 03$
 Which is in English Miles ————— $5203 \overline{) 725}$

And yet it seem: to be distant scarce an Hundred Paces.

P R O B L E M VI.

To compute the Altitude of the Equator, from the greatest and least Meridian Altitude of the Sun found. Fig. 183.

In

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In this Scheme *A B C* is the Meridian, *D* the Center of the World, *A D C* the Horizontal Line; *B D E* the Equinoctial, its Pole *L*; *F D G* the Ecliptick; *H G* the Tropick of Cancer; *F I* the Tropick of Capricorn; *H A* the greatest, *F A* the least Meridian Altitude of the Sun; *B A* the Altitude of the Equator; *H F* the distance of the Tropicks: Therefore

| | | |
|--|-----------|---------|
| 1. From the greatest Altitude ——— | <i>HA</i> | 61 : 59 |
| Subtract the least ——— | <i>FA</i> | 14 : 57 |
| There is left ——— | <i>HF</i> | 47 : 02 |
| 2. This Remainder divide in the middle in <i>B</i> , | | 23 : 31 |
| Add this half to the least Altitude <i>FA</i> , | | 14 : 57 |
| And it makes ——— | <i>BA</i> | 38 : 28 |

Which is the Altitude of the Equator, under the Elevation of the Pole of London.

P R O B L E M VII.

To find the Altitude of the Pole for any place: There are divers ways, but Learners may note these.

1. From the Altitude of the Equator, being found.

| | |
|--|---------|
| From <i>AKA</i> , the Quadrant of a Circle, 90°: Or | 89 : 60 |
| Subtract <i>BA</i> , the Altitude of the Equator ——— | 38 : 28 |
| And there will be left <i>KB</i> , the Latitude of Lond. | 51 : 32 |

The same quantity as the Altitude of the Pole at London sought, *L C*.

2. From the twofold Meridian Altitude of any Star that sets not.

For Example, of the last in the Tail of the great Bear.

| | |
|--|---------|
| Suppose its greatest Altitude to be <i>MC</i> | 88 : 54 |
| Its least Altitude <i>NC</i> ——— | 8 : 14 |
| The Difference <i>M N</i> ——— | 80 : 40 |
| To the half of this <i>N L</i> ——— | 40 : 20 |
| Add the least Altitude <i>NC</i> ——— | 8 : 14 |
| It makes the Altitude of the Pole sought <i>LC</i> — | 48 : 34 |

P R O B L E M VIII.

From the greatest and least Meridian Altitude of the Sun found, to collect the distance of the Tropicks, the

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the greatest Obliquity of the Ecliptick; and the greatest Declination of the Sun.

For these 3 depend mutually on one another, and may be found in this manner.

| | | | | |
|--|----|----|---|----|
| From the greatest Meridian Altitude of ☉ | HA | 61 | : | 59 |
| Subtract the Least | FA | 14 | : | 57 |
| It leaves the distance of the Tropicks | HF | 47 | : | 02 |
| The half of this | BF | 23 | : | 31 |

Is the greatest Obliquity of the Ecliptick, and the greatest Declination of ☉.

P R O B L E M IX.

To observe the Declination of the Sun, and of all the Stars.

In the Globe.

Bring the place of the Sun, or of any Star to the Meridian, in the degrees of which the Declination of the Sun, or of the Star from the Equator, whether it be South or North, is exposed to the Eyes.

In the Heaven.

1. Observe the Meridian Altitude of the Sun or Star.
2. Subtract it from the Altitude of the Equator, if it be greater, or this being lesser, Subtract it from that, and you will have the Declination sought.

Example.

| | | | |
|--|----|---|----|
| The Meridian Altitude of the Sun is | 49 | : | 15 |
| From this Subtract the Altitude of the Equator | 38 | : | 28 |
| It leaves the Declination of the Sun | 10 | : | 47 |

Note.

If any Fixt Star be so far North, that it exceed our Zenith by its Meridian Altitude, and so cannot be taken from the Meridian, do thus.

1. Take

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1. Take the Altitude of it from the North.
2. Take away this from a Semicircle, and you will have the Meridional Altitude of the Star from the Horizon.
3. From this Subtract the Altitude of the Equator, it leaves the North Declination of that Star.

Example, Fig. 183.

Lets find the Northern Altitude CM, of the
Right Shoulder of Cephæus M. ————— 77 : 40
Therefore from the Semicircle CMA 180° Or 179 : 60
Take away the Altitude North CM ————— 77 : 40
It leaves MA the Altitude South ————— 102 : 20
From this take away the Altitude of the Equator BA 38 : 28
It leaves MB ————— 63 : 52
The Declination of the Star North.

P R O B L E M X.

The greatest Obliquity of the Ecliptick being given 23° : 31', to find the Declinations of each degree of the Ecliptick, by Trigonometrical Calculation.

Fig. 184. In this Scheme d f g h is the Solstitial Colure, and the same is the whole Meridian Circle : The Equator e a g, its Pole f; d a h the Ecliptick; f c b the Circle of Declination, or any Meridian; a c an Arch of the Ecliptick, of which point C the Declination c b is sought; and let c b be 17° of $\odot$, and the Arch a c 47°.

Make it then.

| | |
|--|--|
| <i>As a d the whole Sine</i>
100000.
<i>So the Sine of a c 47°</i>
73135. | <i>Is d e the Sine of the greatest Obliquity 23 : 31</i>
39902.
<i>Is b c the Sine of Declination sought.</i>
29182 whose Arch is 16 : 58 |
|--|--|

N O T E.

By this Artifice is made the Table of Declination of every part of the Ecliptick, from which, without much Labour, the Declination of every degree of the Ecliptick

Ecliptick may be had: The Method which is used, not only in the Table of Declination, but in many others, we will briefly shew.

1. Seek the given Sign, either in the Front or in the Foot of the Table; but the degree of the Sign in the side, either in the left, if you have found the Sign in the Front, or to the Right Hand, if in the Foot of the Table.

2. Which being done, the Declination sought will appear in the Angle, which is common to the Sign and Degrees given.

Of finding the part Proportional.

If to any degree of the Ecliptick given, there do also Scruples adhere, as there doth for the most part, the part Proportional must be sought due to the adhering Scruples, in this manner.

1. Take the difference between the Degree given, and the next following, which difference is always 1° or $60'$, to be put in the first place in the Rule of Proportion.

2. Take also the difference between the Declinations of both these Degrees found in the Table; which difference will obtain the second place in the Rule of Proportion.

3. In the third place put the Scruples adhering to the given Degrees, and work after the usual manner.

4. The Quotient either add to the Declination of the whole Degree given, if it increase in the following; or Subtract it from the same if it decrease, and you will have the Declination sought to the given Degrees and Scruples.

Example

Example I.

The Declination is sought.

To $12 : 15$ of γ .

The Declin. of $13 : 0$ of γ is ——— $15 : 46$

The Declin. of $12 : 0$ of γ is ——— $15 : 27$

The Diff. of $1 : 0$ or $60'$ is ——— 19

Say then

If $60'$ give $19'$, what gives $15'$? Answer, $4' : 45''$.

Therefore since the following Declinations increase

To the lesser Declination ——— $15^\circ : 27' : 0''$

I add the Proportional part ——— $04 : 45$

And it makes the Declination sought $15 : 31 : 45$

For the given $12^\circ : 15'$ of γ .

Example II.

The Declination is sought to $15^\circ : 20'$ of κ .

The Declination of 15° of κ is — $5^\circ : 56'$.

The Declination of 16 of κ is — $5 : 52$.

The Difference of 1° or $60'$ is — $0 : 24$.

Say then

If $60'$ give $24'$, what doth $20'$ give? Answer, $8'$.

Then because the following Declinations decrease

Therefore from the Declination of ——— 15° of κ $5^\circ : 56'$

I Subtract the part Proportional ——— 08

And there is left ——— $5^\circ : 48'$ the Declination

Of ——— $15 : 20$ of κ

This Method of finding the part Proportional, is

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 to be diligently noted and practised, since in all Tables almost, the like Operation is required.

P R O B L E M X I.

From the Suns Declination given, to find its true place in the Ecliptick.

If nevertheless the part of the Year be also known, or in which Signs beyond or on this side the Cardinal points the Sun is conversant, since before and after these points the Declinations are equal.

1. *In the Globe.*

Under the number of the Declination given, place the Ecliptick in the Meridian from the Equator, towards the North or South, taken on that part, in which the Sun is supposed to be conversant; the agreement of which, with the number of Declination, will shew the true place of the Sun.

2. *By Trigonometrical Calculation.*

Let it be done in the former Scheme 184.

| | | |
|---|---|--------------------------|
| <i>As de the Sine of the</i> | <i>To the Sine of cb the declin.</i> | <i>So the whole Sine</i> |
| <i>greatest Declination given, for Exam. of 6°: 10'</i> | <i>or Radius d a</i> | |
| <i>23°: 31. 39902.</i> | <i>10742.</i> | <i>100000.</i> |
| <i>To the Sine of the Arch</i> | | |
| <i>sought c a</i> | | |
| <i>26920.</i> | <i>which arch in the Canon of Sines is found 15°: 37'</i> | |

And that you may know whence, and how this Arch found is to be numbred, note what follows.

1. If

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1. If the Sun be conversant in the Spring or Autumnal Quadrant, then the Arch found shews the place of the Sun, numbred from the beginning of γ or α .

2. But if she be in the Summer or Winter Quadrant, then the Arch found must first be taken away from 90° : And then at last indeed it shews the Suns place from the next Solstice, either to the Summer or Winter, that is, either from α or γ ; numbred according to the order of the Signs.

3. *From the Table of Declination of the Ecliptick.*

The finding of the Suns place is not done by the Lateral, but by the Areal entrance in the Table.

For the Declination given is to be taken from the Area of the Table, but the degree of the Ecliptick responding or answering to it, is to be sought in the Side.

In this manner.

Given, for Example, the Declination of the Sun — $5^\circ : 55'$.

Which being sought in the Table of the Suns Declination, or of parts of the Ecliptick, I find to answer it in the left side 15° , either of γ or α ; and in the right side 15° , either of ϖ or κ .

Therefore 15° of the Ecliptick is certain, but which of these 4 Signs the Sun possesses, must be estimated from the Ratio of Time.

Of finding the part Proportional.

For if the given Declination of the Sun be not precisely found in the Area of the Table, the part Proportional is to be sought again.

G g

In

In this manner.

*Given the Declination of the Sun $6^{\circ} : 10'$
What degree of the Ecliptick answers to this Declination?*

In the Table I find.

1. The Declination next greater— $6^{\circ} : 19'$
The next lesser— $5 : 56$
The difference is— 23 which is

The first number in the Rule of Proportion.

2. The second place obtains 1° or— $60'$
3. For the third place or number, we must seek the difference between the Declination given $6^{\circ} : 10'$
And the next lesser— $5 : 56$
Which difference is— 14

Say then:

If $23'$ give $60'$, what doth $14'$ give? *Answer,*
 $36' : 12''$.

Therefore I add this number found to the degree of Declination of the Ecliptick, next lesser to our given, which in this place is 15° . so it will make

Of the Declination given— $6^{\circ} : 10' : 0''$

The responding degree of the Ecliptick $15 : 36 : 12$

But of which Quadrant or Sign this Degree is, must be estimated from the time, as aforesaid.

(And because we have spoken before of Lateral Entrance in the Table of Declination, and of finding the part Proportional, the same also must be understood concerning this Areal entrance; to wit, we must proceed in the like manner also in many other Tables: Therefore Studying brevity, we shall not add

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add Examples hereafter, but leave this to the Studious in Astronomy.)

P R O B L E M XII.

From the Altitude and Declination of the Sun on the Meridian, in a given place, to transfer the same to any other Hour, before or after the Meridian or Noon time.

After Noon, add, before Noon, subtract to or from the place of the Sun, for every Hour $2' : 28''$, for every Minute $2'' : 28'''$, and you will obtain the true place of the Sun, very near.

P R O B L E M XIII.

From the Altitude of the Equator given, and from the declination of the Sun, or of any part of the Ecliptick, to produce the Meridian Altitude of the same.

Add the Declination of the parts of the Ecliptick, or of the Sun, conversant in them in Northern Signs, to the Altitude of the Equator; in Southern Signs subtract it, and you will produce the Meridian Altitudes of all the parts.

Example.

What is the Meridian Altitude of the Sun, if she be conversant in 10° of Aries?

Answer, Since the declination of this degree is $3^\circ : 58'$

Add the Altitude of the Equator ————— $38 : 28$

And you will have the Meridian Altitude ————— $42 : 26$

Another Example.

How much is the Meridian Altitude of the Sun, being in $20^\circ : 0'$ of γ ?

Answer, when the Altitude of the Equator is ————— $38 : 28$

And the Declination of the Degree given ————— $22 : 2$

This Subtracted from that, leaves ————— $16 : 26$

The Meridian Altitude sought.

G g 2

Problem

P R O B L E M XIV.

To find by the Quadrant from Heaven, the distance of two Stars in any great Circle, described from the Center of the Earth, or from the Eye of the Spectator.

I.

1. If both the Stars are in one Vertical Circle, observe the Altitude of both.

2. Subtract the lesser from the greater, the residue is the distance of those two Stars in degrees of a great Circle.

II.

But if they consist not in the same Vertical, yet are so posited, that by the Superficial Plane of the Quadrant you may behold both together.

1. Apply the Sights of the Quadrant to the side, and by them behold one of the Stars, of which you desire to know the distance.

2. The Quadrant remaining unmoved, direct the Sights also to the other Star; which degree of the Quadrant then cut, will shew you the distance of those two Stars.

P R O B L E M XV.

To find the Eastern and Western Amplitude of the Sun, and of the Stars.

1. In the Globe.

Bring the Eastern and Western place of the Sun, or of any Star, to the Eastern or Western Horizon, and the Arch of the Horizon intercepted between the rising or setting of the Sun or Star, and the Equator, will shew the Eastern or Western Amplitude.

2. Under

2. *Under Heaven.*

1. The Sights being fixed to the Quadrant, one of the sides being parallel to the Plain of the Horizon, direct it according to the Meridian Line.

2. Then turn the Sights towards the Sun or the Star, rising or setting, until the Sun Beams extend through the hole of one of the Sights into the opposite hole; or the Star be seen thro' both.

3. The Arch of the Horizon intercepted between the Sights and the side of the Quadrant, Eastern or Western, will be the Eastern or Western Amplitude.

3. *By Trigonometry.*

We must proceed thus, to wit, there being given

1. The Elevation of the Equator — $38^{\circ} : 28'$

2. The Declination of the Sun, or of any part of the Ecliptick, or also of a Star, which for Example is — $17 : 48$

In Fig. 185 A B C is the Meridian, D E F the Equator, G H I the Ecliptick, A E C the Horizon, B H L the Arch of the Circle of Declination; H the degree of the Ecliptick, or any Star given, H L its declination; D A or F C the Altitude of the Equator, E H the Eastern Amplitude sought.

Let it be made then

| | | |
|--|---|-----------------|
| As F E the Sine of the
Elevation of the Equator
$38^{\circ} : 28'$ ————— | To the Sine H L of the
given declination $17 : 48$ or whole Sine E C | So the Radius — |
| 62206 | 30569 | 100000 |
| To E H the Sine sought: | whose Arch $29^{\circ} : 26'$ is the | |
| 47141 | Eastern Amplitude sought E H. | |

From this Foundation are made, Tables of the Eastern Amplitude of the parts of the Ecliptick in an Oblique Sphear, to some one Quadrant of it; from which also the others are known: But in a Right Sphear the Eastern Amplitude and the Declination of any part of the Ecliptick agree in all things: And

in a Parallel Sphear, an half part of the Ecliptick arises not, and an half part never sets.

P R O B L E M XVI.

To find at any time given, the Azimuth Arch of the Horizon, passing between the Vertical thro' the Sun, or some Star, and intercepting either the next Meridian or the Equator.

1. *In the Globe, being directed to a time given.*

Let down the Vertical Circle thro' the place of the Sun, or the given Star, which immediately makes known its distance in the Horizon, from the Equator or next Meridian.

2. *Under Heaven.*

1. Again apply one side of the Quadrant to the Meridian Line, the other to the Line of East or West of the Equinoctial, distant from it by a Quadrant.

2. Direct the Sights towards the Sun or Star; and because when the Sun or a Star is elevated above the Horizon, neither the Beams of the Sun passes thro' the holes of the Sights, neither can a Star be seen thro' them, therefore to the foremost Sight affix a Style; but that Sight must be so directed, that either the shadow of the erect Style may fall upon the Diameter of the Sight, in the observation of the Sun; or the Star may be seen thro' the neer hole and the top of the erect Style: For so the Sight will shew the distance of the Vertical Circle drawn thro' the Sun or Star, being numbred from the Coast, either next to the East or to the West, or to the North or to the South.

3. *Trigo-*

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3. Trigonometrically.

The Method of finding the Suns Azimuth is thus.

1. If the Sun be in γ or α , and there be given
for Example

1. The second Hour from Noon, being resolved into degrees of the Equator ————— $30^{\circ} : 0'$
2. The Altitude of the Sun ————— $34 : 9$
3. The Complement of it ————— $55 : 51$

I say

| | |
|---|--|
| As the Sine of the complement
of the Altitude $55^{\circ} : 51'$ ——— | } To the Sine of degrees of the Equa. $30^{\circ} : 0'$
50000 |
| 82757 | |
| So the whole Sine or Radius
100000 | } To the Sine of the Azimuth sought
60417 |
| | |
| To which answers, the Arch of the Horizon $37^{\circ} : 10'$ | |
| And the Complement of it ————— $52 : 50$ | |

2. If the Sun be in some Northern sign, and there be given again

1. The Second Hour after Noon, of Degrees of the Equator ————— $30^{\circ} : 0'$
2. The declination of the Sun ————— $16 : 21$
3. Its Complement ————— $73 : 39$
4. The Altitude of the Sun ————— $48 : 52$
5. Its Complement ————— $41 : 08$

Make it

| | |
|--|--|
| As the Sine complement
of the Altitude $41^{\circ} : 8'$ ——— | } To the Sine of $30^{\circ} : 0'$
50000 |
| 65781 | |
| So the sine of the complement of declination
$73^{\circ} : 39'$ ————— | } To the Sine of the Azimuth sought
72935 |
| 95956 | |
| To which answers, the Arch of the Horizon $46^{\circ} : 50'$ | |
| And the Complement of it ————— $43 : 10$ | |

3. If the Sun be in any Southern Sign, and there be given

G g 4

1. The

1. The second Hour after Noon, of degrees of the Equator ————— 30° : 0'
2. The Suns declination ————— 11 : 29
3. Its Complement ————— 78 : 31
4. The Suns Altitude ————— 23 : 30
5. Its Complement ————— 66 : 30

I Say

| | | |
|----------------|---------------|---|
| As the Sine of | To 30° of the | } So the Sine of the compl. 78° : 31
97998 |
| the complement | Equator ————— | |
| 66° : 30' — | 50000 | |
| 91706 | To which an- | |
| To the Azimuth | swers 32° 18' | |
| sought — | And its com- | |
| 53430 | pl. is 57 42 | |

P R O B L E M X V I I .

To find the right Ascensions of the Sun, and of every continual part of the Zodiack, viz. begun from the Vernal or Spring Equinox.

I. *In the Globe.*

For it being accommodated to the Site of a right Sphear (or to the Meridian, being used instead of a right Horizon in an Oblique Sphear) bring the end of the given Arch to the Eastern Horizon, and the point in which the Horizon and the Equator cut each other mutually, will shew the right Ascension of the given Arch.

So the Sun being in 40° of Π , being conveyed to a right Horizon, the Section of the Horizon and of the Equator, will shew the right Ascension of it 62°.

In Fig. 14 I A B C is the Colure of the Solstices, A D C the Ecliptick, E D F the Equinoctial, whose Pole

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Pole is G; the Equinox in D: Let L be the point of the Ecliptick, G L K the Circle of Declination, ad-
ing the Office of the Horizon: C F the greatest Obli-
quity of the Ecliptick; C G its Complement, D L the
Arch of the Ecliptick given, D K its right Ascensi-
on sought. Then

2. Trigonometrically, from the given.

1. The greatest Obliquity of the Ecliptick F C ———— $23^{\circ} : 31'$
2. And the Complement of it C G ———— $66^{\circ} : 29'$
3. The Arch of the given, which for Example is D L $30^{\circ} : 00'$
4. Its Complement L C ———— $60^{\circ} : 00'$

Let it be made.

| | | | | |
|---|---|---|---|---|
| As G C the Sine
of the Comple-
ment of Obliquity
$66^{\circ} : 29'$ ———— | } | To the Tangent of L C the com-
plement of the Arch 60° ————
173205 | } | So G F the Radius
100000 |
| 91694 | } | To the Tangent of
the Complement
K F of the Ascen-
sion sought ———— | } | whose Arch is ———— $62^{\circ} : 6'$
And the Compl. of it $27 : 54$ is the Ascension sought. |
| 188906 | | | | |

N O T E.

1. In this manner is made the Table of Right Af-
censions to the first Quadrant of the Ecliptick, viz.
of the Spring.

2. The Ascensions of the following Quadrant of
Summer are had, if you take away the Ascensions
of the first Quadrant from the last to the first, from
 180° .

3. The Ascensions of the other Semicircle are ob-
tained, if you add 180° to each Ascension of the
former Semicircle.

4. From these kind of Tables, being made, how
the Ascensions are found out, you may easily gather
from what is said before.

5. If the Right Ascension of any parted Arch be
sought, the Labour is doubled; and the Ascension
must

must be sought as well at the beginning as at the end of the Arch, that being subtracted from this, (360° being added, if necessity require) leaves the Ascension sought.

6. If you are willing to find the Mid Heaven, as well in a Right as an Oblique Sphear, seek the right Ascension of the Degree of the Ecliptick given, or the Degree of the Ecliptick answering to the right Ascension, and you have your desire.

P R O B L E M XVIII.

To find the right Descension of any Arch of the Ecliptick, continual or parted.

In a right Sphear, the Descension of any Arch of the Ecliptick, is equal to the Ascension of the same, therefore this being given, you must needs know that.

P R O B L E M XIX.

To find the Ascensional Differences between the Right Ascension and the Oblique.

1. *In the Globe.*

You will have the Ascensional Difference, if you first seek the right Ascension, and after that, the Oblique; subtract the lesser from the greater, it leaves the Ascensional Difference.

2. *By Trigonometry, there being given.*

1. The Altitude of the Equator, for Example, ——— $38 : 28$

2. The declination of the given Arch of the Ecliptick. $16 : 56$

But the Arch given 47° : or 17 of γ , that is, 30° of γ and 17° of γ added, make 47° .

In Fig. 185 the Altitude of the Equator is CF, the Declination HL; the right Ascension of the point H, is KEL, but KE the Oblique Ascension of the

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the same, and E C the Ascensional Difference which is sought.

Say therefore

| | |
|--|---|
| As the Tangent
of the Altitude
of the Equator
38° : 28' — | To the Rad. EF } So the Tang. of the decli. HL 16° : 56' |
| 79449 | 100 000 30446 |
| To the Sine of
the Ascensional
Difference — | whose Arch 22° : 32' is the Ascensional
Difference sought. |
| 38322 | |

By this Art may be made the Table of Ascensional Differences, to any Elevation of the Pole, or of the Equator, and in every degree of the Ecliptick: Whence the Ascensional difference to any degree of the Ecliptick, and the declination of it may be drawn forth, yet not neglecting the part Proportional, when there is need.

P R O B L E M X X.

To find the Oblique Ascension of an Arch of the Ecliptick given.

1. *In the Globe.*

By directing it to the Elevation of the Pole, as is said before, in seeking the right Ascension.

2. *From the Ascensional Difference given.*

In Southern Signs, add the Ascensional Difference to the right Ascension: In Northern, subtract the same from the same, and it produces the Oblique Ascension,

Example.

| | |
|---|---------|
| The Right Ascension 47° is ———— | 44 : 32 |
| The Ascensional Difference is ———— | 20 : 08 |
| Which Subtracted, leaves ———— | 24 : 24 |
| The Oblique Ascension of the given Arch under
the Elevation of the Pole 48° : 30'. | |

3. *From*

3. *From the Tables of Oblique Ascensions.*

From the Right Ascensions given, and from the Ascensional Differences, are made the Tables of Oblique Ascensions; from which these are drawn or taken forth, as of Right Ascensions; of which we have Treated before.

P R O B L E M XXI.

To find the Oblique Descension of an Arch of the Ecliptick given.

1. *In the Globe.*

Transfer the end of the given Arch to the Western Horizon, under the Elevation of the Pole; and the point of Section of the Horizon and the Equator, will shew what you seek.

2. *By the Ascensional Difference.*

Do the contrary of it, which we have said before, in seeking the Oblique Ascension; for in Northern Signs you must add; in Southern you must take away the Ascensional Difference from the Right Ascension, to produce the Oblique Descension.

3. *From the Tables of Oblique Ascensions.*

1. Take the Arch opposite to the given Arch, which is done either by adding a Semicircle or 180° to the given Arch, if it be less than a Semicircle; or by Subtracting, if greater.

2. Seek the Oblique Ascension of this opposite Arch, which will be the descension of the given Arch, yet by taking away a Semicircle, if the Ascension be greater than a Semicircle, or by adding one, if it be lesser.

Example.

| | |
|---|-----------|
| <i>Under the Elevation of the Pole</i> — | 48° : 30' |
| <i>To seek the Oblique Descension of an Arch of</i> — | 143 : 00 |
| <i>Add a Semicircle</i> — | 180 : 00 |
| <i>It makes the opposite Arch</i> — | 323 : 00 |
| <i>Whose Oblique Ascension is</i> — | 341 : 37 |
| <i>Take away a Semicircle</i> — | 180 : 00 |
| <i>There is left</i> — | 161 : 37 |
| <i>The Oblique Descension of the Arch</i> — | 143 : 00 |

Another

Another Example.

| | |
|--|---------|
| Given an Arch of the Ecliptick | 323 : 0 |
| Take away a Semicircle | 180 : 0 |
| It makes the opposite Arch | 143 : 0 |
| Whose Oblique Ascension is | 129 : 7 |
| Add a Semicircle | 180 : 0 |
| And it produces | 309 : 7 |
| The Oblique Descension of the given Arch | 323 : 0 |

N O T E.

*In finding the Oblique Descension of a parted Arch.
For Example of the Sign γ .*

Seek the Oblique Ascension of its opposite Sign μ .

Which will be found $41^{\circ} : 11'$.

And this is the Descension of the Sign γ .

P R O B L E M XXII.

To find the Arch of the Ecliptick, which answers to the given Ascension, either Right or Oblique, or that arises with the determined point of the Equator, above the Horizon.

1. *In the Globe.*

Transfer the end of the given Ascension to the Horizon, on the East part.

The point of Section of the Horizon and the Ecliptick, shews the end of the agreeing Arch of the Ecliptick, if indeed the given Arch be continual; but if parted, it will be necessary to find the responding degree of the Ecliptick, as well in the beginning as the end of Ascension; and the Arch intercepted between those Degrees, is that which is sought.

2. *In the Tables, the Method is thus.*

1. Enter the Table, either of Right or Oblique Ascensions, under the Elevation of your Pole, Area-wise, and in it seek the Ascension given.

2. If

2. If the Sign be found precisely in the Front of the Table, the Left Hand side will shew the degree of the Ecliptick ascending.

3. If your Ascension given be not found precisely, you must seek the part Proportional in this manner.

Find the Arch or Degree of the Ecliptick, answering to the Right Ascension— $235^{\circ} : 20'$

The next greater will be found— $235 : 44$

But the next lesser— $234 : 41$

The difference of which is— $01 : 03$

Or $63'$, the first number of the rule of Proportion.

The difference of the degrees of the Ecliptick of these Arches obtains the second place, which is always 1° or $00^{\circ} : 60'$

In the third place you must put the given Arch— $235 : 20$

And the next lesser found in the Table $234 : 41$

The difference, which is— $000 : 39$

Hence I say:

If $63'$ make $60'$, what makes $39'$: Answer $37' : 34''$

Which add to the Arch of Ascension of the Ecliptick of the next lesser to the given, to wit, to 27° of m .

And it will make the Arch of the Ecliptick $27^{\circ} : 37' 44''$ of m .

And in what manner the Arch of the Ecliptick is found, that answers to the right Ascension given; in the same manner also is sought the Arch or Degree of the Ecliptick, passing through the Meridian with the given Arch, or with the Degree of the Equator, and occupying the Mid Heaven; since as aforesaid, the Right Ascensions and Mid Heavens differ not.

P R O B L E M XXIII.

To find the Arch of the Ecliptick, agreeing to the Oblique Descension given.

1. In the Globe.

Transfer the end of the continual Descension to the

the Western part of the Horizon, and in the Section of the Horizon and the Ecliptick you will find what you seek.

2. From the Tables.

You may find the Arch of the same in this manner.

1. Convert or turn the Oblique Descension into the opposite Ascension, by adding a Semicircle if the given Descension be less than a Semicircle, but by subtracting a Semicircle if greater.

2. By the foresaid Precept seek the Arch of the Ecliptick, agreeing to the Ascension of this opposite.

3. Add again to the Arch found, 180° , if it be less than a Semicircle; subtract it if greater: That which it produces is the arch of the Ecliptick, agreeing to the Oblique Descension given.

Example.

| | |
|---|-----------|
| Given under the Elevation of the Pole | 48° : 30' |
| The Oblique Descension, of parts | 161 : 37 |
| And the congruent Arch of the Ecliptick is sought | |
| Add the Semicircle | 180 : 00 |
| And they make for the opposite Ascension | 341 : 37 |
| To this agrees, of the Ecliptick | 323 : 00 |
| Take away a Semicircle | 180 : 00 |
| It leaves the Arch of the Ecliptick | 143 : 00 |
| Agreeing to the Descension given | 161 : 37 |

Another Example.

| | |
|---|----------|
| Given the Oblique Descension | 309 : 07 |
| And the congruent Degree of the Ecliptick is sought | |
| Take away a Semicircle | 180 : 00 |
| And it makes for the opposite Ascension | 129 : 07 |
| Of this the Arch of the Ecliptick is | 143 : 00 |
| Add a Semicircle | 180 : 00 |
| And it will make the Arch of the Ecliptick | 323 : 00 |
| Agreeing to the Descension given | 309 : 07 |

N O T E.

If the Arch of Descension given be discreet or parted, then we must work with the beginning and the end, as hath been said oftentimes.

Problem

PROBLEM XXV.

To find the right Ascension and Descension of any Star, without the Ecliptick.

1. *In the Globe.*

1. If the Star, whose right Ascension you seek, be there depicted, transfer it either to the right Horizon, or to the Meridian of any Oblique Sphere; and the degree of the Equator which passes the Horizon or the Meridian with the Star, is the right Ascension of it.

2. But if the Star, whose Ascension you seek, be not in the Globe, expect or wait while it arrive at the Meridian: Then consider whether in the same Moment of time, any of the Stars painted on the Globe obtain the Celestial Meridian, the which constitute to the Meridian of the Globe.

The degree of the Equinoctial in like manner touching the Meridian, will shew the right Ascension, as well of your Star as of that noted in the Globe.

3. If no Star be Culminate together, do thus.

1. Take the Altitude of some Star known to you, (which will be done more exact by those that are nearest to the Horizon, than those that have great Meridian Altitude) observing the Coast in which that Star is situate.

2. Dispose the Globe so, that that Star in the same Coast, may obtain under the Quadrant of the Vertical Circle, the same Meridian Altitude also in the Globe, which it hath in the Heaven.

3. Then in this posture, the degree of the Equinoctial which touches the Meridian, will be also the right Ascension of thy Star, being in the Meridian; to which also the Descension of the same is equal.

Another way.

1. Let there be given, 1. The Suns place at the Noon, which next preceeds the time of Observation.

2. The

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2. The right Ascension of this place of the Sun.
3. Then having ready at hand a long Pendulum Clock that goes truly, set exactly to the motion of the Sun the same day at Noon.
4. In this observe how many Hours from Noon, the Star, whose Ascension you seek, arrives at the Meridian.
5. Convert those Hours into Degrees and Scruples of the Equator, (by assuming for one Hour 15° ; for 1' Fifteen Minutes, &c.) add them to the right Ascension of the Sun, and you will have the right Ascension of the Star which you seek.

P R O B L E M XXVI.

From the given Altitude of the Equator, and from the Declination and right Ascension of any Star, to find the Ascensional difference of it.

For Example, of Arcturus; The right Ascension is $209^{\circ} : 29'$
 The Declination of the same $21 : 18$
 Under the Altitude of the Equator $41 : 30$

Then Say

| | |
|---|---|
| As the Tang. of the Altitude
of the Equator $41^{\circ} : 30'$ | } To Radius or Sine of 90°
100000
To the Sine of the Ascensional difference
44068 |
| 88473 | |
| So the Tangent of the decli-
nation $21 : 18$ | |
| 38988 | |

The Arch of which $26^{\circ} : 9'$ is the Ascensional difference between the Right and Oblique Ascension of the given Star.

P R O B L E M XXVII.

To find the Oblique Ascension and Descension of any Star.

1. In the Globe.

The place of the Star given being known in the Globe, turn the Globe until the Star fall in with
 H h the

the Oriental Horizon, if you seek the Ascension, or with the Occidental, if you seek the Descension: For then the Section of the Horizon and the Equator will shew the Ascension or Descension of the Star, numbred from the Spring Equinox; and indeed Oblique, being posited under a certain Oblique Sphere.

2. *From the Ascensional difference given.*

1. *For finding the Oblique Ascension.*

In Stars declining towards the North, subtract the Ascensional Difference from the right Ascension of them; but in Stars declining towards the South, add it to them, and you will produce the Oblique Ascension.

Example.

1. Of Arcturus, being Northern; The right Ascension is 209 : 29
 Subtract the Ascensional Difference ————— 26 : 09
 It leaves the Oblique Ascension at Tubing ————— 183 : 20
2. Of the Greater Dog, Southern, the right Ascension is 97 : 07
 Add the Ascensional Difference ————— 19 : 10
 It makes the Oblique Ascension in the Latit. of Tubing 116 : 17

2. *For finding the Oblique Descension.*

Act contrariwise, viz. To the right Ascension of Northern Stars add the Ascensional Difference: From the right Ascension of Southern Stars subtract it, and you will have the Oblique Descension.

Example.

1. Of Arcturus, to the right Ascension ————— 209 : 29
 Add the Ascensional Difference ————— 26 : 09
 It makes the Oblique Descension ————— 235 : 38
2. Of the Dog Star, from the right Ascension ————— 97 : 07
 Take away the Ascensional Differenc. ————— 19 : 10
 It leaves the Oblique Descension ————— 77 : 57

Problem

P R O B L E M XXVIII.

To compute the Longitude of any Star from the Vernal Equinox, and the Latitude from the Ecliptick.

1. In the Globe.

Let fall the Circle of Latitude by the given Star, even to the Ecliptick, from the North Pole if the Star be Northern, but from the South Pole if it be Southern: This Circle of Latitude will determine the Longitude of the Star in the Ecliptick; but by the Arch intercepted between the Star and the Ecliptick, the Latitude of it is determined.

2. By Trigonometry, from having given.

1. The greatest Obliquity of the Ecliptick.
2. The Declination of the Star.
3. The right Ascension of the same.

In Fig. 140 the Meridian is $a b c k$, the Equator $g l f$, its Pole d : The Ecliptick $a l c$, its Pole b ; the point of the Vernal Equinox l , the Circle of Declination $d i e$, the Circle of Latitude $b i h$; the Star given i , which for Example is the Goat Star:

Whose Declination $e i$ is ————— $45^{\circ} : 32'$
 The right Ascension $e l$ ————— $72 : 11$
 The Obliquity of the Ecliptick, or the Angle at $a l g$ $23 : 31$
 The Angles at h and c being right Angles ————— $90 : 00$

And we are to seek the Longitude of this Star $l h$, and its Latitude $b i$.

1. Seek the Arch $n e$, in this manner:

As the Radius } To ag the Tang. of } So the sine of right Ascension $l g$ $72^{\circ} : 11'$
 100000 } Obliquity $23^{\circ} : 31'$ } 95204
 43516 }
 Is the Tang. of the Arch $n e$ 41428 : which is $22^{\circ} : 30'$
 H h 2 2. From

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2. From this is found also the Arch in .

If, to wit, from ie the Declination of the Star North ————— $45^{\circ} : 32'$

You take away the Arch found $ne 22^{\circ} : 30'$

There remains in ————— $23^{\circ} : 02'$

3. The Angle lne is found in this manner.

As the Sine of the side ne , $22^{\circ} : 30'$

38268

So the Sine le of right Ascen. $72 : 11$

95204

To the Sine of the Angle eln , $23^{\circ} : 31'$

39902

To the Sign of the Angle lne

99266

Whose Arch is $83^{\circ} : 4'$, that is the Angle lne .

4. From this Angle lne being given, is also given the Angle hni , opposite by the Cross, to wit, equal in like manner to $83^{\circ} : 4'$.

5. The Side nl is found in this manner.

As the Sign of the Angle eln $23^{\circ} : 31'$

39902

So the Sine of the Angle nel — $90 : 00$

100000

To the Sine of the Side ne $22^{\circ} : 30'$

38268

To the Sine of the Side nl .

95997

Which is found to be $73^{\circ} : 3'$.

6. From these arises the Latitude of the Star hi , by Saying.

As the Sign of the Angle ihn 90°

100000

So the Sign of the Angle hni $83^{\circ} : 4'$

59268

To the Sine of the Side in $23^{\circ} : 2'$

39126

To the Sine of the Latitude hi

38739

Which Latitude is $22^{\circ} : 47' : 31''$

7. Lastly, from these is produced also the Longitude of the Star, in this manner.

As the Tang. of the Angle hni $83^{\circ} : 4'$

822343

To the Sine of the side hn

5117

To the Radius

100000

Whose Arch is

$2^{\circ} : 56'$, which bring added to

the Side nl , $73^{\circ} : 33'$, makes the Longitude

of the given Star hl $76^{\circ} : 29'$

So the Tang. hi of Latit. $22^{\circ} : 47'$

42002

Problem

P R O B L E M XXIX.

From the right Ascension of a Star given, to find the Degrees of the Ecliptick, together with the Rising, Setting, and its passing through the Mid Heaven.

1. *In the Globe.*

Roll down the Star to the right Horizon, either Oriental or Occidental, or also to the Meridian of any Sphere; the Section of the Ecliptick and of the Horizon, or of the Meridian, will shew the Degree of the Ecliptick sought.

2. *By the Tables.*

With the right Ascension given of any Star, make an Areal entrance in the Table of right Ascensions, and find the Degree and Sign of the Ecliptick in the Side and Front agreeing to the same; using the part Proportional also in the Inquiry of it, when there is need, after the manner shewn before in Problem the 22, at Page 461.

P R O B L E M XXX.

From the Oblique Ascension of any Star given, to find the Degrees of the Ecliptick, together with the Rising, or also Setting.

1. *In the Globe.*

Move forward the Star to the Oblique Horizon, either Oriental or Occidental, the Degree will appear in the Ecliptick, there indeed together with the arising, but here together with the setting.

2. In the Tables.

1. For obtaining the Degree of the Ecliptick, with its arising.

Dispatch the Oblique Ascension given in the Table of Oblique Ascensions of parts of the Ecliptick, and it will shew the Degree of the Ecliptick arising, if you work in the same manner as before in Problem 22, using also the part Proportional, if need be.

2. But to obtain the Degree of the Ecliptick, together with the Setting.

Work according to the Method shewn in Problem the 23, at Page 462.

1. To wit, by taking the Oblique Descension of the given Star.

For Example.

- | | | |
|---|------|------|
| Of Arcturus, which is | 235 | : 38 |
| 2. From this take away a Semicircle | 180 | : 00 |
| And there will be left | 055 | : 38 |
| 3. Of this seek the degree of the Ecliptick | } 85 | : 18 |
| arising together with it, which is | | |
| 4. That you may have the opposite, add | 180 | : 00 |
| And you will produce | 265 | : 18 |
| Or of Sagittary | 25 | : 18 |
| Setting together with Arcturus, where the Pole is elevated 48° : 30'. | | |

Another Example.

- | | | |
|---|-----|------|
| 1. The Oblique Descension of the Dog Star is | 77 | : 57 |
| 2. The opposite Degree there being added | 180 | : 00 |
| Is | 257 | : 57 |
| 3. The degree of the Ecliptick arising together | 236 | : 44 |
| 4. That you may have the opposite, take away | 180 | : 0 |
| And there will be left | 56 | : 44 |
| Or of ♄, the Degrees | 26 | : 44 |
| viz. by taking away 30° the degrees of ♄, is left 26° : 44' | | |

Problem

P R O B L E M XXXI.

Some unknown Star being seen in Heaven, to design the Site and Place of it in the Globe, Artificially.

1. *From the right Ascension and Declination of it being found.*

The Degree of the right Ascension being found, number it in the Equinoctial, in beginning from Aries; and the Degree found, adjoin to the Meridian Circle.

2. In the same Meridian number also the given Declination of the Star, in proceeding from the Equinoctial, towards the North Pole of the World, if the Declination of the Star be North; towards the South Pole, if it be South; the term or end of this Numeration in the subjected Globe, will shew the true place of the Star.

2. *From the Azimuth being known, or the distance of it from some Coast, and from the Altitude of the same.*

1. The Globe being accommodated to the Position of the Heavens, and so in it remaining.

2. Seek the Azimuth observed in the Horizon of the Globe, and adjoin to it the Quadrant depending from its Vertical point.

3. In the Degrees of this Quadrant seek the Altitude observed of the Star, and the Term of this Numeration will shew in the Superficie of the Globe, the place of the Star which you seek.

P R O B L E M XXXII.

Any Star being given, to know in what time'tis Cosmical, that is, arising together with the Sun.

H h 4

1. In

1. *In the Globe.*

Stay the given Star at the Oriental Horizon, and the degree of the Ecliptick arising together, shews what is sought; if you consider in what time the Sun enters that degree.

2. *From the Tables of Ascension.*

1. Seek the Right or Oblique Ascension of the given Star.

2. By it inquire the degree, together with it arising; in the time which the Sun retains that degree, the Star arises Cosmically.

Example.

The Oblique Ascension of Arcturus is ————— $183 : 20$

The degree of the Ecliptick rising with it is ————— $2 : 20$

Which the Sun occupies about the 15 of September, then therefore Arcturus rises Cosmically.

P R O B L E M XXXIII.

A Star being given, to know what time 'tis Cosmical, that is, she sets with Sun rising.

1. *In the Globe.*

Move the Star to the Occidental Horizon, and over against it consider the Oriental Degree of the Ecliptick, which when the Sun occupies, the Star sets Cosmically.

2. *By the Descension.*

1. Assume the Descension of the given Star, and by it the Degree of the Ecliptick, together with it descending.

2. When the Sun enters the opposite Degree of this, the given Star sets Cosmically.

Example.

The Descension of Arcturus is ————— $235 : 38$

And together with it sets ————— $25 : 18$

The opposite Degree is ————— $25 : 18$

Therefore.



Therefore this Degree being obtained by the Sun, about the 6 day of June, Arcturus sets Cosmically.

PROBLEM XXXIV.

A Star being given, to observe in what time 'tis Acronycal, that is, rising at Sun setting.

1. *In the Globe.*

Bring the given Star to the Oriental Horizon, and then consider the Degree posited in the West, that is it, which when the Sun occupies, the Star arises *Acronically*.

2. *By the Ascension.*

1. Seek the Degree of the Ecliptick, arising with the Ascension of the Star.

2. When the Sun enters into the opposite of this, the Star rises *Acronically*.

Example.

The Ascension of Arcturus is ————— 183 : 20

The degree of the Ecliptic together arising $\approx$ 2 : 20

The opposite ν ————— 2 : 20

Therefore in this the Sun being about the 14 of March, Arcturus rises with Acronical rising.

PROBLEM XXXV.

A Star being given, to know when 'tis Acronical, or sets together with Sun setting.

1. *In the Globe.*

Place the Star in the Western Horizon; and the Degree of the Ecliptick touching the Western Horizon in like manner, will shew both the place of the Sun, and the time in which the Star sets *Acronically*.

2. *By the Descension.*

Seek the Descension of the given Star, and the degree of the Ecliptick, setting together.

In

In the time which the Sun occupies this Degree, that Star sets *Acronically*.

Example.

The Oblique Descension of Arcturus is $\text{---} 235^{\circ} : 38'$

The Degree of the Ecliptick together descending $225 : 18$

Which the Sun enters about the 7 of December, in which time Arcturus sets *Acronically* with the Sun setting.

P R O B L E M XXXVI.

To inquire the Heliacal Rising and Setting of any Star.

1. *In the Globe.*

Place the Star, whose Heliacal rising or setting you desire to know, in the Eastern Horizon, if you seek the rising; in the Western if you seek the setting; and note the Degree of the Ecliptick, together with the rising or setting.

2. Assume so many Degrees in the Vertical Circle beneath the Horizon, as the Term of Apparition of that Star requires.

3. Apply the end of Numeration to the Ecliptick. Then what Degree of the Ecliptick the Vertical Circle shews, in that position of the Globe; that is it which the Sun ought to possess, that the given Star may arise or set *Heliacally*.

2. *By Trigonometrical Calculation.*

1. For finding the Heliacal Rising, for Example of Arcturus, Fig. 157.

In this Scheme let there be given.

1. The Degree of the Ecliptick D arising together with Arcturus E, which is $2^{\circ} : 24'$ of --- , or of the Ecliptick $182^{\circ} : 24'$
2. The Oblique Ascension of this $\text{---} 183 : 20$
3. From which take away a Quadrant $\text{---} 90 : 00$
There is left the right Ascension of Mid Heaven $93 : 20$
4. To this answers the culminate degree of the Eclipt. $3 : 04$

Or

| | | | |
|--------------------------|--|----|------|
| 5. | Of of the Ecliptick | 93 | : 04 |
| 5. | Which taken from the Degree co-arising, leaves the Arch | 89 | : 20 |
| D G | | | |
| 6. | The Declination of the degree of the Ecliptick culminating | 23 | : 26 |
| is Northward | | | |
| 7. | Which being added to the Altitude of the Equat. | 41 | : 26 |
| Constitutes the Arch G H | | 64 | : 52 |

N O T E.

If the Declination had been South, then you must have Subtracted it from the Altitude of the Equator.

These things being thus had, I say.

As D G the Sine of } To the Radius 90°
 the Arch 89° : 20 } 100000
 99993

So the Sine of the } To the Sign of the Angle at D.
 Arch G H 64 : 56 } 90588
 90582

Which Angle is 64° : 57'

Moreover, because the Term of appearance of Arcturus, a Star of the 1. Magnitude is 12°.

Therefore again I say.

As the Sine of the } To the Sine of the Side 12° : 0'
 Angle — 64° : 57' } 20791
 90588

So the whole Sine } To the Sine of the Arch sought
 or Radius 90 : 00 } 22951
 100000

Which Arch is 13° : 16', in numbring from the Degrees of the Ecliptick, together from the East, to wit, from 2° : 24' of ♈, that is, from the Sun in 15° : 40' of ♈, Arcturus rises Heliacally, and issues out from the Sun Beams, that arising in the Morning it may be seen.

2. For the finding the Heliacal setting of the same Arcturus.

1. The Degree of the Ecliptick setting together is
 1 25° : 18.

This

- This is of the Ecliptick ————— 265 : 18
 2. The Oblique Descension of this ————— 235 : 38
 3. Add a Quadrant ————— 90 : 00
 It makes the right Ascension of Mid Heaven 325 : 38
 4. The degree of the Ecliptick, together with the Culminate,
 is 23° : 18' ²³.
 Or of the Ecliptick ————— 323 : 18
 5. Whence Subtract the setting together ————— 265 : 18
 There is left ————— 58 : 00
 6. The declination of the degree Culminate is ————— 13 : 46
 7. Which, because 'tis South, must be taken from the Altitude of the Equator 41° : 30'.
 And it leaves ————— 27 : 44

These being given, I say.

As the Sine of } To the whole Sine 90°
 the Arch 38° : 0' } 100000
 84805
 So the Sine of the } To the Sign of the Angle sought
 Arch — 27 : 44 } 14874 which is 33° : 17'.
 46536

Again.

As the Sine of the } To the Sine of the Side 12°
 Angle — 33° : 17' } 20791
 54874
 So the whole Sine } To the Sine of the Arch sought
 or Radius 90 : 0 } 37889
 100000

Which will be found 22° : 16', by numbring back again from 25° : 18' of 4, setting together.

That is, the Sun being in 3° : 2' of 4 : Arcturus in the Evening is involved or hid by its Beams, that after Sun setting, 'tis not any more seen.

PROBLEM XXXVII.

To find the distance of two Meridians.

There are divers ways, but the most used and the most certain, is that which is sought from Lunar Eclipses; for Lunar Eclipses in one and the same Moment of a Natural Day, absolutely considered, are

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are beheld by all Men in the same quantity, above whose Horizon the Moon is risen; altho' that Moment of time in another place obtains another Numeration, to wit, greater among the more Eastern Inhabitants, lesser among the Western, according to the diversity of Meridians. For Example, the Meridian in one place at the Hour 12, will be in another place at 11, before Noon, or 1 after Noon.

If therefore two Observators of Eclipses in two several places, compare their Observations, to wit, what Hour of the day, or Moment of Hour, they beheld the beginning, the middle or the end of every Eclipse; from this difference of number of Time, the difference of Meridians may be found, if the difference of Time be converted into Degrees and Minutes of a great Circle, by the Method shewn before in the Chronological Part. For Example, Fig. 188.

An Eclipse of the Moon, which Ptolomy witnesses was seen at Arbela of the Assyrians, the 5th. Hour after Sun set, appeared at Carthagena the second Hour after Sun setting; The difference therefore of Time is 3 Hours, which make 45 Degrees: Therefore by so many Degrees distant is the Meridian of the Carthagenians, and the Arbelans of Assyria: And since those always number less from the Meridian, which are more West than those more East, it follows that Carthagena is 45° more West than Arbela of Assyria.

From this diversity of Meridians, it happened that the Lusitanians, who Sailed about the whole Orb towards the West, returning home even as by the East, they numbred the Fourth day of the Week, which the Lusitanians that staid at home, numbred the Fifth.

For the Meridian of these remained in its place; but the Meridian of those Sailing in the Ship, from whence they numbred the beginning of the Natural Day,

Day, they carried about as it were with themselves; or rather oftentimes passed under one and another Meridian: And in it they had the Sun daily arriving more slower to the Meridian of the Ship, until these Moments of slowness constituted an whole day, which because they could not number them; as it were they lost them. It would happen contrariwise to them, who should Sail towards the East, and return home by the West; for by meeting with the Sun by the way, they would at last number one day more than those to which they returned.

II.

Theoretical Problems.

The proper motion of the Stars may be found many ways, to wit, 1. Either from beholding of them in the Heaven, by the use of fit Instruments. 2. Or from the Theories of the Heavenly Motions, in some measure made subject to our Eyes. 3. Or from perpetual Astronomical Tables, accommodated to all the Forms of Theories. 4. Or from Yearly Almanacks, shewing the Heavenly Motions to each day of the whole Year. We with Artists, furnished with divers Instruments, shall use the first way, leaving the last to the Writers of *Ephemerides*, who for the most part premise the Method of using the same; concerning the Second and Third we shall only touch upon some few things.

For since the Forms of Theories at this time are almost so many as there are Artists in *Astronomy*, and although we would choose some one from the many, yet many Tables are required, which neither the Compendium of *Astronomy* will admit, nor the Occupations of the Studious, who make haste to superior Faculties as they call them; that notwithstanding concerning these excellent things, (in which not only the chief Wisdom of God, but also the admirable

mirable Sagacity, which God hath instilled into Humane Nature, to the finding out of these things, may be seen) they may have some, although but a small Taste, we shall shew Examples in the Theory of the Sun, to which we shall premise some things concerning the Fixed Stars: But concerning the motion of the Moon, we shall not subjoin any but popular or common things, to the end that the Wisdom of the Studious in this *Lunar Astronomy* be not exceeded, as sometimes 'tis wont to be done by Men and Maid Servants of Husbandmen, being oftentimes exercising concerning the Night in the day time, and observing the motion of the Moon.

I.

Of the proper motion of the Fixed Stars, &c:

P R O B L E M I.

To compute the proper motion of the Fixed Stars to any given time.

To this two things are necessary.

1. The *Epocha* or *Radix*, as well of the time, as of the place of the proper motion of the Fixed Stars, or the Longitude of them from the beginning of γ , and their Latitude from the Ecliptick.

2. The Table of their motion, from which may be known how much they are distant at all times from this *Epocha*.

You will find both these at the end of this our *Synopsis*, in adjoined Tables: And 1. a Catalogue of the more Famous Fixed Stars, according to the Opinion of Tycho, with their Longitude, reduced to the beginning of the *Proleptick Julian Period*.

2. A Table of Motion, as well of each as of collected Years; from whence you may know how the
Longitude

Longitude of any Star contained in our Catalogue ought to be computed to any given time: It will be sufficient to shew by one Example only.

Lets seek what was the Longitude of Arcturus in the compleat Year of the Christian Epocha--1650.

That you may have the Year of the Julian
Period, add _____ *} 4713*
And they make Years _____ *6363*

Then in our Catalogue of Fixed Stars you will find.

| | Sigs | ° | ' | " |
|--------------------------------------|------|------|------|-----------|
| The Radix of this Star of Arcturus | 3 | : 19 | : 12 | : 57 |
| In the following Table you will find | | | | |
| For the Years | 6000 | 2 | : 25 | : 0 : 0 |
| | 300 | 0 | : 04 | : 15 : 0 |
| | 60 | 0 | : 00 | : 51 : 0 |
| | 3 | 0 | : 00 | : 02 : 33 |
| The Sum of these is | | 6 | : 19 | : 21 : 30 |

Which was the Longitude of Arcturus, in the Year of the Christian Epocha 1650, compleat.

N O T E.

1. If some Year be not found in the Table, it must be gathered by parts. For Example, the Year 56, by 40 and 16.
2. As often as the Sum arises beyond a Circle, you must cast away an whole Circle, that is, 12 Signs or 360°.
3. The Latitudes of the Stars, are appointed to remain the same always: Therefore they want not Reduction to other Times, but as they are expressed in the Catalogue, so they are wont to be used for any Time.
4. The præcession of the Equinoxes, if you would know it to any time, seek the Longitude of the bright

bright Star in the Vertex, or top of the Head of γ : Whence Tycho numbers the Equinoctial Point; for by how much this is distant from $\circ \gamma$, by so much is the Precession of the Equinoxes judged.

II.

Of the proper Motion of the Sun.

P R O B L E M II.

To find the mean, middle, or equal Motions of the Longitude, and of the Apogæon of the Sun, to any given time.

For to do this, we have reduced the *Rudolphine* Tables to the beginning of the *Julian* Period, and indeed to the Midnight preceeding the Calends of *January*, under the Meridian of *Tubing*.

Therefore if to any one, there be offered the time numbred from the *Christian Epocha*, he must add first to this *Epocha* 4713 Years.

But if the time be given before the *Christian Epocha*, that must be taken from the Years 4714, that the time of the *Julian* Period may be had, accommodated to our Tables.

Which being done, enter the Tables of the middle or mean motions of the Sun, &c.

1. Prefix the *Epocha* of the *Julian* Period.
2. Take the Years, Months and Days of the given time, not current, but compleat; and to them (by collecting the parts, if the whole be not extant in the Tables) as also to the Hours and Minutes, ascribe their motion taken from the Tables.
3. Notwithstanding in the *Bissextile* Year, you must take the day current for the compleat: the Sum collected, (casting away the whole Circles) will shew the equal motion of the Sun to the given time.

Example.

Lets seek the middle motion of Longitude and Apogæon,

pogzon, to the Year compleat of the Christian Epoch
 cha ————— 1650.

Which is of the Julian Period ————— 6363.

Ending at Midnight of the Calends of January, of the Year anteceeding ————— } 1651.

| | Mot. Long. ☉
Sig. ° ' " | Moti. of Apogzon ☉
Sig. ° ' " |
|------------------------|----------------------------|----------------------------------|
| Radix of Julian Period | 8 : 1 : 59 : 38 | 11 : 17 : 39 : 54 |
| Years compleat, 6000 | 1 : 15 : 20 : 24 | 3 : 12 : 42 : 45 |
| 300 | 2 : 16 : 1 | 5 : 8 : 8 |
| 60 | 27 : 12 | 1 : 1 : 38 |
| 3 | 16 : 9 | 3 : 5 |
| The Sum | 9 : 20 : 19 : 24 | 3 : 6 : 35 : 30 |

NOTE.

The Hours and Minutes in computing the motion of Apogzon, are neglected, by reason of their smallness.

PROBLEM III.

From the middle motion of Longitude and Apogzon of the Sun given, to find the Yearly Anomaly of the same.

Sig. ° ' "

From the motion of Longitude, for Example, 9 : 20 : 19 : 24
 Take away the motion of Apogzon ————— 3 : 6 : 35 : 30
 It leaves the Anomaly ————— 6 : 13 : 43 : 54

If the motion of Longitude happen to be the lesser, so that the motion of Apogzon cannot be subtracted from it, you must add to the motion of Longitude, an whole Circle, or 12 Signs.

PROBLEM IV.

To delineate the Theory of the Sun: To which, two Orbs or Circles are required.

See

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See Fig. 189. 1. The *Concentrick Circle* is abc described from the Center of the World, resembling the *Ecliptick*, or way of the Sun, divided into 4 Quadrants, by drawing the two Diameters, their Extrems shewing the 4 Cardinal Points, viz. α , γ , ϵ , δ .

2. The *Eccentric Circle*, incompassing indeed the Center of the World, but having its Center diverse from the Center of the *Ecliptick*, or of the World: which is found in this manner.

1. In the Concentrick Orb or *Ecliptick*, assume a little beyond the beginning of *Cancer*, the point of *Apogæon* g , which at this time is conversant between 6 and 7 Degrees of α .

2. From this point of *Apogæon*, through the Center of the *Ecliptick* e , draw the Line of *Apogæon*, which is gef .

3. In this Line of *Apogæon*, from the Center of the World towards α , assume the Line of Eccentricity ei , which according to the Opinion of *Tycho Brahe*, is esteemed of parts 358400, of which the Eccentric Semidiameter is 10000000.

4. The Extream of this little Line i , is the Center, from which is described the Eccentric under the *Ecliptick*, incompassing the Center of the World, which in our Scheme is mno : This then is the Theory of the Sun, by which, and by the Benefit of the middle motions of Longitude, of *Apogæon*, and of the Annual *Anomaly*; we may find at any time given, by the Canon of Sines, the true motion of the Sun, and the distance of it from the Earth, in the following manner.

P R O B L E M V.

From the middle motions of the Sun given, to find the true motions of the same by Trigonometry.

I i 2

1. From

1. From the point of *Apogæon g*, number according to the order of the Signs, the Annual *Anomaly*, to the end of the Year 1650, found before, not accurately, but only near, which is the Arch *g f h*, that is, of $6^{\circ} : 13^{\circ} : 43' : 54''$.

2. Through the end of Numeration, or the point *b*, from the Center *e* draw the right Line *e h*, which will be the Line of middle motion of the Sun.

3. And from the Eccentrick Center *i*, draw *i k* parallel to *e h*, which in the point of Intersection of the Eccentrick *s*, will shew the place of the Sun in the Eccentrick, and its middle motion under the Ecliptick in *k*.

4. Through the point *s*, even as the Center of the Sun, from the Center of the World *e*, draw the right Line *e s p*, which will be the Line of true motion of the Sun, shewing its true place under the Ecliptick in *p*.

5. Then the *Prosthaphæresis*, or the difference of the mean and true motion of the Sun, will be the Arch *p h*, or the Angle *p e h*, equal to which, are the Angles *p s k*, and *i s e*, as is known to Geometricians; of which this latter *i s e*, is now to be found, from some things given and known, in this manner.

In Triangles, by *Prosthaphæresis*, *i s e* are known thus.

1. The side *i s* the Semidiameter, of Eccentrick parts 100000 : 00
2. The Eccentrick Side of the same parts ———— 3584 : 00
3. The Sum of these Sides ———— 103584 : 00
4. The difference of the same ———— 96416 : 00
5. The Angle *e i s* equal to the Angle *f e h*, which is the excess of the Annual Anomaly above a Semicircle, now is $13^{\circ} : 43' : 54''$
6. The Sum of the other Angles *i e s*, and *i s e*, which are the complement of it to a Semicircle, or two right Angles $5 : 16 : 15 : 6$
7. The half of this Sum, which will be found — 2 : 23 : 08 : 3
8. The Tangent of this half which may be computed from the Tables of Sines, if you want the Tangents } $83^{\circ} 40' 58''$

Having

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Having these things, I say by the Golden Rule of Proportion.

| | | |
|--------------------------------|---|----------------------------------|
| <i>As the Sum of the Sides</i> | } | <i>To the difference of them</i> |
| 103584 : 00 | | 96416 : 00 |
| <i>So the Tangent of half</i> | } | <i>To the Tangent</i> |
| 830405 : 86 | | 772941 : 87 |

Signs ° ' "

But the Arch of this Tangent is—2:22:37:41
 To which the aforesaid half being added 2:23:08:03
 It makes the Angle i e s ————— 5:15:45:44
 And Subtracted from the same half } 0:00:30:22
 25:23°:8':3", leaves the Angle s i e }
 Which is the Prostaphæresis sought.

Then

Signs ° ' "

To the mean Anomaly ————— 6:13:43:54
 Add this Angle ————— 0: 0:30:22
 It makes the true Anomaly ————— 6:14:14:16
 Add the Motion of Apogæon ————— 3:06:35:30
 It produces the true place of the Sun 9:20:49:46

Briefer thus:

* To the middle motion of Longitude 9:20:19:24
 Add the Prostaphæresis ————— 30:22
 It produces the true place ————— 9:20:49:46

*We must note at the Sign **

This Prostaphæresis is not always added to the Annual Anomaly, or to the middle motion of Longitude, but then only when the Line of true motion preceeds the Line of mean Motion: Or when the

mean Annual Anomaly is greater than a Semicircle, as in our Example: But when the Line of middle Motion preceeds or surmounts the Line of true Motion, and this Line of true Motion follows the Line of middle Motion, according to the order of the Signs, which happens in the former Semicircle, in which the Annual Anomaly is less than a Semicircle, then the Prosthaphæresis is not to be added, but to be taken away from the mean Annual Anomaly, or from the mean motion of Longitude, that the true Motion may be produced.

By this Artifice are made the Tables of *Prosthaphæresis*, from which, (supposing you use the part Proportional) you may pick out the *Prosthaphæresis* to each Degree or Scruple of the mean Anomaly: Which Tables of *Prosthaphæresis*, although they are not perpetual, yet their Use continues a long time, until there be discovered some mutation in the Eccentricity, or from other Celestial Causes, which, by reason of the extream slowness of this Mutation, cannot be made but after some Ages: The Table of this kind of *Prosthaphæresis* you may see at the latter end hereof.

P R O B L E M VI.

To find the distance of the Sun from the Center of the Earth, to any given time, according to this Theory.

The distance of the Sun from the Earth to a proposed time, is in the aforesaid Theory of the Sun, in the Angle *s i e*, the Side *s e*, which you may find, it you say.

| | | |
|--|---|------------|
| As the Sine of the Angle | } To the Side is the Eccentric Semidiameter | |
| <i>i e</i> , $14^{\circ} : 14' : 17''$ | | 100000. 00 |
| 24595. 12 | } To the Side <i>s e</i> sought | |
| So the Sign of the Angle | | 96516. 87 |
| <i>e</i> is, $13^{\circ} : 43' : 56''$ | | |
| 23738. 44 | | |

That

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That is, of parts of which the Eccentric Semidiameter is assumed 100000. 00

Which Eccentric Semidiameter, I shall now make of Semidiameters of the Earth, according to the Opinion of Tycho, 1166: It will be easie to apply this distance to the Semidiameters of the Earth, in this manner.

| | | |
|--------------------------|---|--|
| As the Sine of the Angle | } | To the Side s e, which is according to Tycho |
| ies 14° : 14' : 17" | | 1166 |
| 24595. 12 | } | To the Side sought s e, according to Tycho |
| So the Sine of the Angle | | 1221 |
| e is 13° : 43' : 56" | | |
| 23738. 44 | | |

P R O B L E M VII.

To find the time of Equinox, or the Suns Ingress into $\odot \gamma$.

1. Turn your Year given, into the Year of the Julian Period, by adding Years 4713 after the Christian Epoch; or by Subtracting the Epoch before the Christian given back again, from the Years of the Julian Period 4714.

2. Consider whither the Year be Bissextile or Common, and if Common, how far 'tis from Bissextile.

3. Then to this Year, in the Table of Anticipation of the Equinox, choose a convenient Epoch or Radix, which prefix to your Calculation, with a Line drawn under.

4. To this subjoin the Anticipations of the Equinox to each current Year given, being taken from the Table, which collect into one Sum, and Subtract it from the prefixt Epoch: The remainder will shew the day of the Equinox, together with the Hours and Minutes in which the Sun enters into $\odot \gamma$, by numbring from the Calends of January.

Example I.

The time of the Equinox is sought to the Year of
Li 4
Christian

Christian Epocha Biffextile ————— 1652
 The Years of the Julian Period being added — 4713
 Makes the Year of the Julian Period ————— 6365
 To which agrees the Epocha ————— 6365

| | | D. | H. | ' | " |
|---|----------|-----|------|------|------|
| Biffextile | Compleat | 118 | : 13 | : 59 | : 15 |
| <hr/> | | | | | |
| And they anticipate the Equinox | | | | | |
| | 6000 | 46 | : 21 | : 00 | : 0 |
| | 300 | 2 | : 8 | : 15 | : 0 |
| | 60 | | 11 | : 15 | : 0 |
| | 5 | | | 56 | : 15 |
| <hr/> | | | | | |
| The Sum of these | | 49 | : 17 | : 26 | : 15 |
| being subtracted from the superior leaves | | 68 | : 20 | : 33 | : 0 |

Therefore the Equinox happens in the Year 1652, from the Calends of *January*, 69 Days Current, that is *March* 9, the Hours being numbred from Midnight 20, or 8 Afternoon, and 33'.

P R O B L E M V I I I .

Given the Ingress of the Sun into $\odot \gamma$, To find the time of the Suns Ingress into the other Signs.

To the time of the Ingress of the Sun into $\odot \gamma$, from Midnight Current, add from the Table of Ingress of the Sun in the other Signs, the Days, Hours and Minutes ascribed to each Sign; and you will produce the Current day from Midnight, together with the Hours and Minutes, in which the Sun enters the given Sign.

Example.

In the Year 1652 $\odot$ enters $\odot \gamma$, the current day from the Calends of *January* 69 D. : 20 H. : 33' : 0"

When doth she enter $\odot \nu$?

Answer, add from the Table 276 : 5 : 40 : 37

They make ————— 346 : 2 : 13 : 37

That is, $\odot$ enters $\odot \nu$, 346 days from the Calends of *January*, that is, *December* the 11th. the second

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second Hour, and 13 Minutes and 37" after Mid-
night.

III.

Of the proper Motion of the Moon.

P R O B L E M IX.

To judge from beholding the Moon, whether it be New or Full Moon, or in what Quarter.

1. If the Light of the Moon be not deformed by any Gap or Bunch, but that the Superficie shews perfect Circular as to Sight, and she shine all Night with her whole Orb, 'tis said to be Full Moon.
2. If she shine with an half Body, as if cut in the middle, she is posited in a Quadrature, of which, in the first she increases, in the other she decreases.
3. If the Moon be hid, and appear not any where in the whole Heaven, either 'tis said to be New Moon, or nigh to be, or hath been.

P R O B L E M X.

To know whether the Moon increases or decreases.

1. Consider the part of the Moon in which 'tis deficient, if it looks towards the East, she increases; if to the West, she decreases.
2. The Moon being beheld in the Evening of the day, increases; being seen in the Morning of the day, she decreases.
3. Country Men say, if the side of the Moon be deficient towards the Left Hand, 'tis an Argument of the Moons increasing; if to the Right Hand, of decreasing: Hence the little Verse.

Dextera decrescit, sed crescit Luna Sinistra.

P R O B L E M XI.

To know when the Moon rises above the Horizon, or sets beneath the same.

1. The Moon being New, she arises and sets with the Sun.

2. At

2. At Full, she arises at Sun setting, and sets at Sun rising.

3. She begins to grow or increase, when she arises after Sun rising, and sets after Sun setting.

4. Beginning to decrease, she rises a little after Sun setting, and sets before its rising.

5. In the first Square increasing, she rises at Noon time, and sets at Midnight.

6. In the second Square decreasing, she rises at Midnight, and sets at Noon time.

Hence the other times of rising may be gathered, if you remember that the rising of the Moon in each day is 48', or one Hour, flower or later, less by a fifth part of an Hour, that is 12 Minutes.

P R O B L E M XII.

To know which part of the Night the Moon shines.

1. The Moon increasing, in the first place, she begins to shine the former part of the Night, also after the Square, she begins to shine the latter part of the Night.

2. Decreasing, first of all, she leaves off to shine the fore part of the Night, also afterwards, she leaves off shining the latter part of the Night.

P R O B L E M XIII.

To know the Age of the Moon by the Epacts and Golden Number.

How the Golden Number is sought, and from it, the Epacts are found, is shewn before, and shall also be Taught hereafter in Chronological Problems.

And that you may acquire the Moons Age.

1. Write the Epact of the given Year, which for Example, suppose 1652, whose Epact is— 29.

2. To

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2. To this subjoin the day of the Month given,
for Example, *July* the ————— 20.
 3. Add also the number of the Months from
March, being numbred inclusive, viz. ————— 5.
 4. The Sum is ————— 54.
 5. Hence take away the Epact ————— 29.
 - There remains ————— 25.
- To wit, the Age of the Moon at the given day.

P R O B L E M XIV.

From the Age of the Moon given, to find by Popular Calculation, how long the same shines every Night.

I. Case.

If the Age of the Moon be under 15.

1. Multiply the Age of the Moon by ————— 4.
 2. The Product divide by ————— 5.
- The Quotient will shew how many Hours and Minutes the Moon will shine that Night.

Example.

How long shone the Moon the 3. of July 1652?

The Age of the Moon is ————— 8.

The Quadruple of it ————— 32.

Which divided by 5 makes ————— $6\frac{2}{5}$.

That is, 6 Hours and 24 Minutes.

II. Case,

If the Age of the Moon be above 15.

1. Double the Excess, whereby the Age of the Moon exceeds 15.
2. Subtract the Double from the Age of the Moon.
3. Multiply the remainder by 4.
4. Divide the Product by 5, and the Quotient shews that you seek.

Example.

Example.

The 20. of July 1652, the age of the Moon is—25.

The excess above 15 is—10.

The double of this—20.

Subtracted from the age of the Moon, leaves—5.

This residue multiplied by 4, is—20.

Which divided by 5, makes—4.

So many Hours the Moon shines the 20. of July after Midnight.

See Fig. 190: Wherein more briefly thus.

In the said Scheme, seek in the second Circle, the Age of the Moon at the day given, and you will find in the Third Circle the Hours, in the fourth the Minutes, in which the Moon will enlighten the Night of that day.

PROBLEM XV.

To find the mean times of the New Moon, or the mean time of the Conjunction of the Sun and Moon of any given Month.

For even as the Motions of the other Planets, are some true or unequal, but others are mean, or middle and equal; so also the Lunar Syzgies or Conjunctions are distinguished, but chiefly the Conjunctions and Oppositions of the Sun and Moon, of which, some are mean, others are true.

A middle or mean Conjunction, is when the Lines of mean Motions of the Sun and the Moon; but the True, when the Lines of the true Motions meet in the same place of the Zodiack.

So 'tis a mean Opposition, when the Lines of mean Motions of the Sun and the Moon, consist in opposite places of the Zodiack: But 'tis a True Opposition, when the Lines of true Motions, consist in opposite places of the Zodiack.

The space of time in which the Conjunctions and Oppositions of the True, may be before or after the Means,

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Means, extends it self to 13 or 14 Hours: But the greatest distance of the True and Mean Quadratures may be 18 or 19 Hours, if we follow the Prutenick Tables; therefore to collect the mean or middle of New Moon.

1. Change your given Year into the Year of the Julian Period.

2. Enter the Table of Lunations with compleat Years, not current; and prefix the *Epocha* of the Julian Period, beginning from Midnight: To it subjoin the time of *New Moon* to the given Years and Months compleat.

3. The collected Sum take away from the Number next greater, found in the Canon of Syzgies.

That which comes forth, is the mean time of *New Moon* of your proposed Month.

I. Example.

When happened the mean time of *New Moon* of the Month of January, of the Year 1653, of the Julian Period 6366.

From the Table of Lunations or Changes.

| | | D. | Ho. | Mi. | Sec. | | | | |
|-------------------------------|---|------|-----|-----|------|----|----|----|----|
| I take out the Radix | — | 19 | : | 1 | : | 50 | : | 25 | |
| | { | 6000 | 5 | : | 5 | : | 01 | : | 55 |
| | | 300 | 16 | : | 12 | : | 03 | : | 20 |
| For the Years | | 60 | 3 | : | 07 | : | 12 | : | 40 |
| | | 5 | 25 | : | 15 | : | 12 | : | 45 |
| The Sum | | 69 | : | 17 | : | 21 | : | 06 | |
| III. Syzgies next greater are | | 88 | : | 14 | : | 12 | : | 10 | |
| From which Subduct the Sum | | 69 | : | 17 | : | 21 | : | 06 | |
| It leaves | | 18 | : | 20 | : | 51 | : | 04 | |

The *New Moon* of January, in the Year 1653.

But from the first *New Moon*, given at the beginning of the Year, all the other Aspects of the ☾ and



☉ may be collected through the whole Year, if you add.

| | D. | Ho. | | |
|------------------------------|----|------|------|------|
| To the first New Moon, being | 18 | : 20 | : 51 | : 4 |
| For the Sextile Aspect * | 4 | : 22 | : 07 | : 20 |
| For the first Square □ | 7 | : 09 | : 11 | : 01 |
| For the Triangle Aspect △ | 9 | : 20 | : 14 | : 41 |
| For Full Moon ♂ | 14 | : 18 | : 22 | : 02 |
| From Full Moon for Triang. △ | 19 | : 18 | : 22 | : 02 |
| For the second Square □ | 22 | : 03 | : 33 | : 02 |
| For Sextile * | 24 | : 14 | : 36 | : 42 |
| For the other New Moon ♂ | 29 | : 13 | : 44 | : 3 |

Another Example.

Seek what day the New Moon happened in August 1653.
Of the Julian Period ————— 6366.

| | D. | H. | M. | S. |
|---------------------------------------|------|------|------|-----------|
| The Radix ————— | 19 | : 1 | : 50 | : 25 |
| Tears { | 6000 | 05 | : 5 | : 01 : 55 |
| | 300 | 16 | : 12 | : 03 : 20 |
| | 60 | 03 | : 07 | : 12 : 40 |
| | 5 | 25 | : 15 | : 12 : 46 |
| Of the single Epact of July | 05 | : 06 | : 51 | : 38 |
| The Sum is | 75 | : 00 | : 12 | : 44 |
| The next greater Ill. Syzgies is | 88 | : 14 | : 12 | : 10 |
| Hence take the Sum | 75 | : 00 | : 12 | : 44 |
| It leaves the New Moon of August | 13 | : 13 | : 59 | : 26 |
| Add ————— | 14 | : 18 | : 22 | : 02 |
| And you'll produce the next full Moon | 28 | : 08 | : 21 | : 28 |

P R O B L E M XVI.

Given some New Moon or Full Moon, to know whether will be Eclipsed.

1. To the time of New Moon or Full Moon, found by the foregoing Problem, inquire from the Table, the

the motion of *Anomaly*, or of the reason of the Moons Latitude.

2. From this *Anomaly* of Latitude, *Ptolomy* hath assigned the following Eclipses.

1. *An Eclipse of the Moon may happen.*

| | | | Sig. | | |
|---------------|---|---------------------------|------|------------|--------------|
| About $\odot$ | } | If the Anomaly | { | be above | 5 : 14 : 48 |
| | | of the Latitude | | be beneath | 6 : 15 : 12 |
| About $\odot$ | } | of the Moon in $\odot$ if | { | above | 11 : 14 : 48 |
| | | the middle or mean is | | beneath | 0 : 15 : 12 |

2. *An Eclipse of the Sun may happen.*

| | | | Sig. | | |
|---------------|---|--------------------------|------|------------|--------------|
| About $\odot$ | } | If the Anomaly of the | { | be beneath | 0 : 20 : 40 |
| | | Latitude of the Moon | | be above | 11 : 18 : 38 |
| About $\odot$ | } | In $\odot$ the middle is | { | beneath | 6 : 11 : 22 |
| | | | | above | 5 : 9 : 20 |

For your better understanding, consider Fig. 191, in which C D E is the Zodiac; G K I L the Orbit of the Sun, intersecting the Lunar way, which G F I H shews, in G and I: And G is the Node ascending, or the Head of the Dragon; I the Node descending, or the Tail of the Dragon.

Therefore since no Eclipses of the Lights of the Sun and Moon can happen, unless these Luminaries are united or opposite, either in these Nodes, or nigh them; the discovery of which is, if the Anomaly of Latitude of the Moon, (which *Ptolomy* indeed begins from the North Limit F, but we from the ascending Node G) be terminated either in those Nodes, or nigh them: The reason of the given Rules is manifest.

Example I.

In the Year 1652 happened a Full Moon, the 14. of March Current, 19 Hours from Midnight, or 7 Hours after Noon, and 8 Minutes and 30 Seconds.

The

The motion of the Sun at this time is $2^{\circ} : 49' : 21''$

The motion of Longitude of $\odot$ — $2^{\circ} : 49' : 21''$

The Anomaly of $\odot$ Latitude Sig. $21 : 15 : 35$

And because it exceeds — $5 : 14 : 48 : 0$

Hence I gather, that the Eclipse of $\odot$ will be about 8.

Another Example.

The same Year happened the mean New Moon, the 29. of March Current, 13 Hours from Midnight, or 1^h : 30' : 31" after Noon.

Sig.

The Sun's motion at that time is $0 : 17 : 22 : 33$

The Moon's motion — $0 : 17 : 22 : 34$

The Anomaly of Latitude — $0 : 06 : 35 : 43$

And because it is beneath — $0 : 20 : 40 : 00$

I understand the Eclipse of the Sun will be about 8.

There are also other ways of knowing Eclipses happening at New Moon and Full Moon, but because this of Ptolomeys is the most simple and easie, we have prescribed it to Learners in this place, which otherways, some prefer as more exact.

III.

Chronological Problems.

Of Astronomical Time.

PROBLEM I.

From the Sun's place known, to find the Diurnal and Nocturnal Arches of the Sun, and from these, the length of the Artificial Day and Night.

I. From

1. *From the Globe or Material Sphear.*

1. The Globe or Sphear being elevated so much as the Latitude of your Place, or the Altitude of the Pole requires.

2. The Degree of the Sun being known, the Ascension will be known; as also the Ascension of the opposite Degree.

3. Subtract that from this, a Circle or 360° being added if need be, and it will leave the Diurnal Arch; the Complement of which to a Circle, will be the Nocturnal Arch.

4. These divided by 15, the Times or Degrees of the Equator, will shew in the Quotient, the Quantity of the Day or Night numbred in Hours.

2. *From the Tables of oblique Ascensions of your Place.*

1. From these, find the Ascension of the Degree of the Place, as also of the opposite Degree.

2. With these operate as was said but now.

Example I.

Sought, what is the Length of the Day, the Sun being in $\odot$ $\S$ the 10 of June?

1. The opposite Degree is $\odot$ Υ , whose Ascension is— $299^\circ : 29'$.

The Ascension of $\odot$ $\S$ will be found ——— $60 : 31$.

This taken away from that, will leave ——— $238 : 58$.

For the Diurnal Arch of the Sun H. M. Sec.

2. Which being changed into Hours and Minutes, is— $15 : 59 : 52$.

Example II.

What is the Length of the Day, the Sun being in $\odot$ Υ ?

The opposite Degree is $\odot$ $\S$, its Ascension ——— $60^\circ : 31'$.

A Circle being added, they make ——— $420 : 31$.

K k

The

The Ascension of ♄ VS is _____ *299° : 29'.*
This taken away from that, leaves _____ *121 : 02.*
The Diurnal Arch, which being changed into } *H.*
Hours and Minutes, is _____ } *8 : 4 : 08.*

3. *By the Ascensional Difference.*

1. The Ascensional Difference between the right and oblique Ascension of the Sun, in Northern Signs, add to 90°, or a Quadrant of the Equator: In Southern, take it away from the same; that which is produced or left, is the Semi-diurnal Arch.

Which being made known, and turned into Hours and Minutes, shews the Length of the Day.

Example.

The Ascensional Difference of ♄ a Northern Sign, is 29° : 29'.
Add a Quadrant _____ *90 : 00.*
It makes _____ *119 : 29.*
The double of this is _____ *238 : 58.*
Which makes Hours 15, Minutes 55, Seconds 52.

2. *The Ascensional Difference of ♄ a Southern Sign, is* 29° : 29'.
Which taken from 90° leaves _____ *60 : 31.*
The double of which is _____ *121 : 02.*
Which makes Hours 8, Minutes 4, Seconds 8.

N O T E.

1. As the Complement of the Diurnal Arch to a whole Circle, is the Nocturnal Arch; so the Complement of the Length of the Day to 24 Hours, is the Length of the next Night.

2. If the Diurnal Arch, or the Day of any Star be sought, or the Time in which 'tis conversant above the Horizon.

1. *Assume the oblique Descension of the same, for Example, of Arcturus, which is* _____ } *235 : 38.*
 2. *To this add a Semi-circle* _____ *180 : 00.*
That

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9.

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*The Complement to 12, that is ————— 04 : 02.
Shews the Hour of Sun-rising.*

P R O B L E M III.

To learn from the Globe, what Hour of a Solar Day or Night, any Star arises and sets.

1. Accommodate the place of the Sun of the given Day, together with the Hour Index, to the Meridian.

2. Turn the Globe from hence, until your Star first come to the Oriental Horizon, and afterwards to the Occidental also.

So the Hour Index will shew you there, the Hour of the Stars Rising; but here, the Hour of its Setting.

P R O B L E M IV.

To find out by the Globe, the Beginning, End, and whole Duration of the Morning and Evening Twylight of the Sun.

1. The place of the Sun, and the Hour Index being brought to the Meridian, and hence, both being turned again towards the East, until in the Circle of Altitude, or of the Vertical, (which the Globe should be furnished with) either the place of the Sun beneath the Oriental Horizon, or the opposite Degree above the Occidental Horizon, acquire 19° ; here, Degrees of Altitude, there, Degrees of Depression.

2. Either being done, both will be done, and the Hour Index will shew the beginning of Morning Twylight; the Hour of Sun-rising shews the end of the same, the time between, shews the whole Duration

3. This whole Duration, add to the Hour of Sun-setting, and it produces the end of Evening Twylight, from which subtract the Hour of Sun-setting, it produces the whole Duration of Evening Twylight, equal to the Duration of Morning Twylight.

P R O B.

P R O B L E M V.

From the observed Altitude of the Sun in the day, or of any Star in the Night time, to find the Hour of the day or Night from them, by the help of the Globe.

Bring the place of the Sun which it obtains that day, together with the Hour Index, to the Meridian; from hence, turn the Globe either towards the East, or towards the West, (according as you observed the Altitude of the Sun or Star, in this or that Coast) until the Degree of the Sun, or the Center of the Star, obtain the same Altitude in the Vertical Circle of the Globe, which you found in the Heaven; for then the Hour Index will shew you the Hour, either before or after Noon.

P R O B L E M VI.

To erect a Scheme or Figure of the Heaven, by help of the Globe, that is, to divide the Heaven into 12 parts, as it were Mansions or Houses, that you may know what Sign or what Star may be found in any of the Houses, at any given moment of time.

This may be exactly done by the Tables of Ascension; nevertheless, though not so truly, it may also be done by the Globe, in the following manner.

1. Describe a Circle on Paper, divide it through the Center into 12 equal parts, which will shew the 12 Houses of Heaven; so, that part that belongs to the Right Hand, is accounted the Seventh; the upper part the Tenth, the lower part for the Fourth, the intermediate parts constitute the intermediate Houses.

2. Direct the Globe exactly to the Coasts of the World, and to the Elevation of your Pole, and to the given moment of time; to wit, bring the Degree of the Sun of that day, to the Meridian, together with the Hour Index, and from hence to the gi-

ven point of time, which the Hour Index will shew in its little Circle.

3. Hence turn your Eyes to the Western Horizon, and observe the Degree of the Equator there constituted, which is the Oblique Ascension of the Seventh House, and from it, number towards the upper part of the Meridian in the Equator 30° ; and at the end of Numeration, transfer the Semicircle of Position, (with which Semicircle the Globe ought to be furnished) which will shew you in the Ecliptick, the Cusp, or beginning of the Eighth House.

4. From hence proceed, and number another 30° in course; and also do the same continually, in going forward to a Third, a Fourth, a Fifth and Sixth, one after another; and as often as you come to 30° , so often apply the Circle of Position to the same, which in the Ecliptick, will shew the points or beginnings, of the Ninth, Tenth, Eleventh, Twelfth, and at last in the Eastern Horizon, the Horizon it self, in the point of Intersection of the Ecliptick, will shew the Horoscope or Cusp of the first House.

5. The Six Houses existing above the Horizon, being found in this manner, you will have the other Six, under the Earth or Horizon, if you assign the opposite Degrees of the Ecliptick, to the opposite Cusps or Points; which if you ascribe to the Lines of Houses of your Circle, you will have the Scheme or Face of the Heaven, to the proposed moment of time: neither doth any thing more remain, unless, that also you would inscribe the most noted Fixed Stars, and the proper and true Motions of the Planets, each to its House, to the given Moment of time, computed from an *Ephemerides*.

P R O B L E M VII.

To make Sun Dials by the aid of the Globe.

Sun Dials are various, but the most common and easie, and of which the rest for the most part require the Knowledge, are *Horizontal Dials*, which are made on a Plane lying parallel to the Horizon; and the *Vertical*, which behold the Meridian direct,
or

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or infist at Right Angles perpendicularly of the Meridian Line; Concerning these therefore we shall only Treat, how they may be made, by help of the Celestial Globe.

1. *Of the Horizontal.*

If you would make this:

1. On an Horizontal Plane, or some Paper, describe Two Lines, cutting each other at right Angles through the Center, of which Lines, that which tends downwards is the Meridian Line, the other, which cuts this crosswise at right Angles, from the Right Hand to the Left, by its Extrems, East and West, shews the Equinoctial points.

2. From the point of Interfection, describe a Circle, divide it either really into 360° , or having a Semicircle already divided, divide it into like parts.

(For if a Semicircle be made of any durable matter, as Brass, or Dutch Paper, or Vellum, and divided into 180° , you may, by the help of it, in any Circular Line, assume what parts you please, if you accommodate that Semicircle so to your Periphery, that the Diameter answers to the Diameter, and the Center to the Center; and to any Degree of it which you desire, impress a small mark or point on your Paper, and afterward, draw from the Center through this point, either an apparent or an occult Line, which will design and cut off also, the same number of parts in your Periphery)

3. Then in the same Circle, the right Line tending downwards, is the Line of the Twelfth Hour, or Twelve a Clock; the other which cuts this crosswise through the Center at right Angles, shews the Sixth Hour before Noon and after Noon, always being distant by a Quadrant of a Circle, from the Line of the 12th. Hour.

And the rest of the Hour Lines are unequally distant from the 12th. Hour, and to one another, which may be found in this manner.

4 1. Bring under, the Colure of the Equinoxes

in the Globe, to the Meridian, together with the Hour Index.

2. Move forward $\circ \text{ } \nu$, through which the Colure of the Equinoxes passes, towards the West, so long, until 15° of the Equator, or one Hour in the Hour Circle, pass the Meridian.

3. Afterwards number from the Meridian, and note the Degrees of the Horizon, which the Colure of the Equinoxes touches; and the same Degrees, you must also number from the Meridian Line in the lower part of your Circle, as well towards the East as towards the West, and through the end of Numeration, draw a Line from the Center, which will indeed towards the West, be the Line of the 11th. Hour before Noon, but towards the East, the Line of the first Hour after Noon.

4. After this, move forward again the same $\circ \text{ } \nu$ 15° , or the Hour Index again to the next Hour, and your Colure will shew you in the Horizon, the distance of the Hour, 10 before, and 2 after Noon, from the 12th. Hour, or the Meridian Line.

5. And so go on until you come to the Sixth Hour, being 90° Degrees distant from the Meridian Line.

6. And these 6 Hours being found on both sides, the other also in the upper Circle may be had: for by how much the 7 and 8 before Noon, are distant from the Line of the 6 Hour; the 5 and 4 after Noon, are so much distant also from the same 6 Hour; also the 7 and 8 after Noon, are distant the same also, as the 5 and 4 before Noon.

And more Hours, our Region of Hours admit not, because the longest day extends not it self, more than from 4 in the Morning to 8 in the Evening.

5. All these things being performed, the Style or Thread must be fixed in the Center, and elevated above the Meridian Line, according to the Altitude of the Pole of your place: Then the Sun shining, the shadow of it will shew the Hours, if so be the East or West

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West side of the Dial be applied to a Meridian Line, drawn on the Plane of the Horizon.

N O T E.

Because the Colure in the Globe, doth not so exactly determine the Degrees of the Hour distances, you may consult the following

Table

Of Hour Arches, for an Horizontal Dial, computed for the Latitude of London, where the Pole is elevated, or whose Latitude is $51^{\circ} : 32'$.

| Hours | Quarters | Horary distance |
|--------|----------|-----------------|
| 12 : 0 | 0 | 0 |
| | 1 | 2 : 56 |
| | 2 | 5 : 53 |
| | 3 | 8 : 51 |
| 11 : 1 | | 11 : 51 |
| | 1 | 14 : 53 |
| | 2 | 17 : 58 |
| | 3 | 21 : 07 |
| 10 : 2 | | 24 : 19 |
| | 1 | 27 : 37 |
| | 2 | 31 : 00 |
| | 3 | 34 : 29 |
| 9 : 3 | | 38 : 04 |
| | 1 | 41 : 46 |
| | 2 | 45 : 35 |
| | 3 | 49 : 32 |
| 8 : 4 | | 53 : 36 |
| | 1 | 57 : 48 |
| | 2 | 62 : 07 |
| | 3 | 66 : 34 |
| 7 : 5 | | 71 : 06 |
| | 1 | 75 : 45 |
| | 2 | 80 : 27 |
| | 3 | 85 : 13 |

From this Table the distances of the Hours, and Halves, and Quarters of Hours, may be had exactly.

II. *Of the Vertical Dial, be- holding the Meridian or South ex- actly. For Latitude $51^{\circ} : 32'$*

The Method of making this Dial, is almost the same in all things with the former, these only excepted.

I. When you move forward the Globe for each Hour, thro' 15° of the Equator, we num- ber the removing of the Equi- noctial Colure, not in the Ho- rizon, as before, but in the Circle of Altitude, considered from the Vertical point, to the true point of West; again in a Retrograde manner, we num- ber the Hours before Noon to- wards the East, because the shadowy Body or Style, always casts its shadow in the part op- posite to the Luminous Body, or the Sun.

2. We cannot proceed beyond the Sixth Hour, before and after Noon, since this kind of Dial doth not contain more than 12 Hours, 6 before Noon and 6 after.

3. The Style must be fixed in the Center, and elevated above the Hour Line of 12, so much, that the Angle at the Center, answer not the Elevation of the Pole, as before, but the Altitude of the Equator; and so the Style will shew the Hours, if the Plane of the Dial directly behold the South, and the Line of the 12th. Hour, stands perpendicularly on the Meridian Line, drawn on the Plane of the Horizon.

4. And for the more exact taking the distances of the Hour Lines, note the following Table.

Of Arches of Hours, for a Vertical Dial, computed for the Latitude of London $51^{\circ} : 32'$.

| Hou. & Half | Ho. dist. |
|---------------|-----------|
| 12 | 0 |
| $\frac{1}{2}$ | 4 : 41 |
| 11 | 9 : 28 |
| $\frac{1}{2}$ | 14 : 27 |
| 10 | 19 : 45 |
| $\frac{1}{2}$ | 25 : 31 |
| 9 | 31 : 53 |
| $\frac{1}{2}$ | 39 : 02 |
| 8 | 47 : 08 |
| $\frac{1}{2}$ | 56 : 20 |
| 7 | 66 : 42 |
| $\frac{1}{2}$ | 78 : 03 |
| 6 | 90 : 00 |
| Sty. Heig | 38 : 28 |

O F

Politick Time.

P R O B L E M VII.

To change the Civil Hours, cut in two in the middle, from Noon and Midnight, into continual Hours.

If you assume the beginning of the day from Noon, the Hours before the following Noon, add to 12: If from Midnight you assume them, add the Hours after the following Noons.

P R O B L E M VIII.

Any certain Jewish Hour of a day being given, and numbred from Sun rising, how much is it in our equal Hours, numbred from Noon or Midnight.

The length of the given day being foreknown, how much it is with us, for Example, of 16 Hours

in the day of the Summer Solstice; if we seek how many of our equal Hours, answer 5 of the *Jewish* unequal ones, when the Proportion is the same of the unequal Hours given, to the equal ones sought, as is of 12 unequal Hours of the whole day given, to the equal Hours of the same day; we must say by the Golden Rule of Proportion.

Unequal Hours Make equal | *What make unequal.*

12. | 16. | 5.

The Operation being made, it produces $6\frac{2}{3}$, or 40', the Numeration being made from Sun rising.

To this, if you add the Hour of Sun rising, which is the 4th. Hour from Midnight, the fifth *Jewish* Hour, will be with us the Tenth, with 40 Minutes.

P R O B L E M IX.

Given a day of a Jewish Month, to know to what day of which Julian Month it agrees.

This thing is slippery and uncertain, as to the Years and Months assigned in Holy Scripture, as is also said before.

1. Because 'tis not certainly known what kind of Year the *Hebrews* used, before and after the Captivity of *Babylon*, even to the second Destruction of *Jerusalem*.

2. But appointing them to be composed of some Lunar Year, and bound to those, used from the Times of *Moses*, and understood the same always in Holy Scripture, as some will have it; nevertheless because also 'tis uncertain.

3. Whence they drew the beginning of their Months? Whether from some Calculation of Lunar Motion? Or, as the Ancient *Hebrews* witness, from the first appearance of the Moon after *New Moon*, which varies according to the diversity of Times and of places; and sometimes it shewed it self conspicuous the first day, sometimes the second, sometimes also

also at last on the Third day from its Conjunction with the Sun. And

4. Moreover, also the reason of Intercalation, which the *Lunar* Years required, if we believe the *Jews*, they were tyed to no certain Laws, for they did not intercale, only when the defect of the *Lunar* Year from the *Solar*, required the intercalation of one Month, but also, if the *Jewish* Passover happened when the Fruits were not ripe, or the Trees not Budded, or the Bridges being overturned by Floods of Water, denied them passage to the Feast of the Passover, or the Lambs and Kids for Sacrificing to the Feast of the Passover, were not at their full growth, as appears from the *Rabbins*, or *Jewish* Doctors.

From these Causes the beginning of the *Jewish* Years, and consequently also the days of the rest of the Months; cannot be so defined, that there may not arise a doubt, not only of some days, but also by an whole Month, by reason of the intercalation used or omitted, unless some manifest Character from some other place help us, which might lead us to the certain knowledge of the Day or Month of the *Jews*.

But if any one should believe all the Years in the Bibles, to have been *Lunar*, and to begin from the first New Moon next after the Vernal Equinox, and hence would institute, any comparing of the *Jewish* Day, with the same day in the *Julian* Year, he must do thus.

1. He must reduce the *Jewish* Year to the *Julian* Year.

2. To this *Julian* Year, he must seek the day of the Vernal Equinox.

3. Likewise the *Julian* Month of New and Full Moon, in which the Equinox happened.

4. If the Full Moon of that Month happened before the Equinox, the following New Moon, will be the New Moon, of the first *Jewish* Month *Nisan*:

But

But if it happened after the Equinox, that Full Moon it self will be the Full Moon in *Nisan*, and the preceeding New Moon, the New Moon in *Nisan*; which if you begin the Mean, the space of two days after New Moon, you will wander not far from the Truth: And from the first day of the Month *Nisan*, being given in the *Julian* Year, 'tis easie to get the beginning of the other Months, by Altern Addition of 30 and 29 Days.

Example.

In the Year of Christian Epocha— — — — — 34.

'Tis sought how many days of which Month, was the 14 of the Jewish *Nisan*, in which Christ Suffered for the Salvation of Mankind? Answer.

1. The Equinox in that Year happened the 22 day of March current, 2 Hours after Noon.

2. The foregoing New Moon happened the 8 day of March, 8 Hours after Noon, and the Full Moon following was the 23 of March, 2 Hours after Noon, whence truly it may be accounted for the Paschal New Moon, because it followed the Equinox: But because in that time it was the common Opinion of all, that the Equinox should fall on the 25 of March, therefore the Jews, neither would have that New Moon for the Index of the Month *Nisan*, nor this Full Moon for the Full Moon of the Passover.

3. But the following New Moon happened the 9th. of April, 8 Hours before Noon.

4. Which if therefore, April the 9th. the Moon was supposed first seen, the first of *Nisan* happened on the 10 of April, and the 14 of *Nisan* on the 23 of April, which was the Sixth day of the Week, and the day of Passion of our Saviour Jesus Christ.

P R O B L E M X.

To change the Year of Christian Epocha used, into the Year of the Julian Period, and the contrary.

To

To the Year of the Christian Epocha given, add Years 4713, and you will have the Year of the *Julian* Period.

If you Act on the contrary, and from the Years of the *Julian* Period, you take away 4713 Years, you will have the Years of the Christian Epocha.

Example.

The present Year ————— 1700.

How much is it of the *Julian* Period?

Answer, add ————— 4713.

And you will produce ————— 6413.

Again this very Year, how much is it

Of the Christian Epocha? Take away 4713.

You will find the Year ————— 1700.

P R O B L E M X I

From a given Year, numbred back again from the Christian Epocha, to know how much it is of the *Julian* Period, and the contrary.

Take away the Year before the Christian Epocha given, from 4714, it leaves the Year of the *Julian* Period sought.

On the contrary, the Year of the *Julian* Period, being given beneath 4713, take away from the Years 4714, and you will have the Year Current before the Christian Epocha.

Example.

The Year before our Epocha ————— 2308.

How much is it of the *Julian* Period? Answer

From the Years ————— 4714.

Take away. ————— 2308.

There are left the Years of the *Julian* Period ————— 2406.

Again, this Year of the *Julian* Period, how much is it before the Christian Epocha? Answer.

From the Years ————— 4714.

Take away ————— 2406.

And it leaves Years ————— 2308.

Problem

P R O B L E M XII.

To know the interval or space, between two Years of the Julian Period given.

Subtraet the lesser from the greater, the residue shews what you seek.

Example.

The Year of the Julian Period in which Rome was Built 3962.

How much is it distant from the Year of the Julian Period 3203.

In which the Israelites went forth from Egypt?

Answer, Take this from that, and there is left, Years 759

P R O B L E M XIII.

To discern the Bissextile Year from the Common Years.

In the Years of the Julian Period.

Divide the Years given by 4, Unity in the remainder, denotes the Bissextile Year, 2 the Second, 3 the Third, and 0 or nothing denotes the Fourth from Bissextile.

In Years of the Christian Epoch.

Divide the given Years by 4, nothing remaining, is a Sign or Token of the Bissextile Year, if 1 remain, it is the next, 2 the Second, 3 the Third from Bissextile.

**O F
Ecclesiastick Time.**

P R O B L E M XIV.

To find the Cycles of the Sun, of the Moon, and of Indiction.

1. In Years of the Julian Period.

The given Years } 28. } The residue } Of the Sun.
divide by — } 19. } shews the } Of the Moon.
 } 15. } Cycle. } Of Indiction.

Or if 0 remain, it is a compleat Cycle, and the last to be had for the Year of the Cycle.

2. In

2. *In Years of the Christian Epocha.*

To the given Years $\frac{\quad}{\text{add}}$ $\left\{ \begin{array}{l} 9. \\ 1. \\ 3. \end{array} \right\}$ For the Cycle $\left\{ \begin{array}{l} \odot \\ \text{Indict.} \end{array} \right\}$ The Sum $\left\{ \begin{array}{l} 28. \\ 19. \\ 15. \end{array} \right\}$ Divide By

The remainder is the Cycle sought; if 0 remains the Cycle is assumed whole.

P R O B L E M XV.

From the Cycle of the Sun given, to find the Dominical Letter.

1. *In the Julian Year this is a perpetual Method.*

With the number of the Cycle of the Sun, enter the following Table, in which the first or uppermost Rank or Column, is the number of the Cycle: The Second, of the *Dominical Letter*: The third, points out the days of the Week of the Calends of *January*.

| | | | | | | | | | |
|--------------|--------------|---------------|---------------|--------------|--------------|---------------|---------------|--------------|--------------|
| 1
GF
2 | 2
E
4 | 3
D
5 | 4
C
6 | 5
BA
7 | 6
G
2 | 7
F
3 | 8
E
4 | 9
DC
5 | |
| 10
B
7 | 11
A
1 | 12
G
2 | 13
FE
3 | 14
D
5 | 15
C
6 | 16
B
7 | 17
AG
1 | 18
F
3 | |
| 19
E
4 | 20
D
5 | 21
CB
6 | 22
A
1 | 23
G
2 | 24
F
3 | 25
ED
4 | 26
C
6 | 27
B
7 | 28
A
1 |

The Common Years have one *Dominical Letter*, the Bissextile two, of which, the former hath place from the beginning of the Year, to the 25 of *February*, the other, from hence to the end of the Year: The reason is this, because the Intercalated day, to be put in after the 24 of *February*, acquires not another Letter, but that which the precedent day hath,
it

it retains; and hence the following Lords Day obtrudes another Letter than the antecedent had,

In the Gregorian Year.

The Cycle of the Sun is indeed always the same with the Cycle of the *Julian* Year, but the Dominical Letters are neither the same, nor do they always agree unchangeably with the Years of the Cycle; whence for other Ages, other Tables are to be made, but the following will be in use to the Year of Christian Epocha 1700.

A Table of the Solar Cycle, of the Dominical Letter, and the Week days of the Calends of January.

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| I | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| CB | A | G | F | ED | C | B | A | FG | |
| 6 | 1 | 2 | 3 | 4 | 6 | 7 | 1 | 2 | |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | |
| E | D | C | BA | G | F | E | DC | B | |
| 4 | 5 | 6 | 7 | 2 | 3 | 4 | 5 | 7 | |
| 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| A | G | FE | D | C | B | AG | F | E | D |
| 1 | 2 | 3 | 5 | 6 | 7 | 1 | 3 | 4 | 5 |

P R O B L E M XVI.

From the Cycle of the Sun given, and the Dominical Letter, and a certain day of any Julian Month; to know what day of the Week that day is.

1. Seek the given day in some common Almanack, *Julian* or *Gregorian*, and behold what Letter it hath ascribed to it: Or if you have no Almanack at hand, you must

L I

compute

compute how many days from the Calends of *January*, the given day is ? the number of which, if you divide by 7, the remainder will shew the Letter of that day in its Natural Order; for if 1 be over and above, the Letter of that day is A, if 2, 'tis B, and so on. 2. And the Letter of the day being known, if you compare it with the Dominical Letter of that Year, you will easily come to the knowledge of the Week day, by going forward or backward.

Example.

Christ was Born in the Year of the Julian Period 4707, of which the Cycle of the Sun was 3, the Dominical Letter D, the 25 of December, whose Letter was B; which also may be found, if the 25 of December, that is, the day from the Calends of January, being 359 Days, be divided by 7, the remainder is 2, the Index of the Letter B.

If now from the Dominical Letter D, you number back again even to B, you will find that the 25 of December in that Year, was the Sixth day of the Week.

Briefer thus.

To the day given, numbred from the Calends of January, add the number of the day of the Week which the Year begins from this Cycle, Unity being first taken away: Divide the Sum by 7, the residue will shew the Week day.

Example.

*The Year of the Julian Period ————— 4707.
Of which the Cycle of the Sun is ————— 3.
It begins from the Week day ————— 5.
Take away Unity, and to the day from }
the Calends of January ————— } 359.
Add 4, that they make days ————— 363.
Which divided by 7, leaves ————— 6.
To wit, the Week day of Christs Nativity.*

Problem

P R O B L E M XVII.

Given the Dominical Letter, likewise the Cycle of the Moon, or the Golden Number; to find the Paschal Term or Easter, and from it, the Lords Day of the Feast, of the Old Style.

With the Golden Number, enter the following Table, which shews the Feast of Easter to any Golden Number given, with its Letter.

A Table of the Terms of Easter.

| Cycle of ϵ | Terms of Easter | Cycle of ϵ | Terms of Easter | Cycle of ϵ | Terms of Easter. |
|---------------------|-----------------|---------------------|-----------------|---------------------|------------------|
| 1. | Apr. 5. D | 8. | Apr. 18. C | 15. | Apr. 1. G |
| 2. | Mar. 25. G | 9. | Apr. 7. F | 16. | Mar. 21. E |
| 3. | Apr. 13. C | 10. | Mar. 27. B | 17. | Apr. 9. A |
| 4. | Apr. 2. A | 11. | Apr. 15. G | 18. | Mar. 29. D |
| 5. | Mar. 22. D | 12. | Apr. 4. C | 19. | Apr. 17. B |
| 6. | Apr. 10. B | 13. | Mar. 24. F | | |
| 7. | Mar. 30. E | 14. | Apr. 12. A | | |

The next Lords Day after this Letter ascribed to the day of the Month, which the Dominical Letter foreknown will shew, will be the Feast of the Paschover or Easter, in the Old *Julian* Kalendar, unless the Letter of the Term be the same as the Dominical Letter; for then the Feast of Easter is referred to the following Lords Day.

Example:

What day is Easter Celebrated in the Year of Christian Epocha ————— 1654.

Answer, The Cycle of the Sun of this Year is — 11.

The Dominical Letter ————— A.

The Golden Number ————— 2.

Which shews the Feast of Easter is March the 25th. with the Letter G, which next follows the Dominical Letter A: I say therefore the Feast of Easter was Celebrated March the 26th.

P R O B L E M XVIII.

From the Golden Number given, to find the Epacts.

1. *In the Julian Year, by a perpetual Method.*

Enter the following Table, and to the Golden Number you will find the Epacts subscribed.

| | | | | | | | |
|-------------|--------|------|-----|-------|------|-------|------|
| Gold. Numb. | 1. | 2. | 3. | 4. | 5. | 6. | 7. |
| Epacts | XI | XXII | III | XIV | XXV | VI | XVII |
| Gold. Numb. | 8. | 9. | 10. | 11. | 12. | 13. | 14 |
| Epacts | XXVIII | IX | XX | I | XII | XXIII | IV |
| Gold. Numb. | 15. | 16. | 17. | 18. | 19. | | |
| Epacts | XV | XXVI | VII | XVIII | XXIX | | |

2. *But in the Gregorian Year.*

The Method of finding the Epacts, is the same by the Golden Number, which in both Years is always the same; but the Tables are not perpetual, and the following will last to the Year of the Christian Epocha 1700.

A Table of Epacts in the Gregorian Year.

| | | | | | | | |
|-------------|-------|------|-------|------|-----|------|------|
| Gold. Numb. | 1. | 2. | 3. | 4. | 5. | 6. | 7. |
| Epacts | I | XII | XXIII | IV | XV | XXVI | VII |
| Gold. Numb. | 8. | 9. | 10. | 11. | 12. | 13. | 14. |
| Epacts | XVIII | XXIX | X | XXI | II | XIII | XXIV |
| Gold. Numb. | 15. | 16. | 17. | 18. | 19. | | |
| Epacts | V | XVI | XXVII | VIII | XIX | | |

P R O B L E M XIX.

Given, either the Golden Number for the Old Julian Year, or the Epacts of the new Gregorian Year: To find the Feast of Easter of the Christians, in both Calendars.

1. *For*

Part III. Ch. V. Its use in Chronological Probl. 517

1. For the Feast of the Julian Year.

1. With the Dominical Letter, which is also foreknown, enter the following Table.

2. In the Column of Golden Numbers, to which the Dominical Letter is prescribed, seek the Golden Number of the Year.

3. In which rank of Numbers you find the Golden Number, over against it, under the Title of Feast, you will find the Month and Day of Celebrating *Easter*.

2. For the Feast of the Gregorian Year.

The Method is altogether the same, unless that instead of the Golden Number, we use the Epacts of the *Gregorian* Year, under the Dominical Letter and Title of Epacts.

A TABLE OF THE FEAST OF EASTER.

| Dominic.
Letter | Golden
Numbers | Epacts | Feasts |
|--------------------|-------------------|---------------------|-----------|
| A. | 2. 5. 13. 16. | 23. 21. 19. | March 20. |
| | 7. 10. 15. 18. | 18. 16. 15. 13. 12. | Apr. 8. |
| | 1. 4. 9. 12. | 10. 8. 7. 5. | Apr. 9. |
| | 3. 6. 11. 14. 17. | 4. 2. 1. 29. | Apr. 16. |
| | 8. 9. | 27. 26. 24. | Apr. 23. |
| B. | 2. 5. 13. 16. | 23. 21. 19. 18. | March 27. |
| | 4. 7. 10. 15. 18. | 16. 15. 13. 12. | Apr. 3. |
| | 1. 9. 12. 17. | 16. 8. 7. 5. 4. | Apr. 10. |
| | 3. 6. 11. 14. | 21. 29. 27. 2. | Apr. 17. |
| | 8. 19. | 26. 24. | Apr. 24. |
| C. | 2. 5. 10. 13. 16. | 23. 21. 19. 18. | Mar. 28. |
| | 4. 7. 15. 18. | 16. 15. 13. 12. | Apr. 4. |
| | 1. 6. 9. 12. 17. | 8. 7. 5. 4. 10 | Apr. 11. |
| | 3. 11. 14. 19. | 2. 1. 29. 27. | Apr. 18. |
| | 8. | 26. 24. | Apr. 25. |
| D. | 16. | 23. | Mar. 22. |
| | 2. 5. 10. 13. | 21. 19. 18. 16. | Mar. 29. |
| | 4. 7. 12. 15. 18. | 15. 13. 12. 10. | Apr. 5. |
| | 1. 6. 9. 17. | 8. 7. 5. 4. 2. | Apr. 12. |
| | 3. 8. 11. 14. 19. | 20. 27. 26. 24. | Apr. 19. |

| Dominical Letter | Golden Numbers | Epafts | Feasts |
|------------------|-------------------|---------------------|-----------|
| E | 5. 16. | 23. | March 22. |
| | 2. 10. 13. 18. | 21. 19. 18. 16. 15. | March 30. |
| | 1. 4. 7. 12. 15. | 13. 12. 10. 8. | Apr. 6. |
| | 6. 9. 14. 17. | 7. 5. 4. 3. 1. | Apr. 13. |
| | 3. 8. 11. 19. | 29. 27. 26. 24. | Apr. 20. |
| F | 5. 16. | 23. 21. | March 24. |
| | 2. 7. 10. 13. 18. | 19. 18. 16. 15. | March 31. |
| | 1. 4. 12. 15. | 13. 12. 10. 8. 7. | Apr. 7. |
| | 3. 6. 9. 14. 17. | 5. 4. 2. 1. | Apr. 14. |
| | 8. 11. 19. | 29. 27. 26. 24. | Apr. 21. |
| G | 5. 13. 16. | 23. 21. (13. | March 25. |
| | 2. 7. 10. 18. | 19. 18. 16. 15. | Apr. 1. |
| | 1. 4. 9. 12. 15. | 12. 10. 8. 7. | Apr. 8. |
| | 3. 6. 14. 17. | 5. 4. 2. 1. 29. | Apr. 15. |
| | 8. 11. 19. | 27. 26. 24. | Apr. 22. |

P R O B L E M XX.

Being known, 1. The Dominical Letter. 2. The Golden Number. 3. The Feast of Easter. 4. Also the Nativity of Christ, unchangeably fixt the 25 of December: To design the rest of the Feasts of the whole Year, and to adorn the whole Calendar.

I.

The 4. 3. 2. 1. Lords Days next before the Nativity of Christ, are called Advent.

II.

1. The Lords Day following the same, is called the Lords Day after the Nativity, which is none, if the Nativity of Christ fall on the Lords Day.

2. The Lords Days next following the Feast of Easter, are denominated from Singings, and old Rites

Rites and Ceremonies of the Church, in this manner.

1. *Quasimodogeniti.*

3. *Fubilate.*

2. *Misericordia.*

4. *Cantate.*

5. The Voice of Mirth, otherwise also called the Lords Day of Rogation, and of the Cross; of which Week, the 5th. Week day is Celebrated the Feast of Christs Ascension into Heaven.

6. *Exaudi*, or hear ye.

7. The Feast of Pentecost, or the Mission of the Holy Spirit, commonly called *Whitsontide*.

8. The Lords Days of the Holy Trinity, which follows; the Lords Day after Trinity, I. II. III. IV. and so on, in numbring, even to the first Lords Day of Advent.

9. The following Lords Days, next antecede the same Feast of Easter.

1. Palm Sunday. 6. *Invocavit*, or *Quadragesima*.

2. *Judica.*

7. *Esto mihi*, or *Quinquagesima*.

3. *Latate*, or to 8. *Sexagesima*.

Rejoyce.

4. *Oculi.*

9. *Septuagesima*.

5. *Reminiscere.*

III.

The 1. day of *January* is Celebrated the Feast of the Circumcision of Christ, and begins the new Year.

Sometimes the Lords Day following this day, (rejoycing only in the Name) is called the Lords Day after the Circumcision: But it is not, if the Feast of Circumcision fall on the Lords Day, or the second day of the Week. The Sixth day of *January* is kept the Feast of *Epiphany*, or the Manifestation of Christ to the *Gentiles*, every where of Three Kings: The Lords Days following this Feast are

called the 1. 2. 3. &c. after *Epiphany*, to *Septuagesima* Lords Day, of which sometimes there are more, sometimes fewer, according as the Feast of Easter is, sooner or later.

Therefore the Months of the *Julian* or *Gregorian* Year being designed, with their days, which either the Common or the Bissextile Year requires, and the Feasts, as well movable as immovable, which we have also reckoned up before, with their agreeing days ascribed; also the usual days to the rest, with their Names every where put to them, will describe an whole Ephemerides or Calendar, or as the *Arabs* call it, an Almanack for the whole Year, that is, you will have described the Number and Distribution of the Days; to which the more Learned, omitting the Vanities of Astrologers, may add the Celestial Motions.

Therefore we were willing to adjoin this little Compendium of Ecclesiastick Computation, as they call it, to this Astronomick Chronology, that this profitable and necessary Doctrine, (which in old time was first proposed to Youths in Schools, and wholly required by Ecclesiastick Persons) should not altogether be forgotten, but as it were, by way of recovery, be recalled into the Schools, especially in those wherein Youth is Educated, for Ecclesiastick Affairs: For neither is it Laudable, that a Bishop or Presbyter of a Church should be ignorant of the reason of ordaining the Feasts, and request this Doctrine from Astrologers alone; some of which defile it with filthy foolish Trifles.

An APPENDIX

Concerning

Historical Chronology.

TIME is the measure or number of Order, and of succession of things done: As therefore 'tis in vain to use Abstract Numbers, unless something subsist which is numbred; so the Examination of Time is barren and unfruitful, unless it be also accommodated to things done in time.

Hence descends Historical Chronology, in which the things themselves are bound to the times in which they are done, and the times to the things: This many handle largely; we, least we should seem to tire our selves with only a bare Contemplation of time, as before, we have recited some Epochas, whence times flow, so now we shall briefly touch upon the whole Head of all Chronology, that Learners may have as it were sure Forms, to which they may refer, whatsoever occurs to them in Reading of History.

For the determination of Time, is as it were the Life of Histories, without which they are more like Fables than Histories.

And we shall begin from the Fall of our first Father *Adam*, which in Holy Scripture is subjoined immediately to the Week of the Mundane Creation: Whence *Moses* seems to have deduced his Times; he of all Historians, whose Writings have come to us, is the first and the most truest.

We

We use commonly the noted *Julian* Years, which nevertheless in the first Ages of the World, we begin not from the Calends of *January*, but from the end of the antecedent *October*, after the manner then in use: *Therefore follows*

CHAP. I.

In which the whole Course of all Ages, from the Fall of Adam, even to our Time, is divided into X Intervals or Distances.

And they are

| 1. From the Fall of <i>Adam</i> , to the General Flood. | Years of Distance
1656½. | Years of the world collected
1656½. |
|---|-----------------------------|--|
| 2. From the Flood, to the first Promise of Christ made to <i>Abraham</i> , and the calling of him into the Land of <i>Canaan</i> . | 367. | 2023½. |
| 3. From the calling of <i>Abraham</i> , to the going forth of the <i>Israelites</i> from <i>Egypt</i> , and the Law given by <i>Moses</i> . | 429. | 2452½. |
| 4. From this going forth, to the laying the Foundation of the Temple of <i>Jerusalem</i> , Built by <i>Solomon</i> . | 479. | 3931½. |

5. From

5. From the Foundation of the Temple, to the first Destruction of the same, made by *Nebuchadnezzar*, King of *Babylon*; whence the beginning of the *Babylonish Captivity*.

Years of years of the
Distance world collected

427 $\frac{1}{2}$. 4359.

6. From the *Babylonick Captivity*, to the dissolving the same, made by *Cyrus*, the first King of the *Persians*.

65 $\frac{3}{4}$. 4424 $\frac{1}{4}$.

N. B. or note well,

(*Holy Writ attributes 70 years to the Babylonick Captivity; but it understands the Mystical Years, proper to the Jubilees and Sabbaths, of days 343, of which, the 70 Years constitute only 65 $\frac{1}{4}$ of Julian Years.*)

7. From the dissolving the Captivity, to the Edict of the base Born *Darius*, King of the *Persians*, concerning the Restoration of the Temple and City.

118 $\frac{1}{2}$. 4543 $\frac{1}{4}$.

8. From the Edict of *Darius*, to the Passion of Christ.

453 $\frac{1}{4}$. 4996 $\frac{1}{4}$.

9. From the Passion of Christ, to the end of the *Jewish War*, which happened in the Year of the Christian Epocha 71.

33. 5029 $\frac{1}{2}$.

10. From the beginning of the *Jewish War*, to the end of the Year of the Christian Epocha 1652.

1585 $\frac{1}{4}$. 6615 $\frac{1}{4}$.

C H A P. II.

The first distance or interval from the Fall of Adam, to the Flood, distributed into shorter Articles.

| | | Years of distance | Years of the world collected |
|--|-----------------------------|---------------------|------------------------------|
| S <i>Eth.</i> | <i>Adam.</i> | 130. | 130. |
| <i>Enos.</i> | <i>Seth.</i> | 105. | 235. |
| <i>Cainan.</i> | was Born <i>Enos.</i> | 90. | 325. |
| <i>Mahaleel.</i> | in the Year <i>Cainan.</i> | 70. | 395. |
| <i>Jared.</i> | of the Age <i>Malaleel.</i> | 65. | 460. |
| <i>Enoch.</i> | of <i>Jared.</i> | 162. | 622. |
| <i>Mathusal.</i> | <i>Enoch.</i> | 65. | 687. |
| <i>Lamech.</i> | <i>Mathusalem.</i> | 187. | 874. |
| <i>Noah.</i> | <i>Lamech.</i> | 182. | 1056. |
| In the Year of <i>Noahs</i> Age | | 599 $\frac{1}{2}$. | 1655 $\frac{1}{2}$. |
| began the Flood, during Years | | 01. | 1. |
| Whence the Sum of Years is made, from the Fall of <i>Adam</i> , to the end of the Flood. | | | 1656 $\frac{1}{2}$. |

C H A P. III.

Explication the II. Of Distances from the Flood, to the calling of A B R A H A M.

T *His Interval in like manner is explained by the Ages of the Patriarchs in Genesis, Chap. II. and 12, which may be presented in such a Scheme as this.*

Arphaxad

Arphaxad was Born in the Year from the ending of the Flood.

| | | Years of the | Years of the |
|-----------------|---------------------------|--------------|-----------------|
| | | distances | world collected |
| <i>Elah.</i> | <i>Arphaxad.</i> | 35. | 1694. |
| <i>Eber.</i> | <i>Selah.</i> | 30. | 1724. |
| <i>Peleg.</i> | Born in the <i>Eber.</i> | 34. | 1758. |
| <i>Reu.</i> | Year of the <i>Peleg.</i> | 30. | 1788. |
| <i>Sarug.</i> | Age of <i>Reu.</i> | 32. | 1820. |
| <i>Nachor.</i> | <i>Sarug.</i> | 30. | 1850. |
| <i>Thare.</i> | <i>Nachor.</i> | 29. | 1879. |
| <i>Abraham.</i> | <i>Thare.</i> | 70. | 1949. |

And *Abraham* was called, and received the Promise of Christ, from his own Country into the Land of *Canaan*, in the Year of his Age.

Hence the Sum is made

| | |
|------|-------|
| 75. | 2024. |
| 367. | |

N. B.

Some appoint Abraham to be Born not in the 70, but the 130 Year of his Father Thare, but foolishly;
For

1. *They cannot affirm it by any Testimony of Scripture, but only they endeavour it, by the ambiguous and doubtful Ratiocination taken from the Narration of Stephen, Acts 7. and in this manner.*

2. *Since from the Age of Abraham, all the following Chronology of the Bible depends, the whole certainty of it fails, if we recede from the 70 Year of Thare, in which the Scripture consents Abraham was Born.*

For of the 130 Year of Thare, the Scripture mentions not a word that Abraham was Born to him.

3. *Which was to him instead of a Miracle, when he heard from God, that a Son should be Born to him,*

in his Hundred Years, which admiration would not have been raised up in him, if he should not have been Born of his Father within the Hundred Years.

CHAP. IV.

Exposition III. Of the distance from the calling of Abraham, to the going forth of the Israelites from Egypt, and the publishing the Law.

THE beginning and end of this Interval, the Holy Scripture it self determines and expresses the whole of it, which was 430 Years; to wit, in numbring from the Promise made to *Abraham* in the Year of his Age 75, even to the *Exit*, or going forth from *Egypt*, or a little after the publishing of the Law: For so we Read in the Epistle of *Paul* to the *Galatians*, Chap. 3. *At length began the Law, after 430 Years from the Promise made to Abraham.*

Moses consents also to this Testimony of *St. Paul*, who writes in *Exodus* the 12th. *The Habitation of the Children of Israel, in which they remained in Egypt, was 430 Years.*

In which Words, also the time of Pilgrimage of *Abraham*, *Isaac* and *Jacob*, in the Land of *Canaan* and the bordering places, are comprehended.

1. The Apostle *Paul* is a Witness to us beyond exception, who repeats these 430 Years, not from the entrance of the *Israelites* into *Egypt*, but from the first Promise made to *Abraham*.

2. Also *God* himself, not only includes the Children of *Jacob*, or of the *Israelites*, but also the Pilgrimage of the Fathers of *Abraham* and *Isaac*, about the space of 400 Years. *Genesis* 15.

3. So also the Authors of the *Greek* Version, include the Pilgrimage of the *Abrahamites*, as well in *Canaan*

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Canaan as in *Egypt*, in these 430 Years. *Exodus* 12. and 41. Without doubt from Ancient and sure Tradition; so that it is unadvisedly done of those who Argue those false things.

4. To which also the Authority of *Josephus* Assents, who in his *Antiquities*, Book 2. Chap. 13. from the Descent of *Jacob* into *Egypt*, to the Exit of the *Israelites*, numbers 215 Years; whence it follows, that the remaining 215 Years are attributed to the Pilgrimage of *Abraham*, *Isaac* and *Jacob* in the Land of *Canaan*.

5. Which also, lastly the Account of Scripture it self makes appear: For *Cahat*, the Grandfather of *Moses*, who went down with *Jacob* into *Egypt*, Lived 123 Years, his Son *Amram* Died in the 127th. Year of his Age; *Moses* the Son of him, was 80 Years Old when he led out the *Israelites* from *Egypt*. Suppose now, *Cahat* in the Descent of *Jacob* into *Egypt*, and *Amram*, to Beget Children in the last Year of their Age, yet neither will you proceed beyond 350 Years, according to this computation of Time, for 123 and 127 and 80 make but 350 Years; which if now you grant to *Cahat* some Years, in which he Lived in *Canaan*, and if you will take away some Years from the same, *Cahat* and his Son *Amram*, in which, before the end of Life, they Beget Children, that Sum will become much lesser; whence it clearly appears that the 430 Years assigned in *Exodus* the 12th. comprehends also the Time in which *Abraham*, *Isaac* and *Jacob*, were Pilgrims and Sojourners in the Land of *Canaan*, and therefore in *Egypt* also, being forced to change their Habitations for the cause of satisfying Hunger.

All which if weighed in an equal Ballance, no Man will gain-say with reason: These things therefore being presupposed, this distance of Time may be deduced from Scripture thus.

From

| | collect years
of Distance | collect years
of the world |
|--|------------------------------|-------------------------------|
| From the calling of <i>Abraham</i> ,
to the Birth of <i>Isaac</i> , in the 100
Year of <i>Abraham</i> . | 25. | 2048. |
| From the Birth of <i>Isaac</i> , to
the Birth of <i>Jacob</i> , in the Year
of <i>Isaacs</i> Age 60. | 85. | 2108. |
| To the Passage of <i>Jacob</i> into
<i>Mesopotamia</i> , in the Year of his
Age 77. | 162. | 2185. |
| Hence to the Birth of <i>Joseph</i> ,
in the Year of Bondage of <i>Ja-
cob</i> 14. | 176. | 2196. |
| From hence to the Exaltation
of <i>Joseph</i> , in the Year of his
Age 30. | 206. | 2229. |
| To <i>Jacobs</i> going into <i>Egypt</i> ,
in the Second Year of the Fa-
mine, or of the Exaltation of
<i>Joseph</i> 9. | 215. | 2238. |
| To the Death of <i>Joseph</i> , in
the Year of his Age 110, Years
71. | 286. | 2309. |
| To the Birth of <i>Moses</i> , Years
63. | 349. | 2372. |
| To the exit from <i>Egypt</i> , in
the Year of <i>Moses</i> his Age 80. | 429. | 2452. |

C H A P. V.

*Interval the IV. From the Exit or going out of
Egypt, to the Building the Temple of Jerusa-
lem, demonstrated from Scripture.*

From

FROM the going forth of the *Israelites* out of *Egypt*, to the Building of the Temple of *Jerusalem*, the Holy Scripture numbers in the whole, 480 Years. 1. of *Kings*, Chap. 6. Although some think to use a Limitation to this Sum, in distinguishing between the beginning and ending of the exit from *Egypt*, which happened many Years after it, when all the *Israelites* had possessed their Rest, whence they add 100 Years more to the Sum of 480 Years, relying on the Account of the Book of *Judges*; which times of Bondage and Freedom, of Rest and Troubles; according to which the Chronological Account instituted, seems to number a part.

Nevertheless, to us, from so clear Writings of Holy Scripture, which numbers 480 Years from the exit to the Temple, it seems not adviseable to depart so far; therefore we appoint the Years of Bondage of the *Israelites*, to be included in the Years, in which the Deliverers of the same Common Wealth had been before: Neither doth it trouble us, that the Scripture mentions, that those *Israelites* Lived quietly under the *Judges*; for there is a twofold Rest or Quiet, one, wherein I Live in Tranquility, free from all Offence; the other, in which, although I am affected with divers Injuries, yet those are not opposed by me, but in suffering all things patiently, I enjoy Quiet, such as it is, and such which the *Israelites* are understood to enjoy in the Book of *Judges*: The Quiet to which Troubles are opposite, is that in which we are desirous to cast off the Yoke of Bondage; therefore those 480 Years, which the Holy Writ numbers from the going out of *Egypt*, to the laying the Foundation of the Temple of *Jerusalem*, we explain thus.

| | Years of every
ones Reign. | Years of Reign
collected | Years of the
world collected |
|--|-------------------------------|-----------------------------|---------------------------------|
| <i>Moses</i> Governed in the Desert complete Years | 39. | 39. | 2491 $\frac{1}{2}$. |
| <i>Joshua</i> , and after him the Elders of the People, in the Land of Canaan. | 18 $\frac{1}{2}$. | 57 $\frac{1}{2}$. | 2510. |
| <i>Othaniel</i> . | 39 $\frac{1}{2}$. | 97. | 2549 $\frac{1}{2}$. |
| <i>Eliud</i> . | 80. | 177. | 2629 $\frac{1}{2}$. |
| <i>Deborah & Barac</i> . | 39 $\frac{1}{2}$. | 216 $\frac{1}{2}$. | 2669. |
| <i>Gedeon</i> . | 39 $\frac{1}{2}$. | 256. | 2708 $\frac{1}{2}$. |
| <i>Abimelech</i> . | 3. | 259. | 2711 $\frac{1}{2}$. |
| <i>Thola</i> . | 22 $\frac{1}{2}$. | 281 $\frac{1}{2}$. | 2734. |
| <i>Jair</i> . | 21 $\frac{1}{2}$. | 303. | 2755 $\frac{1}{2}$. |
| <i>Jephtha</i> . | 5 $\frac{1}{2}$. | 308 $\frac{1}{2}$. | 2761. |
| <i>Abesan</i> . | 7 $\frac{1}{2}$. | 316. | 2768 $\frac{1}{2}$. |
| <i>Eleon</i> . | 10 $\frac{1}{2}$. | 326 $\frac{1}{2}$. | 2779. |
| <i>Abdon</i> . | 8. | 334 $\frac{1}{2}$. | 2787. |
| <i>Sampson</i> . | 20. | 354 $\frac{1}{2}$. | 2807. |
| <i>Eli</i> . | 39 $\frac{1}{2}$. | 394. | 2846 $\frac{1}{2}$. |
| <i>Samuel</i> . | 26. | 420. | 2872 $\frac{1}{2}$. |
| <i>Saul</i> 1 King. | 15 $\frac{1}{2}$. | 435 $\frac{1}{2}$. | 2888. |
| <i>David</i> . | 40. | 475 $\frac{1}{2}$. | 2928. |
| To the laying the Foundation of the Temple by <i>Solomon</i> . | 3 $\frac{1}{2}$. | 479. | 2931 $\frac{1}{2}$. |

For in the beginning of the following Year, from the *exit* 480, in the second Month, the Temple began to be Built. 1. Kings, and the 6.

A *brief* Computation from the 13th. Chap. of the Acts of the Apostles may be Instituted, thus.

| | |
|---|------|
| 1. The tarrying in the Desert, after the <i>exit</i> from Egypt. ———— | 39. |
| 2. The Distribution made of the Land, Years ———— | 07. |
| 3. The time of the Judges ———— | 350. |
| 4. The Reign of Samuel and Saul ———— | 40. |
| 5. Of David ———— | 40. |
| 6. Of Solomon, to the laying the Foundation of the Temple ———— | 3. |
| The Sum ———— | 479. |

After which, in the Year 480 from the *exit*, in the second Month, in the Fourth Year of Solomon, the Temple was Built.

CHAP. VI.

Of Interval the V. deduced from the laying the Foundation of the Temple, to the Ruin of the same.

THIS Interval or Distance is sought out from Holy Writ with much difficulty, which omits the lesser Articles of Times; and since Fathers and Sons ruled together, or the *Interregnum* happened between the Years of the Father, and of the Son, or distinguishes not the length.

We proceed thus.

| Solomon af-
ter the Foun-
dation of the
Temple, reign-
ed Years | Years of each
one's Reign. | Years of Reigns
Collected. | Years of the
world collected. |
|---|-------------------------------|-------------------------------|----------------------------------|
| | 37. | 37. | 2968 $\frac{1}{2}$ |
| Rehoboam. | 17. | 54. | 2985 $\frac{1}{2}$ |
| Abiah. | 3. | 57. | 2988 $\frac{1}{2}$ |

M m 2

Assa

| <i>Assa</i> reigned | Years of each
ones Reign. | Years of Reigns
Collected. | Years of the
world collected. |
|--|------------------------------|-------------------------------|----------------------------------|
| 41 Years, but
3 with his Son
<i>Josaphat</i> , alone. | 38. | 95. | 3026 $\frac{1}{2}$. |
| <i>Josaphat</i>
Reigned twen-
ty five Years,
but 3 with his
Father, with
his Son <i>Foram</i>
9, therefore a-
lone, and with
his Father. | 16. | III. | 3042 $\frac{1}{2}$. |
| <i>Foram</i> reign-
ed with his
Father 9 years,
alone 3 Years,
with his Son
<i>Abasiah</i> 1 year,
and so in all,
Years | 13. | 124. | 3055 $\frac{1}{2}$. |
| But 8 Years
from the Confirm-
ation of the King-
dom, was made in
the 24th. Year of
his Father <i>Josaphat</i> . | | | |
| <i>Abasiah</i> reigned
with his Father 1
Year, alone. | $\frac{1}{2}$. | 124 $\frac{1}{2}$. | 3056. |
| <i>Athaliah</i> , a Wo-
man, Reigned vio-
lently, by an Usurp-
ed Reign, years | 6 $\frac{1}{2}$. | 131. | 3062 $\frac{1}{2}$. |
| <i>Josiah</i> Reigned in
all 40 Years, but 3
with his Son <i>Amas-
iah</i> , alone. | 37. | 168. | 3099 $\frac{1}{2}$. |
| <i>Amasia</i> , with his
Father 3, alone
26, in all. | 29. | 197. | 3128 $\frac{1}{2}$. |

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| The Interregnum, in which <i>Ufias</i> of <i>Azaria</i> was Educated, under the Custody of Governors. | Years of every ones Reign. | Years of Reign collected. | Years of the world collected |
|---|----------------------------|---------------------------|------------------------------|
| <i>Ufias</i> , free from his Governors, Reigned Years. | 12 $\frac{1}{2}$. | 209 $\frac{1}{2}$. | 3141. |
| <i>Jotham</i> Reigned with his Son, in all 16 Years, alone | 52. | 261 $\frac{1}{2}$. | 3193. |
| <i>Achas</i> Reigned in all 16 Years, but with his Son <i>Hiskia</i> 3, alone, and with his Father. | 15. | 276 $\frac{1}{2}$. | 3208. |
| <i>Hezekiah</i> reigned in all 29 Years, but with his Son 2, with his Father, and alone. | 13. | 289 $\frac{1}{2}$. | 3221. |
| <i>Manasses</i> Reigned with his Father, and alone, years | 27. | 316 $\frac{1}{2}$. | 3248. |
| <i>Ammon</i> the Son | 55. | 371 $\frac{1}{2}$. | 3303. |
| <i>Josias</i> . | 2. | 373 $\frac{1}{2}$. | 3305. |
| <i>Joachas</i> . | 31 $\frac{1}{2}$. | 405. | 3336 $\frac{1}{2}$. |
| <i>Joiakim</i> . | $\frac{1}{4}$. | 405 $\frac{1}{4}$. | 3336 $\frac{1}{4}$. |
| <i>Jechonia</i> . | 11. | 416 $\frac{1}{2}$. | 3347 $\frac{1}{2}$. |
| <i>Zedekia</i> . | $\frac{1}{4}$. | 416 $\frac{1}{4}$. | 3348. |
| | 11. | 427 $\frac{1}{2}$. | 3359. |

About this time *Nebuchadnezar* Burnt the Temple and the City of *Jerusalem*, and carried away the People into the *Babylonick* Captivity.

| Others from gathering together the Kings of <i>Juda</i> and <i>Israel</i> , collect thus. | Years of every ones Reign. | Years of Reigns Collected. | Years of the world collect. |
|---|----------------------------|----------------------------|-----------------------------|
| <i>Solomon</i> Reigned from the Foundation of the Temple. | 37. | 37. | 2968 $\frac{1}{2}$. |
| From <i>Jerobnam</i> , King of <i>Israel</i> , to <i>Assa</i> King of <i>Juda</i> . | 19. | 56. | 2987. |

M m 3

From

| From Affa to A-
hab King of Israel. | Years of every
ones Reign. | Years of Reign
Collected. | Years of the
world collect. |
|--|-------------------------------|------------------------------|--------------------------------|
| From Affa to A-
hab King of Israel. | 37. | 93. | 3024. |
| From Ahab
to Jofaphat,
King of Juda. | 3. | 96. | 3027. |
| From Jofa-
phat to Joam,
King of Israel. | 17. | 113. | 3044. |
| From Joam
to Abasia, King
of Juda. | 11. | 124. | 3055. |
| From Abasia
to Jehu, King
of Israel. | 1. | 125. | 3056. |
| From Jehu
to Joam, King
of Juda. | 6. | 131. | 3062. |
| From Joam
to Joash, King
of Israel. | 36. | 167. | 3098. |
| From Joash
to Amasia, King
of Juda. | 1. | 168. | 3099. |
| From Amasia to
Jereboam, II. King
of Israel. | 14. | 182. | 3113. |
| From Jereboam
II. to Uzziab, King
of Juda. | 26. | 208. | 3139. |
| From Uzziab to
Peka, K. of Israel. | 51. | 259. | 3190. |
| From Peka to
Abaz, K. of Juda. | 16. | 275. | 3206. |
| From Abaz to
Osea, K. of Israel. | 11. | 286. | 3217. |
| From Osea
to Hiskiah, K.
of Juda. | 2. | 288. | 3219. |

From

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From the Building of the Kingdom of *Israel*, there Reigned in the Kingdom of *Juda*.

| | Years of every
ones Reign. | Years of Reign
Collected. | Years of the
world collected. |
|-------------------|-------------------------------|------------------------------|----------------------------------|
| <i>Hiskiah.</i> | 29. | 317. | 3248. |
| <i>Manasses.</i> | 55. | 372. | 3303. |
| <i>Ammon.</i> | 2. | 374. | 3305. |
| <i>Jofias.</i> | 31 $\frac{1}{2}$. | 405. | 3336 $\frac{1}{2}$. |
| <i>Joachas.</i> | $\frac{1}{4}$. | 405 $\frac{3}{4}$. | 3336 $\frac{3}{4}$. |
| <i>Joiakim.</i> | 11. | 416 $\frac{1}{4}$. | 3347 $\frac{3}{4}$. |
| <i>Jechonias.</i> | $\frac{3}{4}$. | 416 $\frac{1}{2}$. | 3348. |
| <i>Zedekiah.</i> | 11. | 427 $\frac{1}{2}$. | 3359. |

Also this Interval or Space of time may be most briefly accounted thus, from Ezekiel 4.

From Founding the Temple, to the dividing of the Kingdom into the Kingdom of *Israel*, and of *Juda*, Years ————— } 37 $\frac{1}{2}$.

Hence to the Destruction of the Temple — 390.

The Sum ————— 427 $\frac{1}{2}$.

C H A P. VII.

The demonstration of the remaining Intervals, from the Rain of the City until now.

IN the first Destruction of the Temple by *Nebuchadnezar*, the Chronological Account of Scripture is thought to leave off; and perhaps, if we would Account by Civil Years, it would be so: for by this means the Chronology would hardly be continued, unless we join Foreign History with Sacred. But if we admit the Mystical Years lately detected, each of 343 Days, of which, 7 days constitute a Week of days, Seven such Weeks, a Weekly Month, 7 Months one Year, 7 Years one Annal, or great Year, 7 of the great, one greater, 7 of the great-

er one greatest, and 7 of the greatest one Age; but 3 Ages waste all the time of this present World: I say, if we use these Mystical Years, we may proceed without offence from the Fundamentals of Holy Scripture, even to the last Destruction of *Jerusalem*, in this manner.

| | Mystical
Years. | Civil
Years. | Yearsofthe
WORLD |
|---|--------------------|---------------------|----------------------|
| The <i>Babylonick</i> Captivity
dured Years. | 70. | 65 $\frac{3}{4}$. | 3424 $\frac{3}{4}$. |
| Hence to the second Year
of the base Born <i>Darius</i> , and
the beginning of <i>Daniels</i>
Weeks. | 126. | 118 $\frac{1}{4}$. | 3543. |
| To the Restauration of the
Temple, 7 Weeks | 49. | 46. | 3589. |
| Hence to the Passion of
Christ, Weeks 62, Years. | 434. | 407 $\frac{1}{2}$. | 3996 $\frac{1}{2}$. |
| Hence to the confirmation
of the Covenant, Weeks 1.
Years. | 7. | 6 $\frac{1}{2}$. | 4004. |
| Thence to the beginning of
the <i>Jewish</i> War, half of the
greater Week compleat. | 28. | 26 $\frac{1}{4}$. | 4030 $\frac{1}{4}$. |
| Hence to the last destructi-
on of the Temple, and of the
City of <i>Jerusalem</i> . | 4. | 3 $\frac{3}{4}$. | 4034. |
| Hence to the Year of Chri-
stian Epocha 1652. | 1684. | 1581. | 5615. |
| Completed | | $\frac{1}{4}$. | $\frac{1}{4}$. |

C H A P. VIII.

*Another Computation of Years of the World, from the
beginning of things, even to our Times, deduced
by Four Monarchies.*

1. From

| | Years of
Intervals | Years of the
world collected |
|--|-----------------------|---------------------------------|
| 1. FROM the Fall of <i>Adam</i> , to the end of the Flood. | 1656 $\frac{1}{2}$. | 1656 $\frac{1}{2}$. |
| 2. From the Flood, to the beginning of the first Monarchy, constituted by <i>Nimrod</i> or <i>Belus</i> , in <i>Chaldea</i> or <i>Assyria</i> . | 72 $\frac{1}{2}$. | 1729. |
| 3. From the beginning of the first Monarchy, to the first Division of it into <i>Assyrians</i> and <i>Medes</i> , under <i>Sardanapalus</i> . | 1360. | 3089. |
| 4. From this first Division of the Empire, to the new <i>Babylonish</i> Government, or the <i>Æra</i> of <i>Nabonassar</i> . | 127 $\frac{1}{2}$. | 3216 $\frac{1}{2}$. |
| 5. From the beginning of the <i>Babylonish</i> Government, to <i>Nebuchadnezzar</i> the Great. | 123. | 3339. |
| 6. Hence to the beginning of <i>Cyrus</i> , the Founder of the <i>Persian</i> Monarchy. | 63. | 3402. |
| 7. To the Vanquishment by <i>Cyrus Darius</i> King, and the Stability of the <i>Persian</i> Monarchy. | 5. | 3407. |
| 8. To <i>Darius Codoman</i> , the last King of the <i>Persians</i> , overcome by <i>Alexander</i> the Great, and the Monarchy of the <i>Persians</i> Translated to the <i>Greeks</i> . | 225. | 3633. |
| 9. Hence to the Death of <i>Alexander</i> the Great, and the Monarchy of the <i>Greeks</i> cut or rent in 4 parts, viz. The Kingdom of the <i>Syrians</i> ; <i>Asians</i> , <i>Macedonians</i> and <i>Egyptians</i> ; Years. | 6 $\frac{1}{2}$. | 3639 $\frac{1}{2}$. |

10. To

| | Years of
Intervals. | Years of the
world collected |
|---|------------------------|---------------------------------|
| 10. To <i>Demetrius Polyorces</i> , the King of lesser <i>Asia</i> , overcome by the Son in Law of <i>Seleucus Nicanor</i> , and the joining the Kingdom of <i>Asia</i> to the <i>Assyrians</i> . | 136 $\frac{3}{4}$. | 3776. |
| 11. To the Victory by <i>Romanus Persia</i> , the last King of the <i>Macedonians</i> , and the Translating the Kingdom to the <i>Romans</i> . | 19. | 3795. |
| 12. To <i>Tigranus</i> , left Naked by the <i>Romans</i> in the Kingdom of <i>Syria</i> , and the Kingdom reduced into a <i>Roman Province</i> . | 99. | 3894. |
| 13. To <i>Caius Julius Caesar</i> , the Builder of the Fourth Monarchy, the Reformation of the Calendar. | 23 $\frac{1}{4}$. | 3918 $\frac{1}{4}$. |
| 14. Hence to <i>Antonius</i> , with <i>Cleopatra</i> , Queen of the <i>Egyptians</i> , overcome by <i>Augustus</i> , and the <i>Roman Monarchy</i> established. | 14 $\frac{3}{4}$. | 3933. |

15. To

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| | Years of
Interval | Years of
the world | Years of
Ch. Ep. |
|---|----------------------|-----------------------|----------------------|
| 15. To the Christian Epo-
cha. | 30 $\frac{1}{4}$. | 3963 $\frac{1}{4}$. | 0. |
| 16. To <i>Constantine</i> the
Great, the first Christian Em-
peror. | 305 $\frac{1}{4}$. | 4269. | 306. |
| 17. To the end of the
Western Roman Empire, under
<i>Augustulus</i> . | 476. | 4438. | 475. |
| 18. To the Restauration of
the same, under <i>Charles</i> the
Great. | 528 $\frac{1}{4}$. | 4763 $\frac{1}{4}$. | 800. |
| 19. To <i>Rudolphus</i> . <i>Hab-</i>
<i>spurgensis</i> . | 473. | 5236. | 1273. |
| 20. To the end of the East-
ern Empire, possessed by the
<i>Turks</i> . | 179 $\frac{1}{2}$. | 5415 $\frac{1}{2}$. | 1452 $\frac{1}{2}$. |
| 21. To <i>Ferdinando</i> the III.
in the Year 1652 compleat. | 200 $\frac{1}{4}$. | 5615 $\frac{1}{4}$. | 1652. |

The End of the Chronological Appendix.

AN E P I T O M E O F G E O G R A P H Y.

The P R E F A C E.

I. **G**EOGRAPHY is a Doctrine shewing the Reason of Distinguishing, and of Measuring the Earth.

2. The Handling of it may be Instituted in a twofold manner, one of which is *Historical*, and the other *Mathematical*.

3. *Historical*, pursues liberally all the parts of the Earth, and of the Water adjoined to it, and declares whatsoever it hath peculiar or worthy of observation.

4. *Mathematical*, is content with the more general distinction of the Terrene Globe, chiefly respecting the Dimension of it.

5. This our present Epitome will be such, which being known, a way will be prepared to the Studious, by surveying with profit, since it tends to the understanding of Globes and Geographical Maps, as well as to other things which appertain to Geography, delivered by others.

6. For Terrestrial Globes and Geographical Maps are Instruments, in which, all things which are delivered by words in Geography, are subjected also to the Eyes; the use of which also we shall explain, as much as the reason of the Order will permit.

7. And there may be two parts constituted of Geography, of which it Treats.

- ✓ I. *Of the Distinction.*
II. *Of the Dimension.* } *Of the Earth.*

OF GEOGRAPHY. PART I.

Of the Distinction of the Earth.

CHAP. I.

Of the Earth, and the ways of distinguishing it.

1. **T**HE Earth, Geographically considered, is a Body adjoined with Water, constituting one Globe, which from the more excellent part is called *Terrene*.

(We have made good this Description of the Earth clearly before in the Astronomical Part)

2. The Distinctions of it are taken out.

1. *From the integral Parts.*

2. *From the Coasts of the World.*

3. *From the diversity of Habitations.*

4. *From the imagination of conceived Circles.*

Whence, again arises a seven fold diversity, to wit, in the *Ratio* or *Proportion*.

1. *Of a double Hemisphere.*

2. *Of Zones.*

3. *Of Climates.*

4. *Of Shadows.*

5. *Of the Site of the Inhabitants.*

6. *Of the diverse situation of the Globe.*

7. *Of the Longit. and Latit. of places.*

CHAP. II.

Of the I Distinction, from Integral Parts.

I. **I**NTEGRAL Parts are either of Dry or of Wet.

II. *Of*

II. Of Dry, are.

- | | |
|--|--------------------------|
| 1. Firm Earth, or a Continent. | 5. A Promontory or Cape. |
| 2. An Island. | 6. A Mountain. |
| 3. Peninsula, or a place almost surrounded with Water. | 7. A Valley. |
| 4. Isthmus, or a Neck of Land between two Seas. | 8. A Field. |
| | 9. A Wood, &c. |

III. Of Wet, are.

1. The Sea. 2. Sinus, or a Bosom or Gulf of the Sea. 3. A narrow Sea between two Lands. 4. A Port, Haven or Harbour. 5. A Lake or standing Pool. 6. A standing Water. 7. A Fountain. 8. A River. 9. The Mouth of a River, &c.

(All these want not more accurate Definitions and Descriptions, since the Significations of the words themselves, explains the things obvious enough every where; unless that the Sea every where encompassing the Earth, is called the Ocean, the others receiving a Name from the Site, or of the place which it flows nigh to.

C H A P. III.

Of the II Distinction, from the Coasts of the World.

1. **O**F the Coasts of the World, Four are Cardinals.

1. East.

3. North.

2. West.

4. South.

2. Hence the places of the Earth are called.

1. Eastern or Oriental.

3. Northern or Septentrional.

2. Western or Occidental.

4. Southern or Meridional.

3. And indeed, either absolutely without respect, if the whole Globe be conceived in Mind, or restrictly, with respect of certain places: in which Ratio, one Region, in respect of this, may be Eastern or Northern, in respect of another place, it may be Western or Southern.

4. These

4. *These Coasts of the World may be known.*

1. From beholding the Sun rising, thus.

1. From your Face, you have the East.

2. From your backside, the West.

3. On your Left Hand, the North.

4. On your Right Hand, the South.

II. *From beholding the Pole Star in the Tail of the Lesser Bear.*

For then is:

1. The East on your Right Hand. 3. The North before you.

2. The West on your Left Hand. 4. The South on your Back.

CHAP. IV.

Of the III. Distinction, from the diversity of Habitations.

I. **O**F the Earth, which is Inhabited of all Men, four parts may be constituted.

I. *The Earth under the Northern.*

II. *The Earth under the Southern.* IV. *The Old Orb.*

III. *The New Orb America.*

{ Africa.

{ Asia.

{ Europe.

Besides Islands which may be divided into great, mean, small, and least of all.

That you may learn the Site of these, turn your Face toward the North, and you will have.

1. From the Front, the Earth under the Northern.

2. On your Back, under the Southern and Africa.

3. On your Right Hand Asia.

4. On your Left Hand America.

5. Europe is that in which we dwell, and which encompasses us on every side.

CHAP. V.

Of the Earth under the Northern.

THE Sub Northern Earth, is a Tract of the Terrene Globe, nigh to the North Pole, or Northern.

The

The parts of it are, Friezland, Island, Icaria, Grunland, Grocland, Terra Nova, or Spitsberg, Rusten, Nova Zembla; concerning which you may consult the Navigations of the Hollanders, and others.

C H A P. VI.

Of the Sub Southern Earth.

THE Earth under the Southern, is a Tract of the Terrene Globe, nigh to the South Pole, or to the South.

'Tis also called *Terra Magellanica*, from its first finder *Magellanus*, in which are *Terra del fuogo*, the Promontory of the Subaustrale Earth, the Region of *Parrets*; *Lucas*, *Beach*, *Maletur*, *Nova Guinea*, and other Countries, not as yet known.

C H A P. VII.

Of the New Orb or World.

1. AMERICA, also is called *This New Orb*, and is a great Portion of the Earth, situate to us beyond the Ocean, towards the West; found out by *Christopher Columbus*, either first or anew, about 200 Years ago, and denominated by *Americus Vesputius*, who detected the further part of it.

2. Its Terms or Limits are.

1. On the East, the *Atlantick Ocean*, and the *Ethiopic Ocean*.
2. On the West, *Mare Pacificum*, or *Zur*.
3. On the North, *Terra Sub borealis*.
4. On the South, *Terra Sub australis*.
3. 'Tis distributed generally into Northern and Southern, conjoined by an *Isthmus*, of which that is possess'd by the *Mexicans*, This by *Peruvians* or *Dariens*.

4. The North part of *America* at the West side, in passing from the North towards the South East, from thence to the North East, in ascending on the Eastern side, the more excellent Kingdoms and Regions are: The Kingdom of *Ariana*, *Quivirana*, *Tholmia*, *Nova Granada*, *Nova Gallacia*, *Nova Hispania*, *Mexicana*, *Guatinala*, *Fondura*, *Nicaragua*, *Jucatan*, *Florida*, *Virginia*, *Nova Belgia*, *Nova Francia*, *Canada*, *Terra Corterealis*, *Laboratoris*, and *Estotilandia*, more Northern no Body hath yet found out.

5. *America*

5. *America on the South part*, by the compass of the Sea Coast, returning from *Isthmus* to *Isthmus*, contains the Provinces, *Castilia aurea*, *Dariena*, *Papajana*, *Peruvia*, *Cuscana*, *Carea*, *Chicha*, *Patagonum Regionem*, *Chila*, *Brasilia*, *Caribana*, *Guiana*, and *Cumana*: But those Regions which are in the very middle no Man yet knows sufficiently.

(*The lesser Regions of either may be seen in Maps.*)

C H A P. VIII.

Of the old Orb, especially Africa.

1. **A**frica is a great Portion of the old Orb, to us Europeans 'tis situate beyond the Mediterranean Sea, towards the South.

2. Its Limits are,

1. *On the East*, The Arabian Gulf, or the Red Sea.
2. *On the West*. The Atlantick Ocean.
3. *On the North*. The Mediterranean Sea.
4. *On the South*. The Ethiopian Ocean.

3. The Ancients did not penetrate to all the knowledge of this, therefore the Maritime and Mediterranean Regions found out, they number thus; That from the East to the Red Sea the first Region, is called by them, *Arabia Troglodytica*: But to the Mediterranean Sea, the second, *Egypt*: Then towards the West, the third, *Cyrenaica*: From thence the fourth, *Affrica the lesser, or proper*; with the fifth, *Numidia*: Hence the sixth, *Mauritania Casariensis*, & *Tingitana*, to which the Mediterranean adheres; to wit, to *Egypt* towards the South: Seventh, *Ethiopia*: But towards the West, eighth, *Lydia* the broadest extent, between which and the other

N n

Maritim

Maritim Places, there lies the Regions of, ninth, *Garamantum*; Tenth, *Troglodytarum*; and eleventh, *Getulorum*.

4. But at this time, all *Africa* is divided principally into seven Regions, of which these are the names, counted by us, by the coasting of the Sea:

1. *Egypt*. 2. *Barbary*. 3. *Biledulgerid*. 4. *Negroland*. 5. *The true Ethiopia*. 6. *The Empire of the Abyssines*; all bordering upon the Sea, and one situate in the very middle, viz. 7. *The Desert Sarra*.

5. *Egypt* at this day is not divided as in old time, into Governments or Provinces, although the Provinces of the Ancients contain now under themselves somewhat more parts; from the *Mouth or Entrance of Nylus* to both Shoars of it, it extends to an hundred Miles.

6. *Barbary*, is the largest, containing the *Country of Barca*, and the *Kingdoms of Tunis, Tremisen, Fez, Morocco*, and the *Darenses*, so called from the cheif Cities.

7. *Biledulgerid*, having its name from the abundance of Dates, 'tis the longest Tract of Land, from the *Atlantic Sea* to *Egypt*, continued all along thro' the *Mediterranean* places, and is divided into six Kingdoms, *Zanhaga, Zwenziga*, (sometime *Hair*) *Targa, Lempia, Berdoa*, and *Goaga*, having by-names in like manner as the Cities.

8. *Negroland*, or the *Region of Blacks*, it derives its name from the River *Niger*, which washes it; from each side of the Mouths of which *Meroen* (*Gueguere*) is extended the Island of *Nylus*, in which the Kingdoms are in like manner denominated from the Cities: *Gualata, Hoden, Geneboa, Senega, Tombuti, Melli, Bitonia, Guinea, Tonian, Damma, Cano, Cassena, Benin, Zanzara, Guanguara, Borno, Nubia, Biafra, Media*.

9. *True Ethiopia, or the Inferior*, is that Terrestrial

al Tract, which joining to the rest of *Africa* on the North, is encompassed on every side, by the Sea on the West, South and East; the chief Regions of which are, *Congo*, *Monomotapa*, *Zangibar*, and *Ajana*, which again are subdivided into more, concerning which you may consult Globes and Maps.

10. *The Empire of the Abyssines*, is part of *Africa*, which is enclosed on the East by the *Arabian shore*, and by the Regions of *Ethiopia*, *Ajana* and *Zangibar*; on the South, by *Monomotapa*; on the West by *Congo* and *Medar*; on the North 'tis inclosed by *Nubia* and *Egypt*, and 'tis subdivided into many Provinces, viz. *Dasila*, &c.

11. *Sarra*, which we have called the Mediterranean, is all that Space which extends thro' the middle of *Africa*, from the East Kingdom of *Gaoga*, even to the West of *Gualata*, for the most part deserted, whence also it receives its name, and is called the *Desert of Sarra*.

12. Among the Islands of this Region, the most famous are those which at this day are called the *Canaries*, but of the Ancients, called the *Fortunate Islands*.

In this place the Ancient Geographers placed the first Meridian, from which, the Modern have retired back Ten Degrees.

CHAP. IX.

Of ASIA.

1. **A**SI A is the greatest Part or Portion of the old Orb situate indeed to us, towards the East, but stretched out by a long Tract into the North and South.

2. Its Limits are,

1. On the East, *The Ocean of Chinensius.*
2. On the West, *The River Tanais.*
3. On the North, *The Tartarian Ocean.*
4. On the South, *The Indian Sea.*

3. It may be distinguished according to its state at this time, into five vast Empires, to which some Regions situate to the North-west, and the South-east, together with many Islands, may be brought into the number.

4. We shall reckon up *these five Asiatick Empires*, by going round about, from the West by the Tract of the Sea Shore to the West; to wit, the Empire,
1. *Of the Turks.* 2. *Of the Persians.* 3. *Of the Mogul Indians.* 4. *Of the Sinenses.* 5. *Of the Tartarians*, with their dependancies, as shall be shewn in its place.

5. We dispose the Regions of the *Turkish Empire* in *Asia*, into five Classes or Forms; for they are situate, either first between the *Caspian Sea*, and the *Elack Sea*,; or secondly, between this it self, and also the *Egean Sea*, and the *Mediterranean*, even to *Euphrates*; or thirdly, between this River and the furthestmost eastern part of the *Mediterranean Sea*; or fourthly, between the Gulf of *Arabia* and *Persia*; or fifthly, about the Tract of *Euphrates* and *Tygre*.

6. Between the *Black Sea* and the *Caspian Sea*, going down to the Banks of this, from the North towards the South; the Countries in old time were called *Albania* and *Iberia*; of which at this day, that is called the *Eastern Georgia*, this the *Western*; which, notwithstanding, is called *Balbur*, where it extends it self to the *Black Sea*: To the East and North Shore of which, in old time, *Colchis* was situated, which is now called *Mengrelia*.

7. To the second Class or Form is referr'd *the lesser Asia*, which at this day is called *Natolia*; in which,
from

from the East of the *Black Sea*, towards the West, lie the Maritim Regions, 1. *Cappadocia*. 2. *Paphlagonia*. 3. *The Black Sea*. 4. *Bithynia*, to *Phosphorum*, *Hellepontum*, and the *Egean Sea*. 5. *The lesser Phrygia*. 6. *The greater and lesser Mysia*. 7. *Æolia*. 8. *Ionnia*. 9. *Caria*. 10. *Doris*: To the *Mediterranean Sea*, 11. *Lycia*. 12. *Pamphylia*. 13. *Cilicia*; to which adjoins, being situate in the very middle, 14. *Galatia*. 15. *The greater Phrygia*. 16. *Pisidia*. 17. *Lydia*, and 18. *Armenia the lesser*, which touches *Euphrates*: Although at this day these Regions are exprest by other names.

8. To the third Class, or Form, belongs *Syria*; which is divided into six Regions, four of which touch upon the Sea, to wit, in going from the North-east towards the South-west: 1. *Aniobene*. 2. *Phœnicia*. 3. *Palestina*. 4. *Idumea*: But two, viz. 5. *Comagene*, and 6. *Calesyria* extend to *Euphrates*, that bends towards the North, this towards the South and East; concerning which, the greatest part is described, not only in *Prophane*, but chiefly also in the *Holy Scripture*.

9. *Arabia*, constitutes the fourth Class, or Form, which is threefold, to wit, 1. *Felix*. 2. *Petrea*. 3. *Deserta*; of which the first is the greatest, the greater part situate beyond the Tropick of Cancer, and hath its name of *Happy* from the exceeding Fruitfulness thereof: But *Petrea* extends it self to the North and West, as the *Desert* doth to the North and East.

10. To the fifth Class, or Form, belongs, 1. *Chaldea*, (at this time *Chaldar*) touching the *Persian Gulf*. 2. *Mesopotamia*, which according to the *Euphrates*, is called *Aliduli*; by the *Tigrims*, *Diarbech*: 3. *Armenia the greater*, extending to the *Caspian Sea*, whose North part is now called *Tarcomannia*, the middle part *Popul*, the Southern part *Cardi*.

11. The *Persian Empire* approaches next towards the East, whose Provinces may be distributed into two Classes : In the first of which are, 1. *Gedrosia*. 2. *Carmania*. 3. *Persia*, properly speaking. 4. *Susiana*. 5. *Assyria*. 6. *Media*. 7. *Hyrkania*. 8. *Margiana*. 9. *Bactriana* ; which from the South-east join themselves by a Semicircle, even to the North-east : In the other Class, are, 10. *Parthia*. 11. *Aria*. 12. *Paropamisus*, 13. *Arachosia*. 14. *Drangiana* ; which are situated in the very middle, in going from the West towards the East and South, and all of them at this day, have names altogether differing from the Ancients.

12. At this time the *Persian Empire* belongs to the *Indian Mogul*, which possesses a small part of old *Scythia*, and the greatest part of both the *Indies*; situate as well within as without *Ganges*, and hath 44 Regions, accounting seven Series, in numbring always from the North towards the South, which the Map shews : besides those Provinces and Kingdom of *India*, not as yet subject to the Great *Mogul* ; among which these are the most excellent, *Deccan*, *Narsinga*, *Pegu*, *Malacca*, *Camboja*, on the Continent, with great plenty of Islands.

13. The uttermost *Empire of Asia* towards the East, is *Sisense* (with the *Peninsula Corea*) encompassed on every side, by the Sea, by a Wall, and by Mountains, and it hath 16 Provinces, numbred most conveniently from the North towards the South, the names of which the Table likewise will supply, which you shall have at the latter end of this Tract.

14. There remains the *Tartarian Empire*, which contains the most part of all *Old Scythia*, and within and without, the Mountain *Imaus*, among all the Regions of *Asia*, least accurately known, yet is wont to be divided into five parts, of which the 1. is *Cathaya*, nigh the North Gulf : 2. *Old Tartaria*, or the *True*, nigh the River *Tartar*, in the furthest Northern Part of *Asia* : 3. *Turkestan*, nigh the Empire of the *Persians* and the *Moguls* : 4. *Zagataya*, nigh the Shoars of the *Caspian Sea* : 5. *The Tartarian Deserts*, cut into eight parts on this side and beyond the Mountain *Imaus* : which touch upon the utmost part of a Province in obedience to the *Russians*, and they extend even towards *Turkey*.

(Now we proceed to *Palæstina*, and then to *Europe* ; for the Islands of both, the Tables will truly note, which we shall subjoin at the end of this Tract.

C H A P. X.

Of Palestine, or the Land of Canaan.

1. **T**HE names of it, as above, hath been most Ancient; which *Land of Canaan* was possessed by *Canaan* a Grandson to *Noah*; and *Palaestina*, by the *Philistines*, which descended from *Mizraim*, another Grandson of *Noah*, especially treated of in Holy Scripture.

The particular Countries of it were named from each People that inhabited them, which are recited often times in Scripture.

But after that God had instituted Abraham, Isaac, Jacob or Israel, and his Posterity the *Israelites*, Heirs of this most fruitful Land; it was also called the *Land of Promise*; the *Land of Israel*; the *Holy and Blessed Land*; not so much because this People was the peculiar Elect of God; as because the Holy of Holies, Christ the Redeemer of Mankind was Born in this Land, of this People; in which also the most Holy Offering, the Redemption of Mankind was performed.

2. And this Land was situate towards the East, to the Eastern Mouth of the *Mediterranean Sea*, but bending by an oblique Tract on the North part, towards the East; and on the South part towards the West.

The Country was more in Length than Breadth; in length from North to South almost 40 German Miles, that is, 160 English Miles: In Breadth from the West towards the East, about fifteen German Miles, that is, 60 English Miles.

3. The Limits and Terms of it were,

1. On the East, The Mountainous *Arabia*, *Gilead* and *Hermom*.
2. On the West, The Eastern Mouths of the *Mediterranean Sea*.
3. On the North, *Phœnicia*, and the Mountains of *Libanon* and *Antilibanon*.
4. On the South, *Egypt*, *Idumia*, and the Rocky *Arabia*.

4. The Midland, was divided from the North to the South, by the most famous River called *Jordan*, which afforded the first Holy Water of Baptism.

This River flows from the two Fountains or Springs Jor and Dan, at the foot of the Mountain Antilibanon, and glides along the two Lakes on the North part, viz. the Lake Samachonitidem and Cenereth, or Genesareth, which in Holy Writ are called the Sea of Galilea and Tiberias: But at last discharge themselves into the Lake Asphaltis, or the Salt and Dead Sea, where Sodom and Gomorrha sometime were situate, which divides in two in the middle, the Land beyond, even to the ends of Idumea.

Before the Israelites possessed it, the Philistines and the Sidonians inhabited it, from the South towards the North, to the Western Entrance of the Mediterranean Sea: The Hethites inhabited the Midland on this side Jordan, as did also the Amorites, the Jebusites, and the Hivites: Beyond Jordan inhabited the Moabites, the Ammonites, and the Inhabitants of the Country Basan: But after that the Israelites had expelled and slain most of the inhabitants, they overcame this Land by a violent hand, and took it, and it attained another form.

5. It was divided into twelve Regions, between the twelve Tribes, the twelve Sons of Israel, (except the Tribe of Levi, and the Tribe of Joseph, being numbred for two.)

Of the division of which, being made by Lot, this was the Ratio or Proportion.

1. The lower part towards the South, between the Mediterranean and the Dead Sea, was allotted to the Tribes of Judea and Simeon: The next from these, to Dan and Benjamin; to which adjoined Ephraim and half Manasses.

From these towards the North, there followed the Tribe of Issachar, from hence, of Zabulon: The remaining Land even to the Northern Limits, was possessed towards the West by the Tribe of Aser, and towards the East by the Tribe of Nephthalim.

2. Be-

2. Beyond *Jordan* on the North part of the Land, about the ends of it, was possessed by the Country of *Basan*, the other half by the Tribe of *Manasses*; that which followed in descending towards the South, was the Tribe of *Gad*; and lastly the Tribe of *Reuben*, even to the end of the Land of the *Moabites*, who remained in the possession of the Land by Divine Command.

3. The Tribe of the Priesthood of *Levi*, were intermixt among them all, for the conveniency of teaching the Duty of Religion, as also among some of the first inhabitants, who were not driven out, nor put to the Sword.

6. The Neighbourhood of People, and those that possess the Land of *Palestina*, were,

1. On the East, The *Moabites*, the *Ammonites*, the *Assyrians*, the *Chaldeans*, the *Babylonians*, the *Persians*.
2. On the West, the Inhabitants of the next Islands of the *Mediterranean Sea*.
3. On the North, The *Syrians* and the *Phœnicians*.
4. On the South, The *Arabians*, the *Idumeans*, and the *Egyptians*.

7. From the division of the People after the death of *Solomon* the King, *Palestina* was divided into two Kingdoms, one of which was the Kingdom of *Judea*, the other was called the Kingdom of *Israel*. To wit, The Denomination being made from the former part, for the Tribe of *Benjamin* was also under the Kingdom of *Judea*, and the Tribe of the Priesthood of *Levi*; as also of *Simeon*, which was mixt with the Tribe of *Juda*. But the rest, of which there were many, retained the name of *Israelites*.

8. The Metropolis of the Kingdom of *Israel* was *Samaria*; and of the Kingdom of *Judea*, the chief City was *Jerusalem*, built on four Mountains; that towards the East was the Mount *Moriab*, in which the Temple was situate: Towards the West was the Mount *Acra*: Towards the North, the Mount *Bezetha*: And towards the South was Mount *Sion*, with the Royal Ark.

The

The City therefore was very Spacious, which contained in its compass more than a German Mile, or more than Four English Miles.

9. From the time of Christ then the Kingdom of *Israel* was destroyed, although the whole Region was generally called by the Name of *Judea*, yet more especially it was divided into the Country on this side and beyond *Jordan*.

10. That which lay on this side *Jordan*, was subdivided into 3 parts.

1. *Judea*, so called in particular, situate towards the South.
 2. *Galilea*, the Upper and Lower, towards the North.
 3. *Samaria*, lying between *Judea* and *Galilea*.
11. But beyond *Jordan*, was situate *Perea*, towards the South; and *Trachonitis* and *Abilene* towards the North.

CHAP. XL

Of the Regions of Europe, even to Denmark.

1. **E**UROPE is the Western Tract of the Old Orb the better part of which lies under the *Temperate Zone*, towards the North, yet somewhat running out to the *Frigid Zone* of this Coast: But because we Inhabit in it, it will be worth our while to Discourse of it with somewhat the more care.

2. The Limits of it, by which 'tis enclosed, are, 1. The *Mediterranean Sea* to the South. 2. To the West, by the *Atlantick Sea*. 3. To the North, by the *North Sea*, and the *Sub Southern Land*. 4. To the East, by the Rivers *Oby* and *Tanais*, likewise by the *Black Sea*, and by the *Aegean Sea*, where 'tis bounded or separated from *Asia*.

3. The more excellent Provinces of it, which afterwards were subdivided, may conveniently be thus reckoned up: As from *Tanais* and the *Black Sea*, towards the *Aegean Sea*, and so on, going a compass until you return to the same *Black Sea* again, which is the Class of the Exterior Provinces of *Europe*. Then 2. appears the Interior, by proceeding from the same *Black Sea*, towards the West, to *Germany*;

many; and we shall number the Provinces of Europe, in all XV. and those in this order. 1. *Tartaria Præcopensis*. 2. *Turcia Europea*. 3. *Illyricum*, or *Sclavonia*. 4. *Italy*. 5. *France*. 6. *Spain*. 7. *Brittain*. 8. *Denmark* and *Norway*. 9. *Swedeland*. 10. *Russia*. 11. *Polonia*. 12. *Dacia*, or *Transilvania* and *Moldavia*. 13. *Hungary*. 14. *Bohemia*. 15. *Germany*.

4. *Tartaria Præcopensis*, is the furthest part of Europe, towards the Asian Empire, adjacent to the Black Sea, which hath its Bounds on the East, by the River *Tanais*; on the South by the same, with the *Moory Dead Lake* and the *Black Sea*; on the West, by *Borysthene*, a River of Poland; on the North, by the *Russian Empire*.

5. *Tartary*, this same may be divided into 3 parts, of which the most Southernly is, 1. A Peninsula, to wit, *Chersonesus Taurica*, bounded only by the *Black Sea* and a Neck of Land. 2. The most Western, bounded by this Neck of Land and the *Black Sea*; moreover by *Borysthene*, then encompassed by the Barricado also of the lesser *Tanais*. 3. The most Eastern, by the *Moory Dead Lake*, *Meotide*, of *Tanais* the greater and the lesser; and also by those Barricades enclosed even to the Mountains; but a more accurate Geographical Knowledge is here wanting.

6. *European Turk*, is a Tract or space of Land, from the *Black Sea* towards the West, next to certain whole Regions, some of which contain a fifth part only: But we shall only Treat of the whole Regions here, noting the rest afterwards, part of which the Christians as yet Govern.

7. And 'tis bounded on the East by the *Black Sea*, and *Propontidem*, or the Sea betwixt Europe and Asia, with the Arms of both, and by the *Ægean Sea*. The remaining part on the South is bounded by the *Mediterranean Sea*, and by the *Adriatick Gulf*; on the West, by *Illyricum* and *Pannonia*; on the North, by the same, and *Dacia*, by which 'tis enclosed on every side, and participates also of all things with the Empire.

8. The intire parts of this Turkey are 3. 1. *Thracia*. 2. *Mesia*. 3. *Greece*. The former of which at this day is called *Rome*, and is that Tract of Land extended from the *Black Sea* to *Greece* on the West, inclosed by the Seas, by the River and Mountains of *Scardico*.

9. But

9. But *Thracia* hath its Bounds on the East, by the *Black Sea*, *Bosphorus*, *Propontis* and *Hellespont*; on the South, by the *Ægean Sea*, and the *Gulf Strymonicum*, (called the Gulf of *Contesa*) on the West, by the River *Strymonem*; and on the North, after the Mountains *Scardicos*, by the lower *Mæsia*.

10. And the Division of it may be instituted three fold, where,
1. The greatest and Easterly part is, from the *Black Sea* to the River *Hebre*. 2. The Western part is situate between this River and *Strymon*. 3. The least and Southernly part is called *Chersonesus* of *Thracia*.

11. *Mæsia* follows, which is the Tract of *European Turkey*, situate on the Right Hand of the Shoar *Ister*, spreading out to the *Black Sea*, being bounded on the South by *Greece* and *Thracia*; on the West by *Illyricus*; on the North by the *Danube*; and on the East by the *Black Sea*; also it was sometimes called by the Ancients, *Mysia*.

12. It was divided in Old time, and is now, into the Upper and Lower; of which that is the lesser space, and at this day is called *Servia*: This, is near 3 times greater, and is called *Bulgaria*; whence also the Division of both may easily be adjudged.

13. *Greece* now remains, which is a Tract of *European Turkey*, between the *Ægean Sea* and the *Mediterranean*, and also in part enclosed by the *Adriatick Gulf*, and extends even to *Mæsia*; which also on the North is bounded like as *Thracia*, and by the *Ægean Sea* on the East; on the South by the *Mediterranean*; on the West by the same, and the foresaid Gulf, with *Illyricum*.

14. And *Greece* with us is divided into *Macedonia*, *Thessalia*, *Epirum*, *Greece*, properly so called, and *Peloponesus*, which again are subdivided; of which *Macedonia*, the most spacious, is Northern, and in Old Time was divided into 8 parts, which are called. 1. *Albania*. 2. *Mygdonia*. 3. *Bisaltia*. 4. *Paraxia*. 5. *Pelagonia*. 6. *Æmathia*. 7. *Piecia*. 8. *Pelasgia*. Notwithstanding at this day 'tis only divided into 3 parts, of which, 1. The North East is *Jamboli*. 2. The North West is *Albania*. 3. The Southern is called *Comelonitari*.

15. *Thessalia* of *Macedonia* is situate towards the South, and encompassed to 3 Coasts of the World by Mountains, but on the East the Country is bound-
ed

ed by the *Ægean Sea*, which in Old Time was divided into 5 People or Towns, to wit, 1. The *Theſſalians*, properly ſo called. 2. The *Eſſiotans*. 3. The *Pelaſgans*. 4. The *Magneſians*, and 5. The *Phthiotans*: At this day 'tis called by one Name *Janna*.

16. *Epirus*, is a Tract of Land ſituate by the Sea ſide, beneath the *Adriatick Gulf*; in Old Time divided into 7 People. 1. *Moloffans*. 2. *Dryopeans*. 3. *Chaoneans*. 4. *Threſprotians*. 5. *Caſſiopeans*. 6. *Amphilocheans*. 7. *Acarneaneons*. At this day 'tis called *Canina*, (thro' the error of ſome, *Albania*) and is divided from the other part of *Greece*, beſides the bounds of the Sea, by the Mountains *Macedonia* and *Theſſalia*, by the River *Acheloo*.

17. *Greece*, properly ſpeaking, is that Tract which beſides the ſaid Provinces, as yet remains without *Peloponneſus*, ſometimes containing 8 Provinces, which are called. 1. *Ætolia*. 2. *Doris*. 3. *Locris Ozolea*. 4. *Phocis*. 5. *Megaris*. 6. *Attis*. 7. *Bæotia*, and 8. *Locris Epicnemedia*. At this time 'tis generally called *Levadia*, being bounded by the Mountains, by the Seas and *Iſthmus*.

18. *Peloponneſus*, is a *Peninſula*, concerning which we ſhall ſpeak in the Table; in Old Time containing 6 Provinces, viz. 1. was called *Achaja*. 2. *Elida*. 3. *Meſſenia*. 4. *Arcadia*. 5. *Laconia*. 6. *Argolica*, which at this day are called by one Name *Morea*.

19. *Illyricum* of *Greece* lies next, and is that Tract of *Europe* which is occupied on the better part by the *Adriatick Gulf*, on the Northern Shoar, with crooked and winding Streams towards the Eaſt, being bounded on the South by the Gulf it ſelf; on the Weſt by part of *Italy*; on the North by *Pannonia*; on the Eaſt by *Meſia* and *Greece*, whoſe upper part in Old Time was called *Liburnia*, the lower part *Dalmatia*, but at this day that is expreſſed by *Morlachia*, this by its Old Name; although this Appellation alſo is now common to the whole Country, which ſometimes is illy called by ſome *S'avonia*.

20. *Italy* agrees juſt to this order, the Tract of Land ſhooting out between the *Adriatick Gulf* and the remaining *Mediterranean Sea*,

Sea, even to the *Alps*, hath its bounds on the West, by *France*; on the North by *Germany* and the *Adriatick Gulf*; on the East by the *Mediterranean Sea*; and the same also on the South, together with the *Islands*.

21. *Italy* may be divided into the most Ancient, and belonging to the *French*, but both taken together, in Old Time *Strabo* divided into 8 parts, which by the leading of the *Sea Shoars*, we shall thus reckon up, viz. The next is to *Illyricum*, and therefore part 1. *Venetica*. 2. *Picenum*. 3. *Apulia*. 4. *Lucania*. 5. *Campania*. 6. *Latium*. 7. *Thuscia*. 8. *Liguria*; under which divers People are contained; which *Italy* was divided into XI. Regions by *Augustus Caesar*, that which is the middle Division of it, is more rightly remembred.

22. In this Division the Country or Region nigh the principal City, The 1. contained *Latinos* and *Campanos*. From thence 2. To the North East, *Apulos*, *Pucetes* and *Salentinos*. Then 3. To the South, *Calabros*, *Lucanos* and *Brutios*. From hence 4. To the North West, *Ferentanos*, *Mauricinos*, *Pelignos*, *Vestinos*, *Samnites* and others: Moreover, 5. In the same Shoar *Picentes*. 6. *Umbros*. From hence 7. To the South West, *Thuscus* or *Etruscus*. From thence 8. Somewhat towards the North, *Faventinos*, *Fidentinos*, *Padinates*, and others. Hence 9. To the South West, *Ligures*, and others, even to *Padum*; in passing over which, 10. with your Face towards the *Adriatick Gulf*, *Venetos*, *Istros*, *Japyges*. Lastly, 11. To the West occurs the last Region, between the *Alps* and *Padum*, which contains, *Taurinos*, *Salassos*, *Insubres*, *Rhetos*, and others.

23. The newest Division of *Italy* is instituted into Lordships, to wit, 1. *Hispanica*, or belonging to the *Spaniards*. 2. *Pontificia*, or belonging to *Bishops*. 3. *Ducalia*, or belonging to *Dukes*. And 4. *Libera*, free, or not subject; among which the beginning of *Italy* at the *Alps*, and the end of it at *Sicilia*, occupies

cupies the I. Classe of Lordships; to wit, there the State of *Mediolanensis*, this here the Kingdom of *Neapolitan*, in which on the *Thuscan* Shore, are 4 Ports and Cities.

24. To the II. Classe belongs the *Roman Ecclesiastick* State, extending to each Sea; and the *Ecclesiastick* State *Ravennas*, towards the West, situate on the Coast of the *Adriatick Gulf*, together with the District *Beneventan*, near the very middle of the Kingdom of *Neapolitan*.

25. The III. Classe makes, 1. *Piedmont*, towards the *Alps*, and the State *Mediolanens*, belonging to the Duke of *Schaudia*. 2. The Great Dukedom of *Florence*, situate on the *Tuscan* Sea. 3. The Dukedom *Mantua* and *Montferrat*, looking on the left side of the River *Po*. 4. The Dukedom of *Parmens* and *Placentin*, situate on the right side of the River *Po*. 5. Afterwards in the same place the Dutchy of *Mutimens*, (*Modena*) 6. The Dukedom of *Urbis*, situate at the *Adriatick Gulf*, between 2 Pontifical States; to which 7. may be added the County *Mirandula*, which runs towards the aforesaid Dukedom of *Mutimens*, near the River *Po*.

26. The IV. Classe contains; 1. The Lordship of *Venice*, about the *Adriatick Gulf*. 2. *Genoa*, at the *Tuscan* Sea. 3. *Lucens*, in *Thuscia*, situate not far from the Sea; to which at last belong some Appendixes of *Austria*, of *Rhetica*, of *Helvetia* in *Germany*, worthy of remembrance.

27. *France* receives *Italy*, being less spacious now than formerly, considering that the *Neatherlands*, and whatsoever Regions occupy the left side of the *Rhine* were contained, even to the Springs of it almost; whence there is one Division of Old *France* and another of new, and the computation of the bounds of it.

28. And since *France* on this side the *Alps*, (which is also called *Togata*) is *French Italy*, neither doth there belong a word to us of the Soil beyond the *Alps*,

Alps, (both being so called in respect of the City of *Rome*) which some time was divided into *Braccata* and *Comata*; from which, *Braccata* of *Italy* is also next to the *Mediterranean Sea*, by a Line stretched out by *Varro*, to the Springs of *Rhodan*; and from it below *Lugdunum*, stretched out under the Mountains *Arvernus*, to the Mouth of the River *Garumna*.

29. But *Comata Gallia*, or *France*, was three fold, 1. *Aquitania*, now called *Guienne*. 2. *Celtica*. 3. *Belgica*, or belonging to the *Dutch*; among which, it (since at the first *Garumna* also was enclosed by the *Pyrenian Mountains*) is computed by *Augustus* to *Ligerim*: *Celtica* also, (called *Lugdunensis*) spread abroad of late between *Matrona*, *Sequana*, the Ocean and *Garumna*, the Mountain *Gebenna*, and the Rivers *Rhone* and the *Rhine*, afterwards between the Rivers *Matrona*, *Sequana*, and *Liger* or *Loyre*; and also it was bound by the Ocean. Lastly, *Belgica* (in the first place was contained between the *Rhine*, the Ocean, and *Sequana* and *Matrona*) in the next place it was contained between the *Rhine*, the *Rhone*, *Ararim*, *Matrona*, *Sequana*, and also the Ocean; which for the sake of Ancient History, ought not to be unknown.

30. Now we come to *France*, as 'tis at this time, which is a Tract of *Europe*, situate on this side, between the *Alps* and the *Pyrenean Mountains*, situate from thence between the *Mediterranean Sea* and the Ocean, and likewise between the *Hodiern Neatherlands* and the Provinces of the *Rhine*; bounded on the East by *Italy* and *Germany*; on the South by the *Mediterranean Sea*; on the West, besides the *Pyrenean Mountains*, it owns part of the Ocean for its Limits; by which also (the Name of the *English Channel* is at last given to it) it is terminated towards the North.

31. *France*

31. *France* at this time, from the *Lake Lemanno*, from the *Alps* towards the *Mediterranean Sea*, and so on, by going about, until you return to the same Lake, (computing also the Regions not obedient to the King, because they are comprehended by the Bounds of *France*) may conveniently enough be divided into Provinces. 1. The Outermost. 2. Those further in. 3. Those furthest in. The following Designation by Parenthesis, notes the Foreigners.

32. Of *France*, the outermost Regions or Countries are. (1. *Subaudia*, with 2. The Country *Geneva*) 3. *Delphinatus*. 4. *Province*, (in which 5. The County of *Avegnon* And 6. The Principality of *Aurange*) 7. The Region of *Languedock*. 8. *Vasconia*, where *Bearnia*, *Bigorria*, *Comingia*, *Albretia*, *Arminiaca*. 9. The Country of *Guiene*. 10. *Xantoine*. 11. *Pictavien*. 12. *Brittany*. 13. *Normandy*. 14. *Piccardy*. 15. *Artesia*. 16. *Campania*. 17. *Lotharingia*. And there the Episcopacy of *Metensis*, *Thullensis*, and *Verdunensis*. 18. The Dukedom of *Burgundia*, (in the Confines of which, the Counties of *Burgundy* and *Montpelier* are situate) all which 19. *Bressia* at last compleats.

33. There remains the inner and most inward Countries of *France*, of which the former lye to the East. 1. *Borbon*. 2. *Limane*. To the South, the Tract of *Lugdunensis* or *Lions*. And 4. *Arvernica*: And also 5. *Quercia*. To the West, 6. The Regions of *Perigort*. 7. *Limosin*. 8. *Blasen*. 9. *Turone*. And 10. *Andegauen*. To the North, 11. *Lemania*: 12. The Country *Le Beauisse*. 13. The Island of *France*: But the last and most inward are, 14. *Berria*; and 15. *Nivernia*.

34. Passing over the *Pyrenean Hills*, *Spain* presents it self, which is a Tract of *Europe*, the same being encompassed by these Hills towards the West, (and the rest inclosed by the Sea) so that it hath no Terrestrial Bounds besides *France*, from the East to the North, yet in the middle for the most part, 'tis inclosed by Mountains, because it is in a manner inclosed by the *Mediterranean Sea*, and the Straits of *Gibraltar* on the South; and on the West and North by the *Atlantick Ocean*.

35. *Spain* was divided in Old Time into the hither and the farther, of which, that was contained by a very small

Tract of the *Pyrenean Hills*, and by the River *Iberus*; the farther, extended it self to all the remaining space of Land; but afterwards it was also divided in a three fold manner, into *Betica*, *Lusitania*, and *Tarraconensis*; from which it was extended by the River *Ana*, from thence to the Regal City; hence to *Almeria*: *Lusitania* is contained from the River *Durius* to *Camora*, and hence to the Regal City; all the remaining part was called *Tarraconensis*.

36. The Division of *Spain* at this day, if by going about from the *Pyrenean Hills* to the *Mediterranean Sea*, and so on, until you return to them again; may be rightly instituted, (besides the Kingdom belonging to the Isles, containing *Majorcam*, *Minorcam* and *Ebusiam*) into XVII. Kingdoms and Provinces, which also are, either 1. Outward, or 2. inward ones; to wit, in the former Classe XIV. 1. *Catalonia*. 2. *Valentia*. 3. *Murcia*. 4. *Granada*. 5. *Andalusia*. 6. *Extremadura*. 7. *Algarbia*. 8. *Portugallia*. 9. *Gallacia*. 10. *Asturia*. 11. *Biscaia*. 12. *Guipischoja*. 13. *Navarra*. 14. *Aragon*. In the Second Classe are III. viz. from the North towards the South. 1. *Castilia Nova*. 2. *Vetus*. And 3. *Legio*.

37. Hence, going strait towards the North to the Islands, of which, although those (as is said in its place) we shall handle in a peculiar Table; yet it is not impertinent to the matter to exhibit the *Britanick* and *Danish* Kingdoms, (howsoever for the most part, the whole belonging to Islands) because of the frequent resorting, and the daily Travelling thither: And the *Britanick* Empire is an Hoard of many Islands, divided from *France*, on the North part by the *Narrow Sea*, whose bounds to ascribe is not needful, considering that they are not but close to the Sea Side.

38. Of these *Britanick* Islands, that which is particularly called *Great Britain*, (some time *Albion*) is the greatest of all, and is divided into two Kingdoms. 1. *England*. 2. *Scotland*, of which the Southern part of *Britain* is the greater, having *Scotland* on the North, its only Bound: It was divided in Old Time into VIII. Kingdoms, by computation instituted by going about from *London*: They were, 1. *Essex*. 2. *Kent*.
3. *Sussex*,

3. *Suffex*, or the *South Saxons*. 4. The *West Saxons*. 5. The Kingdom of *Mercia*. 6. The Kingdom of *Wales*. 7. The Kingdom of *Northumberland*: And 8. The Kingdom of the *East Angles*: But at this day the Division of it differs.

39. For 'tis divided 1 into *Loegria*, or *England*; and 2 *Cambria*, or *Wales*, and that again into the Upper and Lower; of which the former is towards the North, the latter towards the South East; but *Cambria* is situate towards the Eastern Shoar and the Mediterranean, according to the following Account of the Provinces, which descends from the Confines of *Scotland* towards the South.

40. In *Loegria* or *England*, the Upper or Superior are XVI. Provinces, 1. *Northumberland*. 2. *Durham*. 3. *Cumberland*. 4. *Westmorland*. 5. The Province of *Yorkshire*. 6. *Lancashire*. 7. *Cheshire*. 8. *Darbyshire*. 9. *Nottinghamshire*. 10. *Lincolnshire*. 11. *Rutlandshire*. 12. *Leicestershire*. 13. *Warwickshire*. 14. *Worcestershire*. 15. *Staffordshire*. 16. *Shropshire*.

41. In *Loegria* or *England*, the Inferior or Lower, there are many Provinces, to wit, in number XXII. of which, in beginning to proceed from the East to the North, and towards the South, and so to the West; and from thence again to the North, leaving *Cambria* or *Wales* on the Western side, in ascending they are, 1. *Norfolk*. 2. *Suffolk*. 3. *Cambridgeshire*. 4. *Huntingtonshire*. 5. *Northamptonshire*. 6. *Bedfordshire*. 7. *Hartfordshire*. 8. *Essex*. 9. *Buckinghamshire*. 10. *Middlesex*. 11. *Surrey*. 12. *Kent*. 13. *Suffex*. 14. *Hampshire*. 15. *Dorsetshire*. 16. *Devonshire*. 17. *Cornwal*. 18. *Somersetshire*. 19. *Wiltshire*. 20. *Barkshire*. 21. *Gloucestershire*, and 22. The Province of *Oxford*, or *Oxfordshire*.

42. *Cambria* or *Wales* remains as yet, which hath XIII. Provinces, and they are, in continual proceeding from the West Angle of the inferior or lower *Loegria* or *England*, 1. *Pembrokeshire*. 2. *Camar-*
denshire,

denshire, (Meridunum) 3. *Glamorganshire*. 4. *Monmouthshire*. 5. The Province of *Herefordshire*. 6. *Radnorshire*. 7. *Brecknockshire*. 8. *Cardiganshire*. (Ceretica) 9. *Montgomeryshire*. 10. *Merionethshire*. 11. *Carnarvanshire*. 12. *Denbighshire*; and 13. The Province of *Flintshire*.

43. *Scotland* follows after that, which is the most Northern part of Great Britain, being bounded by *England*, which precedes it on the South; all the remaining part is encompassed by the Sea, and the same is naturally parted by the Hill *Grampius* or *Granzebain*, so that one half of it is 1. Northern. 2. The other Southern, both which contain many Provinces.

44. In the North of *Scotland*, to the North West is, 1. *Strathnavern*, and towards the East, 2. *Cathnes*; hence towards the South, 3. *Southerland*; thence to the West again, 4. *Rosse*, then towards the East, 5. *Artmaneth*, and again to the Western Shoar, 6. *Affine*, and below this 7. *Loquabar*; hence to the North and the East Shoar, 8. *Murray*, and 9. *Bugbuan*; from thence a little lower, 10. *Marsia*, and towards the West, 11. *Batgenothia*, and a little lower towards the South, 12. *Angusia*; hence towards the South West, 13. *Albia*, 14. *Argile*, 15. *Lornia*, 16. *Knapdalia*, and 17. *Cantyria*.

45. And in *Scotland* on the South, which occupies a less space, the Provinces are, 1. *Perth*. 2. *Tisa*. 3. *Menthesia*. 4. *Striveling*. 5. *Lennox*. 6. *Cunningham*. 7. *Cylia*. 8. *Carrick*. 9. *Galloway*. 10. *Nydesdale*. 11. *Liddesdale*. 12. *Anandale*. 13. *Eskdale*. 14. *Merch*. 15. *Tivedale*, and 16. *Laudon*; and in the middle, 17. *Clidesdal*.

46. To this Empire also belongs another Island, which adjoins to *Britain*, towards the West, called *Ireland*, (indeed lesser than *Britain*) but divided into IV. spacious Provinces, of which, 1. The most Northern is *Ultonia* or *Ulster*,

Ulster, which 2. To the South West receives *Connach* or *Connaught*: To this 3. a little lower to the East, is opposed *Lagenia* or *Leinster*, but at the end of the Island, 4. in the Southerly Coast, appears *Momonía* or *Munster*; all which Provinces are again divided into 33 Counties, which to reckon up here we omit.

47. Besides these two most Noble *Britanick* Islands, many lesser belong to this Empire, among which are the Famous Isles of *Orkney*, about 40, bordering on the North of *Scotland*, and 7 still more Northern: Then above *Ireland*, towards the North, to the West side of *Scotland*, are the Western Islands, called *Hebrides*, about 30 in number; neither is there any other Islands belonging, which are nearer to *England*, to wit, between it and *Ireland*, above the Isle of *Anglesey*; and a little below *Menapia*: But in the *Gallick Sea*, besides the Isle of *Wight*, (near to *England*) there are the Isles of *Anglesey*, *Garnsey* and *Jarsey*, with some lesser lying between, resembling the same.

CHAP. XII.

Of the other Regions of Europe, even to Germany.

1. OF from Great *Britain* towards the East, approaches *Denmark* and *Norway*, viz. a Tract of *Europe*, partly Islands, partly Continents; on this side bounded by the *Baltick Sea*, but from hence adjoining to the North Sea; concerning the Bounds of which, it may be better enlarged in each part.

2. And *Denmark* consists of Islands and Continents, but we pass by those, among which *Seelandia* is the most Famous, and the next to this in Magnitude, lying towards the West, is *Fionia*, the rest are lesser: Towards the South of *Seeland* and *Fionia*, is situate *Alsa*, *Aaria*, *Berbia*, *Langlandia*, *Lalandia*, *Falsfria*, *Meunia*; and most Southern *Femria*: Towards the East at this time also look-

ing to *Denmark*, are *Bornholmia*, farther off; and nearer there are *Amagria*, *Southolmia*, *Huena*; (but now looking to *Sweedland*, are *Gotlandia*, *Oesilia*, *Dakria*) To the North are some, somewhat greater, as *Samsia*, *Anutia*, *Lefonia*; with many more smaller, of which they reckon some are of the Continent of *Denmark*, of which we shall now speak.

3. And indeed, the Continent of *Denmark* not long since, was two fold, Eastern and Western; but those which belonged to that, are the Provinces of *Hallandia*, *Scania*, *Blekingia*, now belonging to *Sweedland*; therefore there remains of it the Western Continent, viz. *Chersonesus Cimbrica*, which is divided into *Jutia* and *Holsatia*, of which we shall handle them better in *Germany*; but we shall Treat of it here, even as 'tis divided.

4. And it may be divided into 3 parts, viz. 1. *Wenssuffelia*. 2. North *Jutia*, and 3. South *Jutia*; among which, *Wenssuffelia* or *Wensilia* is most Northern, being reduced into the new form of a *Peninsula*, by the Gulf *Limicus*, and ending in the Tract of *Tiens*, computing many Islands to the same.

5. 1. This North *Jutia* succeeds for its nearness, which is divided into IV. Diocesses, having Names from the chief Cities; of which the first is *Alburgensis*, situate about the beginning of the Gulf *Limicus*. 2. is the *Mediterranean*, and called *Viburgensis*; this at the East Shoar of the Sea, receives 3. *Arbusensis*, but 4. that to the Western Shoar of the *German Sea*, is called *Ripensis*, which nevertheless extends it self to the other Sea also.

6. There remains 2. Southern *Jutia*, which runs between 2 Seas, like as it were in breadth, towards *Holstein*, unless that in the lower part 'tis somewhat enlarged; and although there be no particular Division given of it, yet to the Confine of *Holstein*, the Tract, 1. Of *Eiderstetensem*, and 2. Of *Anglensem*, is annexed to it, on this and that side, together with Islands adjoining on both sides.

7. To

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7. To the same Empire we account also the aforesaid *Norway*, which is a Tract of *Europe*, at the farthest part of the West Shoar of the North Ocean, situate from *Denmark* towards the North, together with Islands belonging to the same; which is bounded on the West by the Ocean; on the North by *Finmarchia*; on the East by the Hills of *Sweedland*, that lye between; and on the South by the same, and *Holland*.

8. And this whole Kingdom, although it be very large, is divided into V. Jurisdctions, which we shall reckon up in going from the South towards the North; and of them the 1. is *Bahusia*. 2. *Aggerhusia*. 3. *Bergerhusia*. 4. *Niderhusia* or *Dronthemensis*. 5. and lastly, *Wardhusia*, with the Maritim Tract *Trunsfoliens*.

9. Hence it leads us, by the order of Connexion, to *Sweedland*, which is a Tract with all the Provinces belonging to this Kingdom of the North part of *Europe*, situate on this side, between *Norway*, on that side, between *Russia* and *Polonia*, ending for the most part at the Shoars of the *Baltick Sea* and the North Ocean; whose Bounds on the West are the Ingress of the *Baltick Sea* and *Norway*; on the North, by the Ocean of its Name, on the East, besides the *White Sea* or Gulf of *Granwick*, 'tis Bounded by the Countries of *Russia* and *Polonia*; on the South, partly by these, and partly by the *Baltick Sea*, which washes *Germany* and *Denmark*.

10. All *Sweedland* may be divided into Countries, beginning from the situation of the Kings Seat. 1. On this side, the Gulf of *Bothnica*. 2. On this side the Gulf *Finnica*, and 3. beyond the Gulf *Finnica*; which we shall reckon up in an orderly manner.

11. The Countries of *Sweedland* on this side the Gulf of *Bothnica* are, 1. *Uplandia*. 2. *Sudermannia*. 3. *Ostrogothia*. 4. *Smalandia*. 5. *Blekingia*. 6. *Scania*. 7. *Hallandia*. 8. *Wästrogotia*. 9. *Nericia*. 10. *West-*

mannia. 11. *Gestricia*. 12. *Dalecardia*. 13. *Helsingia*. 14. *Middelpadia*. 15. *Femtia*. 16. *Angermannia*. 17. *Bothnia*, on the West part. 18. *Lappia*. 19. *Finmar-
chia*, and in the last place, on the North part, 20. *Schrickfinnia*.

12. The Countries of *Sweedland*, beyond the Gulf of *Bothnica*, and on this side the Gulf *Finnica* are, 1. *Bothnia* in the East. 2. *Cajania*. 3. *Finlandia*. 4. *Tavastia*. 5. *Nylandia*. 6. *Carelia*. 7. *Jeskia*. 8. *Savolaksia*, and 9. The remainder of *Lappia*. But the Provinces belonging to *Sweedland*, beyond the Gulf of *Finnica*, are III. 1. *Ingria*. 2. *Estonia*. 3. *Livonia*.

13. Afterwards *Russia* shews it self, which is a Tract of *Europe*, towards the East, and moreover, the further end towards the North; a great portion of *Asia*, if we look upon the ends of the Empire, to wit, comprehending part of *Tartary*, and from thence to the *Caspian Sea*, extending it self from hence almost to the *Black Sea*, and is there called *Tartaria Russia*.

14. The Bounds of it on the East, are the ends of Great *Tartaria*; on the South, the *Caspian Sea*, the Provinces of *Turkey*, and also the Lesser *Tartaria*; on the West, many Provinces of *Poland*, and also of *Sweedland*: Lastly, on the North, 'tis joined by the North Sea, with the *Tartarian*.

15. 'Tis divided into many Provinces, some of which are, 1. Of *Autocracy*, or self subsistence 2. Some are of *Cesar*, or belonging to the Emperor. 3. Others are of the Great Dukedom. 4. Of others they are wont to be called plainly Lordships: We on this side, with this respect, which is *Political*, not *Geographical*, divide the Countries into outer and inner; and we number them from the *Caspian Sea* toward the East, and so on, by going about until we come to the inner Countries.

16. And to the outer Regions or Countries be-
long,

long, 1. *Astracania*. 2. *Mordua*. 3. *Refa*. 4. *Severia*. 5. *Smolenskia*. 6. *Besecia*. 7. *Megrina*. 8. *Vologdia*. 9. *Duina*. 10. *Condora*. 11. *Siberia*. 12. *Obdora*. 13. *Samojeda*. 14. *Jugoria*. 15. *Pega*, and 16. *Russia Tartaria*, or *Tartary*, belonging to *Russia*. But the inner Provinces of the *Russians* are but few, viz. 17. *Casania*. 18. *Ceremesia*. 19. *Sudalia*. 20. *Wlodomiria*. 21. *Rosania*. 22. *Vorptia*. 23. *Muscovia*, and 24. *Jaroslavia*, in which they proceed, by numbring crooked and obliquely, from East to West.

17. *Poland* succeeds *Russia*, which is the most spacious Tract of *Europe*, situate in respect of the *Russian Empire*, partly towards the West and partly towards the South; in as much as 'tis Bounded on the East by *Russia*; on the North by the same and *Sweedland*; on the West by *Germany* and *Bohemia*; on the South by *Hungary*, *Denmark* and *Tartary*.

18. The Provinces of *Poland*, into which this Kingdom is divided, may best be reckoned up, if the access be made from the North towards the South East, and from thence towards the West, and from thence again to the North; in which Circuit is, 1. *Lithuania*. 2. *Polesia*. 3. *Volhinia*. 4. *Ukrania*. 5. *Podolia*. 6. *Russia*. 7. *Poland the lesser*. 8. *Mazovia*. 9. *Poland the greater*. 10. *Pomerellia*. 11. *Prussia Royal*. 12. *Samogitia*. 13. *Scmigallia*. 14. *Curlandia*.

19. From thence we come to *Dacia*, which is a Tract of *Europe*, beneath *Poland*, behind the *Black Sea*, to some place towards the West, extended to the Left on the *Danube Shoar*, at this day subject to divers Lordships; whence the Bounds of it on the North is *Poland* it self; on the East 'tis bounded by *Tartary*, with part of the *Black Sea*; on the South (the *Danube* coming between) by *Thracia*, both the *Mefias*, and part of *Hungary*; which also is the Bounds on the West of *Dacia*.

20. The

20. The Division of it in Old Time, (which it may also obtain at this time) was three fold, to wit, part 1. belonging to the *Alps*. 2. Of the *Mediterranean*. 3. Of *Ripensis*; of which the former situate on the North, is *Moldavia*; next to it, but lesser, is *Wallachia*, to which *Bessarabia* adheres. The *Mediterranean* at this day is called *Transilvania*, possessed partly by the *Germans*, partly by the *Hungarians*, and partly by the *Wallachians*, (of late Invaded by the *Turks*) Lastly, *Ripensis* contains a Portion of *Hungary*, situate not far from *Tibisco*, which also most rightly is ascribed to the same.

21. *Hungary* follows, which is a Tract of *Europe*, partly reckoned by it self, from the ends of *Dacia*, extending to the West, and beyond the South, and running out to both Shoars of the *Danube*, by a sufficient space.

22. The Bounds of it towards the North, are *Poland*, towards the East, *Dacia* and *Servia*, towards the South *Illyricum*; but towards the West, partly *Bohemia*, partly *Germany*, by which 'tis every where inclosed.

23. This *Hungary* in Old Time was called *Pannonia*, when it lay more open, but now at this day 'tis divided with Provinces belonging to it, 1. *Ungharia*, properly so called; then 2. *Slavonia*. 3. *Croatia*. 4. *Dalmatia*. 5. *Bosnia*, and 6. *Rascia*; of which, *Hungaria*, which is situate towards *Poland*, either in Planes or Mountains, is called 1. Superior; and that which lies nearer to the *Danube*, and extends it self towards *Servia*, 2. is called Inferior; but the whole is wont to be divided into 50 Counties, which for brevity we omit.

24. Of the other Provinces belonging to *Hungary*, the next is *Slavonia*, that Kingdom which is extended between *Dravus* and *Savus*, all along to the *Danube*: *Croatia* appears in passing to *Savus*, to which adheres

adheres *Bosnia*, on the South part of *Dalmatia*, looking hither, and in the lower Shoar of the River, then afterwards passing towards the North East, partly by *Savus* and partly by the *Danube*, *Rascia* is joined, and in the said place part of *Dacia*.

25. Now succeeds the *Bohemian* Kingdom, with the Provinces belonging to it, which is a Tract of *Europe*, for the most part inclosed by the Provinces of *Germany*, but that on the East, it touches upon the Confines of *Poland* and *Hungary*: for on the North, it owns for its Bounds, the Countries of *Brandenburg* and *Saxony*; on the West the *Palatine*; but on the South, the Countries of *Austria*.

26. And 'tis divided into, 1. *Bohemia*, properly so called; into 2. *Moravia*, and 3. *Silesia*, and also 4. *Lusatia*; which last, notwithstanding in these Times, is more rightly numbred to *Germany*: Moreover the same *Bohemia* is divided into XV. Districts or places of Jurisdiction, viz. 1. *Elebogenssem*. 2. *Philnsensem*. 3. *Sazensem*. 4. *Rakonizenssem*. 5. *Leitmerizenssem*. 6. *Slanensem*. 7. *Potherensem*. 8. *Braschensem*. 9. *Kaurischimenssem*. 10. *Moldaviensem*. 11. *Zaslaviensem*. 12. *Bechiniensem*. 13. *Konigsgrasensem*, and 14. *Crudimerensem*; instituted in order, from the North to the South, and again subdivided; to which comes 15. The County of *Glazensius*.

27. *Moravia* the curious is not divided, but *Silesia* into the *Upper*, which is situate towards *Hungary*; and 2. The *Lower*, which tends towards *Lusatia*, and the Marchionate of *Brandenburg*: The Division of both is more accurately instituted into XVII. Dukedoms. 1. *Crosnensem*. 2. *Saganensem*. 3. *Glogoviensem*. 4. *Jauraviensem*. 5. *Wolaviensem*. 6. *Lignizenssem*. 7. *Uratislaviensem*. 8. *Schwidnizenssem*. 9. *Munsterbergensem*. 10. *Oelsnensem*. 11. *Brigensem*. 12. *Grotgaviensem*. 13. *Oppoliensem*.

sem. 14. *Carnoviensem.* 15. *Troppaviensem.* 16. *Ratiborenssem.* And lastly, 17. *Teschinensem*: if we Travel towards the South, from the North West, we are wont to number by 7 Classes.

C H A P. XIII.

Of the middle of Europe, or of the last Country Germany.

1. **G**ermany only remains, which requires, (because 'tis our own Country) a more larger Declaration; and 'tis a Tract of *Europe*, between *France*, *Italy* and *Hungary*, as also between the middle of *Bohemia* and *Poland*; by which also the Bounds are every where inclosed, if we add the *Baltick Sea*, and the outmost of *Cimbrica Chersonesus*, and likewise part of the Ocean on the North side of it.

2. And the Division of *Germany* was various, according to the Ancients, also in any place very uncertain, nor but leaning upon Conjectures; but the Division at this time may conveniently be instituted into Provinces, 1. without the Communion of the Empire, and 2. Living within the same: To the former Classe, belong only the *Netherlands* and *Helvetia*.

3. And the *Netherlands* is a Tract of *Germany*, situate at the West and North Angle of it, and extended through the long Series of the Shoars of the *German Ocean*, with some Mediterranean Countries spacioussly extended or stretched out; so that the Bounds of the *Netherlands* on the North, is the Sea Coasts; on the East 'tis bounded partly by the South and West of the *German Empire*, and partly by the circumjacent Provinces of *France*.

4. At this time divided 1 into the free *Netherlands*, and 2 the *Spanish*, of which that is Easterly, this Westerly; that finished by VII. Regal Provinces, 1 of the *Confederates*, 2. this into X.

5. The Confederate Provinces of the *Netherlands*, are numbered from the East towards the West, in a contiguous order; 1. The Lordships of *Groningen*, with *Omlandia*. 2. *Frisia*. 3. *Transsylvania*. 4. *Geldria*, with *Zutania*. 5. The Lordship of *Utrecht*.

6. *Holland*,



6. *Holland*, and 7. *Zeeland*, although not all *Geldria*: but notwithstanding many parts of *Brabant* and *Flanders* belong to the Confederates Orders, so nevertheless they obtain only VII. of the aforesaid Countries, by the right of Majesty.

6. To the Royal *Netherlands*, belong these Maritime Provinces, 1. *Brabant*. 2. The Marchionate of *Antwerp*. 3. *Flanders*, and of the *Mediterranean*. 4. *Artesia*. 5. *Hannonia*. 6. *Namurcum*, or *Namur*. 7. *Limberg*. 8. *Lutzenburg*. 9. *Mechline*, and 10. *Gilderland*; the middle part of which (as 'tis said) as yet is Governed by *Spain*; but *Artesia*, by a late Peace, is Translated from *Spain* to *France*.

7. Proceeding from the North West Angle of *Germany*, to the opposite of it, South and by West, where *Helvetia* offers it self, which is a Tract of *Germany*, bounding *France* and *Italy* from the same; of which that hath its Bounds on the West, this on the South; on the East 'tis bounded by *Sweedland*, and on the North by the same, and *Alsatia*.

8. *Helvetia* in Old Time was divided into IV. Villages, 1. *Urbigenum*. 2. *Tigurinum*. 3. *Aventicensum*, and 4. *Antuatem*: afterwards it was divided in a twofold manner into *Alemanny* and *Burgundy* the lesser; so it is at this day divided into IV. Lands, viz. commonly separated from *Sweedland* towards *France*, into 1. *Turgoviam*. 2. *Zurchgoviam*. 3. *Argoviam*, and 4. *Wislisburgoviam*.

9. The Commodious Division of the rest, is into XIII. 1. The Villages of the *Helvetians*, and 2. The Confederate Allies of *Helvetia*, and also 3. The Common People subject to the Empire. Of the Villages, 6 are outer and 7 inner, in numbring from the West to the North, and thence to the East, being instituted Archwise, and ending in the very middle; where occurs, 1. *Friburg*. 2. *Bern*. 3. *Soloduria*. 4. *Basile*. 5. *Schafhuse*. 6. *Appenzella*. Then 7. *Uria*. 8. *Underwaldia*. 9. *Schwitzia*. 10. *Glarona*. 11. *Lucerna*. 12. *Tugium*, and 13. *Tigurum*.

10. The

10. The Allies of *Helvetia*, are either Districts or singular Burroughs. Of the Districts, the Eminent are, 1. Of *Rhetus*. 2. Of *Vallesius*; of which those bend to the North East, but these to the South West; there is the Bishops See of *Curienfus*, here of *Sittenfus*, being the best of States: But the Burroughs or Walled Towns are, 1. *St. Gall*, towards the Lake *Acronia*. 2. *Rothwila*, in *Suevia*. 3. *Mulhusium*, in *Alsatia*. 4. *Bipennium*, about *Bernam*; nor far from thence, 5. *Neocomum*; and at last at the Lake *Lemannum*. 6. *Geneva*.

11. The subjected People or places in common, (or at least many of the Burroughs) are Provinces or places of Jurisdiction, in number IX. to wit, 1. The County of *Badens*. 2. The free Provinces in *Wagenthal*. 3. *Turgea*. 4. *Sarunetes*. 5. *Rhegusci*. 6. *Luganenses*. 7. *Locarnenses*. 8. *Mendrisii*. 9. *Medio-vallenses*.

12. Hence at length proceeding to Imperial *Germany*, which is all the remaining Tract, extended or stretched out from the *Alps*, as well to the *German* as the *Baltick Sea*, and likewise from *Poland*, *Bohemia* and *Hungary*, even to *Helvetia*, *France* and the *Netherlands*; where the ends of *Germany* easily shew themselves together.

13. And indeed Imperial *Germany* is divided by publick Authority into X Circles, to wit, 1. *Burgundy*. 2. The Upper *Rhine*. 3. The Electorate on the *Rhine*. 4. *Suevia*. 5. *Franconia*. 6. *Westphalia*. 7. *Bavaria*. 8. *Austria*. 9. *Saxony*, as well the Upper as 10. The Lower. Nevertheless, because some Provinces are not comprehended in reckoning of these Circles, we reckon whatsoever lies in a direct Line from the *Helvetian Gulfs* towards the North, and we make the beginning thence, again from the South, by proceeding to the same Coast; and we repeat the same again and again, until we have reckoned up all the

the Regions of Germany, from the South towards the North, into 4 Classes.

14. In the I. Classe occurs *Alsatia*, (the Imperial, or belonging to the Emperor) 2. The Palatinate of the *Rhine*. 3. The Electorate of *Mentz*. 4. The Electorate of *Treves* or *Trier*. 5. The Electorate of *Cologne*. 6. The Dukedom of *Gulick*, of *Cleve*, and of *Mons*,

15. To the II. Classe belongs, 1. *Suevia*. 2. *Franconia*. 3. *Hassia*. 4. *Westphalia*. 5. The Dukedom of *Bremen*. 6. *Holsatia*.

16. To the III. Classe belongs, 1. The Dukedom of *Tirol*. 2. Of *Bavaria*. 3. Of the Upper Palatinate. 4. Of *Thuring*. 5. Of *Brunswick*, &c. 6. And the rest is the Lower Saxony.

17. To the IV. and last Classe belongs, 1. The Archbishoprick of *Salzburg*. 2. The Province of *Austria*. 3. *Misnia*, and Upper Saxony. 4. The Marches of *Brandenburg*. 5. *Pomerania*, and 6. on the side, *Cassubia*.

18. Under *Alsatia*, which the Landgrave attends, (part of which belongs to *France*) are comprehended, 1. The Upper, and 2. The Lower, and also 3. *Brisgoia*; as also 4. *Basile*, the Bishoprick of *Basill*. 5. Of *Stratsburgh*. 6. Of *Spire*, and 7. Of *Worms*: But also many Counties, 1. The Governments of *Stuevis* and *Rhene*. 2. Of *Leiningen*. 3. Of *Hanovien* the Upper. 4. Of *Rappolstein*, and 5. Of *Kriechingen*: And also some other Cities besides these that we have Named.

19. The Palatinate of the *Rhine*, not only 1. The Eighth Electorate at this day, but also 2. *Lutzelstein*. 3. *Birkenfelden*. 4. *Spanheimen*. 5. *Bipontin*. 6. *Simeren*; and in which other Dominions are contained: To which we may join the Countys, 7. *Veldentz*, and 8. *Erpach*.

20. The 3 Electorates, 1. Of *Mentz*. 2. Of *Treves*, and 3. Of *Cologne*, require no curious Subdivision, unless that the two former be distinguished now and then between the Palatines and the *Hassians*; and the latter, although it be neither deficient in them, it is not only encompassed by many Counties, but also numbers to it self the Bishoprick of *Leige*, situate between the *Belgick* Provinces.

21. They conclude this I. Classe with, 1. The Dukedom of *Gulick*, on the left side of the *Rhine*. 2. The Dukedom of *Newburg*, on the right side of the *Rhine*, with the adherent County of

Marches;

Marches; and 3. The Dukedom of *Cleve*, beheld from both sides of the *Rhine*, with the Lordship of *Ravenstein*, which is sited beyond the *Maes*, towards the West.

22. *Suevia* begins the II. Classe, which is the Marchionate of *Baden*, situate not far from the right Shoar of the *Rhine*, and the Dukedom of *Wirtemberg*, (in which these things were written) about both sides of the River *Necker*, as also the Bishoprick of *Constantien*, joining to the Lake *Acronius*; then *Augusta*, by the *Danube*, with many, 1. *Prelacies*. 2. *Counties*, and 3. *Imperial Cities*, which are there. I. *Elchingen*, *Zwifalten*, *Elwangen*, &c. here. II. *Ebersteinen*, *Furstenburg*, *Zolleran*, *Walzburg*, *Helfenstein*, *Leostein*, *Montforten*, the Government of *Limburg*, with others, &c. III. And here the City *Ausberg*, *Ulma*, *Memminga*, *Reutslinga*, *Estlinga*, *Heilbronna*, *Bibracum*, *Lindavia*, *Ravenspurg*, *Halla Suevorum*, *Nerolinga*, *Kauffbyca*, and many others, &c.

23. *Franconia* touches upon *Suevia*, towards the North, a Tract of *Germany*, between the middle of both the Palatinates, which contains I. *Marches*, viz. 1. *Anspacen*. 2. *Culmbacen*, (or *Bareiten*) towards the West. II. Bishopricks. 1. *Eithsteten*, to the South. 2. *Bambergen*; and 3. *Herbipolen*, (which also is a Dukedom of *Franconia*) situate in the middle, to the sides of the Walls. Then III Counties, 1. Of *Oetingen*. 2. *Hobenloben*. 3. *Wertheimen*. 4. *Hanovien* the lower. 5. *Isenburgen*. 6. *Henneberg*. 7. *Schwartzeburg*; together IV. with the Imperial Cities *Noriberg*, *Rothinburg*, *Winshem*, *Suevofurt* and *Francofurt*.

24. *Hassia* or *Hessen* succeeds this, a Tract of *Germany*, between the Provinces of the *Rhine* and the middle *Thuring*, in which are, II. Landgraves, 1. Of *Castell*, and 2. Of *Darmstad*, that Northern, this Southern, declining to the West; but the outmost Provinces of it from the East towards the West, and hence towards the North, are *Buchovia*, there the Prelacy of *Fulden*, from thence the Abbotship of *Hirschfeld*, and the County of *Zigenheim*, likewise *Rhingovia*, and a little beyond *Wetteravia*, and it entertaining the Counties of *Nassau*, *Solmen*, *Salmen*, *Waldecken*, *Witzenstein*, and many others in the District

strict (*Westerwald*) with the Imperial Cities *Fridberg*, *Wetzlar*, *Gelhu*.

25. *Westphalia* joins to *Hassia* or *Hessen*, a Tract of Germany, between the *Rhine* and the middle of the Dukedom of *Brunswick*, divided into many Bishopricks and Counties: of them, the greatest and the mean is *Munster* or *Monasterium*, on the right side, *Minden* at *Visurg*, and beyond is *Verden*, (now two Principalities) but back again towards *Amisium*, is *Osnabruck*, and to the East of *Luppia*, is the Bishoprick of *Paderborn*; from hence the Abbotship of *Corbejen*, from thence of *Essen*: Then the Counties of *Visurg*, are *Schavenburg*, *Hojen*, *Dieppold*, and *Oldenburg*: Of *Amisium*, *Ridberg*, *Thecklenburg*, *Lingen*, *Bentheim*, and *Ostfrisia*, (now a Principality) Of *Luppia* there is *Lippen*, and in passing to *Amisia*, the Lordship of *Ravensperg*, with the adjoining Imperial City *Lemgo*.

26. Then follows the Tract of *Bremen*, now a Dukedom, in old Time an Archbishoprick, between *Visurg* and the Mouth of the *Elbe* and the German Sea, also inclosed by these and the Shoars of the Rivers; and so the middle is contained between the Dukedom of *Lunenburg*, which looks to the East, and between *Westphalia*, which looks to the West and South.

27. Having passed the River *Elb*, you may behold the greatest Province of the North of Germany, the Dukedom of *Holsatia*, and in the furthest part of it towards *Futia*, *Schleswick*: (besides the lesser Lordships of the Governors of *Holsatia*, belonging partly to the King of Denmark, partly to the Duke of *Gottorf*: And the chief parts there, besides certain Islands and Peninsulas, are those which are called at the Ocean, *Ditmarsia*, at the *Elb*, *Stormaria*, at the *Baltick Sea*, *Wagria*.

28. In the III. Classe of German Provinces, from
P P the

the South towards the North, is *Tirol*, a most Noble County, which is next to *Italy*, the *Alps* lying between, and extends it self about either side of the River *Oenius*, even to *Bavaria*.

29. Thence is beheld the Dukedom of *Bavaria*, (now an Electorate) occupying a great Tract of the *Danube*, but the greater on the right side, which part of it is nearer *Suevia*, The 1. is called *Bavaria Superior*, which looks true to the South. 2. *Bavaria Inferior*, containing also III. Bishopricks, *Ratisbon*, and *Passavien* to the *Danube*, *Freisingen* to *Isara*, together with the Imperial Counties of *Ortenburg* and *Ratisbon*.

30. The Superior Palatinate joins to *Bavaria*, together with the Landgrave of *Leichtenburg*, stretching out to some place between *Franconia* and *Bohemia*.

31. Proceeding further is the Landgrave of *Thuring*, and on the North side is offered *Voigtland*; of which the former contains the Principality of *Gotha* and *Isenac*, towards the West, but of *Vinarien*, towards the East, with the Counties *Hohenstein*, *Stollberg* and *Mansfeld*: but the latter (*Voigtland*) is distributed into the Districts of *Saxony* and *Brandenburg*, and of the Lords of *Reussen*.

32. Inferior *Saxony* joins incontinently to *Thuring* and first of all the Dukedom of *Brunswick*, with the Principality of *Halberstadt*, and the Bishoprick of *Hildese*, together with the Imperial Cities *Götting*, *Goslar* and *Nordhus*: but in the former are the Portions, *Zellen*, *Hanover*, *Guelferby*.

33. *Lavenburg* shuts this Classe, and this the greater Dukedom of *Mecklenburg* divides, of which the District in *Swerin*, which is most Western, and *Gustrovien*, which is most Eastern, is the Ducal, together with the Imperial City of *Lubeck*, as also from the by-Name of the Bishopricks of *Swerin*.

34. In the last Classe of the German Province

the Archbishoprick of *Salisbury* stands before the rest between the *Austrian* and *Bavarian* Provinces, yet by means of the Mountains as it were, every where included.

35. Declining from hence a little to the East, is where they begin the Provinces of *Austria*, to wit, 1. The Government of *Italy*, with the Dukedoms. 2. *Carniola*. Then 3. *Carinthia*, and further, towards the North East, 4. *Stiria*. And lastly, 5. The Arch Dukedom of *Austria*; all which they shew with a Site never interrupted: And indeed *Austria*, properly so called, is divided by the River *Anis*, into 1. The Superior, and 2. The Inferior, and both-spaciouly spread about the 2 sides of the *Danube*.

36. From thence afterwards interposes *Bohemia*, it comes to *Misnia* and Upper *Saxony*, which on the West is beheld the Principality of *Anhaltine* and the Archbishoprick of *Magdeburg*; but on the East, concurring with the North Site, is beheld *Lusatia*: and indeed in *Misnia* are found besides, 1. The Electoral Dominion, and 2. The *Saxon Altenburg*, also 3. *Mærsburg*, and 4. *Naumburg*, in Old Time Bishopricks, now Principal Seats: But the Principality of *Anhaltine*, with the County of *Ascan*, is distributed into little Lordships. The Archbishoprick of *Magdeburg* expects shortly a new Conversion. Lastly, *Lusatia* is divided into 1. The Upper, and 2. The Lower, of which that is *Hexapolitan*, this *Tripolitan*; and now also contains the County of *Promnitz*.

37. Moreover, by an orderly Series, follows the Marchionate of *Brandenburg*, which is separated into 1. The Old *Marches*, and 2. The *Middle*. 3. The *New*, and 4. *Uggeren*, and also the District. 5. *Brignick*, from the East to the North; and 6. *Seenberg*, from the East to the South, where joins *Polonia* it self, with the Bishoprick of *Halverberg*; which receives its Name from the River to which it is sited.

38. *Pomerania* follows, with its Collaterals, towards the East *Cassubia*, both of which pretend to the *Baltick Sea*, thence to *Russia*;

that nigh to the *Mecklenburg* Dukedom; from which, although the latter be not subdivided, yet the former, (to wit, *Pomerania*) either, 1. 'tis on this side, or 2. beyond; here, the Bishoprick of *Caninen*, there, some Islands belonging to the same, having first *Rugia*, (a Principality) agreeing to it self.

39. And indeed this Division of the Land or World, taken from the diversity of the Inhabitants, is bounded by us so, that the demonstration of the Globe and of Maps may supply the rest: but the general Knowledge of the Wet and Dry parts of the Earth or World, the 2 Tables annexed at the end of Geography will much promote.

CHAP. XIV.

Of the IV. Distinction of the Terrene Globe, taken from the Circles adscribed to the Terrene Globe by imagination. Fig. 192.

1. **CIRCLES**, by which also the Terrene Globe is distinguished, are obtained from Astronomical Doctrine and from Heaven, and are so applied to the Earth, that they are directly subjected to the Heavenly, and answer in proportion to them.

2. And these are

1. The Rational and Visible Horizon.
2. The Meridian of any place.
3. The Equinoctial, or the Equator.
4. The Parallels of it, to wit, the two Tropicks and the two Polar Circles.

The Descriptions and Distinctions of these Circles, are delivered before in Astronomy, so that there is no need to repeat them in this place.

3. These Circles are divided into two portions: For

1. *The Rational Horizon*, distinguishes the Earth into the upper and lower Hemisphere.
2. *The Visible Horizon*, into the upper part, or that appearing to the sight, and the hidden or lower part.
3. *The Meridian*, distinguishes the Eastern part from the Western, or the fore part from the hinder.
4. *The Equator*, lastly, divides the Terrene Sphere into the Northern half, in which the North pole is; and into the Southern half, in which the South pole is found.

Note, 1. *In the Terrestrial Globe these Circles are found, in the same manner also as in the Celestial.*

2. *The same also are depicted on Charts or Geographical Maps, as well*

well Universal as Particular, which are subjected to one or the other.

3. In others, sited without the Parallels of the Equator, those that stand or are used instead of the Equator and of the Meridians, are shewn by the Columns depicted on the side of the Chart, and divided into Degrees and Scruples or Minutes; of which, those that tend from the East to the West, shew or relate to the Equator, and those from the South to the North, shew the Meridians.

C H A P. XV.

Of the Z O N E S,

1. **T**HE Equator, with the two Tropicks and Polars, distinguish the Earth into Zones.

2. Those are the greater Tracts of the Earth which are included by the 4 parallels of the Equator, encompassing the Earth as it were, by a Girth or Girdle. Fig. 193.

3. And of them, they number 5, one Torrid, two Temperate, and two Frigid Zones.

4. The Torrid, (so called, which is as it were burnt by the heat of the Sun) is the space of Earth contained between both Tropicks, from the Equator passing through the middle of it.

Therefore since the Sun is perpetually conversant between the Tropicks, nor goes not out from them, its Beams indeed for the most part being direct, excites great heat, yet not so much as to make this Zone Unhabitable, as the Ancients believed; for Men Inhabit commodiously enough under the Equator it self also, as is found at this day by many Navigations: for this Zone is made Temperate, (also since it ought to be most hot, by reason of the Suns lying upon them) almost by perpetual Showers of Rain, and by a Cloudy Heaven, and by cold Winds blowing from most high Mountains, which in those Zones are nevertheless perpetually covered with Snow, as is testified by those who have seen it themselves, by many Years experience in those places.

5. The Frigid Zones, (so called, because they Freeze continually with cold Clouds and Ice) are a space of the Earth, contained between either polar, and near the pole of the World.

6. One of these which is included by the Polar and the North Pole, is called Northern; and the other, intercepted by the opposite Pole and the Polar, is the Southern.

Also these Zones, by reason of the great coldness, the Ancients believed them to be not Habitable; but at this day, although under these Poles themselves, no Man as yet hath come, yet 'tis known for certain also that Men Inhabit within the Northern Polar Circle: And the augmented Cold of these Zones is, because they want the presence of the Sun, which is the Fountain of heat, almost the greater part of the Year; and if when its Beams shine, yet those Parallels receive no heat; even as if any one should move his Finger near to the side of a burning Candle, he would also perceive little of heat.

7. The Temperate Zones, are the space of Earth, contained by either Tropick and the next Polar.

8. One of these, which is included by the Tropick of Cancer and the North Polar, is called the Northern Temperate, which we dwell in: The other is the Southern, included by the Tropick of Capricorn, and by the Southern Polar.

And they are called Temperate, because the Sun Beams are less than at Right Angles, nor also are they Parallels, but Oblique in these Zones, making the temper of the Air grateful.

C H A P. XVI.

Of Climates.

1. **T**HE other Parallels of the Equator distinguish the Earth into Climates.

2. These are a space of Earth comprehended between 2 Circles, parallel to the Equator, in which the longest day varies half an hour.

3. Every one consists of three Parallels, of which, one is the beginning, the second the middle, the third determines

determines the end of the Climate; yet so, that always the end of the former is the beginning of the latter.

4. The measure of distance, of each Parallel from one another, is a Quarter of an Hour.

5. And they are numbred from the Equator, as well to the South as to the North, in a like manner.

6. And they are named indeed Northern, from the places through which the middle Parallel of each Climate passes; and the Southern are named likewise from the same.

7. And although all Climates are constituted to the equal increase of the longest day, and so are alike, yet they are unequal, as well in the Ratio of Longitude from East to West, as in the Ratio of Latitude from North to South.

In the ratio of Longitude.

For Parallels of the Equator, by how much they are nearer to the Pole, they become the lesser; whence also the Climates, by how much they are nearer to the Pole, they are made the lesser.

In the ratio of Latitude.

In like manner, by how much they are nearer to the Pole, they are made more narrow; for the Inhabitants of the Earth, by how much nearer they come to the Pole, so much a greater Arch of the longest day is advanced above the Horizon, and so much more also the longest day varies in a small Interval: Whence also it comes to pass, that the parallel passing through the middle of the Climate, is nearer to the end of it than to the beginning.

8. The Ancients made the first or beginning Climate in that place, in which the longest day contained 12 Hours and $\frac{1}{2}$, that is, Minutes 45, and they numbred them even to the longest day of 16 Hours: Whence only 7 Climates was begat by them.

That came to pass from this, because the places near to the Equator and Poles, and the other places further off, were unknown to them; neither could they believe them to be Habitable; otherwise without doubt, they would have placed the beginning of the first Climate in the Equator it self, and have numbred them to those places, in which the length of the longest day was equal to a Natural day, as the Moderns are wont to do.

The chief use therefore of Climates at this day, is to apply them to the understanding and explaining the Writings of the Ancient Astronomers, Geogra-

phers and Physicians; but at this day the use of them is not so frequent; for instead of mentioning of this or that Climate, we usually express the Altitude of this or that Pole, or the Parallel of the Equator, which passes through some place.

CHAP. XVII.

Of the distinction of the Inhabitants of the Earth, in respect of Shadows.

1. **A**CCORDING to the diversity of Zones, diversity of Shadows also happens to the Inhabitants of the same, which their Bodies cast from themselves in the Noon time.

2. Hence some are called *Ascij*, others *Amphiscij*, others *Heteroscij*, others *Periscij*.

3. *Ascij*, are those which have no Meridian Shadow, as those which dwell between the Tropicks in a Torrid Zone, as often as the Sun passes their Vertex.

For the Shadow from an Opaque or Shadowy Body, is always cast in the opposite part to the Luminous Body; therefore since those to which the Sun is Vertical, have not an opposite, unless the Soil or Ground on which they stand, and which they hide with their Feet, therefore they cannot cast a Shadow from themselves.

4. *Amphiscij*, are those whose Meridian Shadow falls as well into the South as into the North.

From this cause they are so called, as it were casting a Shadow on both sides; this happens to them which inhabit in a Torrid Zone, between the Tropicks, when the Sun departs from their Vertex towards the North or South,

5. *Heteroscij*, are those whose Meridian Shadow is always cast towards one part only, either North or South.

That is done among those who dwell in Temperate Zones, the Vertex of which the Sun never touches, much less never passes over or beyond; for they which live in a Northern Temperate Zone, cast their Meridian

ridian Shadow also in the same manner Northward: But they which Live in a Temperate Southern, cast it towards the South.

6. Lastly *Periscij*, are those, as if we should say they carry their Shadows round about, the Shadow of whom is carried round about in the Orb, and falls on all parts of the World.

This happens to those who Live in the Frigid Zones under the Poles, for to those the Sun every Year is turned about above the Horizon, for some days, without setting; by which also the Shadow must needs go about in the opposite part.

C H A P. XVIII.

Of the distinction of the Earth, by reason of the Situation of the Inhabitants.

1. **F**ROM the comparing of the Meridian with the Parallels of the Equator, the Inhabitants of the Earth are distinguished, by reason of their Site or Situation, into *Periæcij*, *Antæcij*, and *Antipodes* or *Antichthones*.

2. *Periæcij*, are they who dwell under the same Meridian and Parallel, but in divers halves of the Meridian, and in opposite points of the Parallel.

Thus our Periæcij are the Americans, which Inhabit in New England.

3. *Antæcij*, are those whose Shoulders are as it were, mutually opposite to one another, who Live under one Meridian, and in the same half of it, but under divers Parallels, equally remote from the Equator towards each Pole.

In this manner our Antæcij, Live in the Southernly Land, not known as yet.

4. *Antipodes*, are those who go as it were with their Feet opposite to others Feet; who dwell under one Meridian, but in points of it Diametrically opposite.

Therefore our Antipodes dwell in the Southern Land, which are such as no Man as yet hath found out.

Nor yet is it to be doubted concerning them, even as *Augustin* doth in Book 16. Chap. 9. de Civitate Dei,

Dei, where he saith, but they also who are the Reporters of Antipodes, that is, Men on the contrary part, where the Sun arises when it sets with us, treading with their Feet by adverse Foot Steps, are in no manner to be believed.

The Example of this Great Man puts us in Mind, that we reject not rashly, that which seems new and incredible to us, lest the same happen to us as to Augustin, which daily Experience convinces; for although our Antipodes are not as yet found out, yet there are known the Antipodes of others; for inner Lybia is almost precisely opposite to India. See Fig. 194.

In this Fig. A B C D is the Meridian, described in the Terrene Globe, from the centre E; the right Line A E C represents the Diameter of the Equator, whose Poles are B and D: The right Lines F G and I K represent the Parallels of the Equator, or rather the Diameters of them, equally remote on both sides from the Equator.

Therefore they that Live in F, have Perizicij dwelling in G, Antacij in I, Antipodes in K.

But they that Live in I, have their Peracij in K, Antacij in F, Antipodes in G.

Therefore 'tis to be noted

That, as the Inhabitants in a right Sphere, under the Equator it self, have no Antacij, but Antipodes between Peracij; so also those which both Live in a right Sphere, and Inhabit under the Poles, are Antacij and Antipodes among themselves, having no Peracij.

C H A P. XIX.

Of the distinction of the Earth, with respect to the divers Positions of the Sphere.

I. **T**HE Equator and the Horizon, according to the divers Positions to one another, distinguish the Earth into a right Sphere, an oblique, and a Parallel Sphere.

2. *A right Sphere*, is that in which the Horizon and the Equator, cut one another at Right Angles, and either of the Poles is raised above the Horizon, the other is depressed below the same.

3. *A Parallel Sphere*, is that in which the Equator is made Parallel to the Horizon, and either of the Poles, with the Vertical Point, falls in with the other opposite to it.

This distinction belongs not only to Astronomy, but also and indeed properly to Geography, for it depends not from Heaven, but from the Earth, in divers places of which also they have an Accident of Heaven, in a diverse manner; which Variety is considered in Geography.

4. *A Right Sphere*, is only so, of two Parallels, one North, the other South; but of oblique, the variety is innumerable, according to every elevation of the Pole, as well towards the North as towards the South.

5. Yet in the whole, are chose 7 positions of Spheres towards the North, the Accidents of which are wont to be considered.

(For what is said concerning these, may be observed also concerning like positions on the opposite Southern part, by only observing the difference of opposites, and may be understood in the same Coast, according to more and less.) Fig. 178.

6. The first Position therefore is of them, who Inhabit in a right Sphere under the Equator it self.

The Accidents of these are

1. They have perpetual Equinox, and not only the days are equal to the Nights, but also the days are equal to all the days, and the Nights to all the Nights.

2. All the Stars, which are in any place in the whole Heaven, arise and set daily; neither is there any which either set not, or always remain hid under the Horizon.

3. The Sun passes Yearly twice through their Vertex, to wit, when 'tis conversant in the Equator it self, and then makes them without Shadow; in the other time, in which it departs into the North and South, it makes them *Amphiscij*.

4. They have indeed always Summer or heat, but twice augmented when the Sun becomes Vertical to them; but more slack when it departs to the sides.

This

(This departing of the Sun makes as it were Winter, yet more fervent, if the Air of the same be temperate among them, than our Summer is; for as much as the Sun when she departs farthest from them, yet she is still nearer to them than when she approaches nearest to us.

5. In this Sphere *Periacij* and *Antipodes* are the same, and to these, the Day and the Night exchange between themselves, and by turns.

8. The Second Position of the Sphere, is of them who dwell in an Oblique Sphere, between the Equator and the Tropick of Cancer. Fig. 196.

The Accidents of these are

1. They have only two Equinoxes, from the Sun being constituted in $\circ$ γ and $\cap$.

2. Some Northern Stars never set; on the contrary, some Southern never rise: and these two Accidents are common to every Oblique Sphere.

3. The Sun also is Vertical to them twice in every Year, from thence she departs farther into the South than into the North, which makes them also partly *Ascij* and partly *Amphiscij*.

4. They have two Summers, by the Sun passing their Vertex; also as it were, two Winters, but unlike, shorter and warmer, (which, yet with respect to ours, may be thought as if they were Summers) by the Sun being conversant in the North; and longer and colder when the Sun departs from their Vertex into the South.

5. *Periacij*, exchange the Noon and Midnight, likewise the time before Noon and Afternoon; yet it may come to pass in the longer days, that they both have the Sun above the Horizon; but to these 'tis Oriental, to those Occidental: The contrary happens if the day be shorter than its Night, to wit, as the Sun exists beneath the Horizon to both, but again to these Eastern, to those Western; But the *Antiacij* change the Stars rising and setting, for those Stars which to these always appear, to those they are never seen, and the contrary: Moreover, notwithstanding

standing they exchange not indeed the vicissitude, but the length of the day and of the Night, for how much the day is there, so much the Night is here; when the days increase with us, the Nights increase with them; also they have Summer and Winter among them in an opposite manner to one another, and 'tis Winter there, when to these 'tis Summer.

Lastly *Antipodes*, even as they have an opposite Position by the Diameter, so all these Accidents are contrary to them.

For when these have day, their *Antipodes* have Night; when the days are at longest to them, they are at the shortest to these, when 'tis Summer to them, 'tis Winter to these: In the same manner also the rising and setting of the Stars is in a diverse and contrary manner; those Stars which are always seen by them, are never seen by these; in what moment a Star rises here, in the same it sets there.

(And this exchanging takes place also in all other Oblique Spheres, by reason of *Periæcij*, *Antæcij*, and *Antipodes*, that it will not be needful to repeat it in what follows)

9. The third Position or Situation is of them, of which the Vertical point falls in with the Tropick of Cancer. Fig. 196.

The Accidents of these are

1. All the Stars which the North Polar Circle contains in its Ambit or Compass, are always appearing; but those that are about the opposite Polar are always hid.

2. The longest day is, from the Sun passing through the Zenith; the shortest, if it be most remote from it to the South.

3. The Sun becomes Vertical to them once only, when she enters *Cancer*, and then they are *Ascij*: And since the Sun ascends not further into the North, they begin to be made *Heteroscij*; and this is the first

first place of them, in which the Meridional Shadow fails.

4. There is one Summer, by the Sun being conversant about the Zenith: Winter to the opposite places, and here also the Torrid Zone leaves of.

10. *The fourth Position* is ours, which inhabit between the Tropick of Cancer and the North Polar, in a Temperate Zone. Fig. 179.

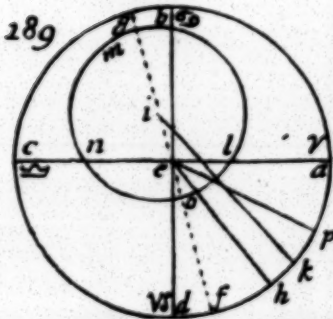
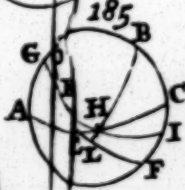
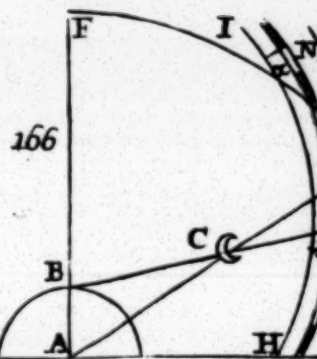
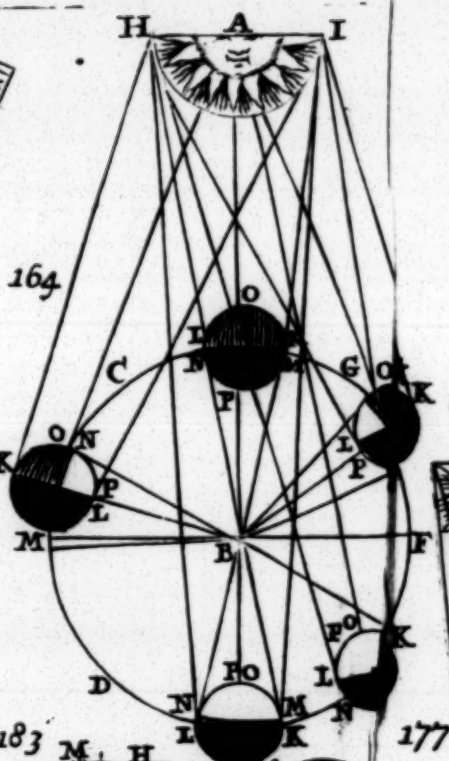
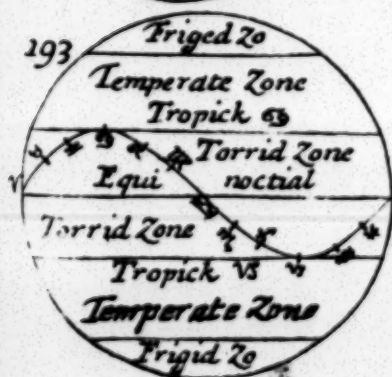
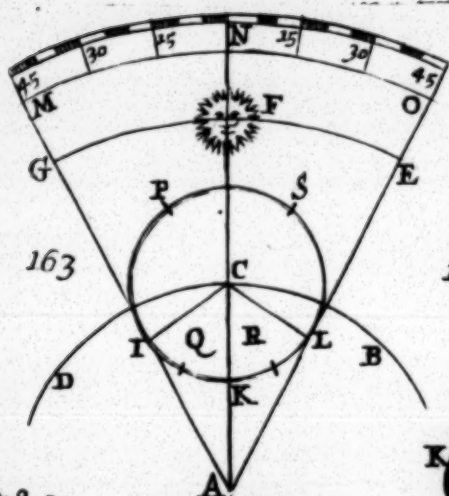
Therefore to us these things happen.

1. Many Stars either never appear, or never set.
2. The greater inequality of the Days and Nights is consistent between 13 and 24 Hours.
3. The Sun never becomes Vertical to us, whence we are neither *Afcij*, nor *Amphiscij*, but *Heteroscij*.
4. We have one Summer and one Winter, but unlike; for places nigh to the Tropick perceive greater Summer and less Cold: It happens contrary among those who come near to the Polar, the Temperateness is mixt in a mean of Cold and Heat.

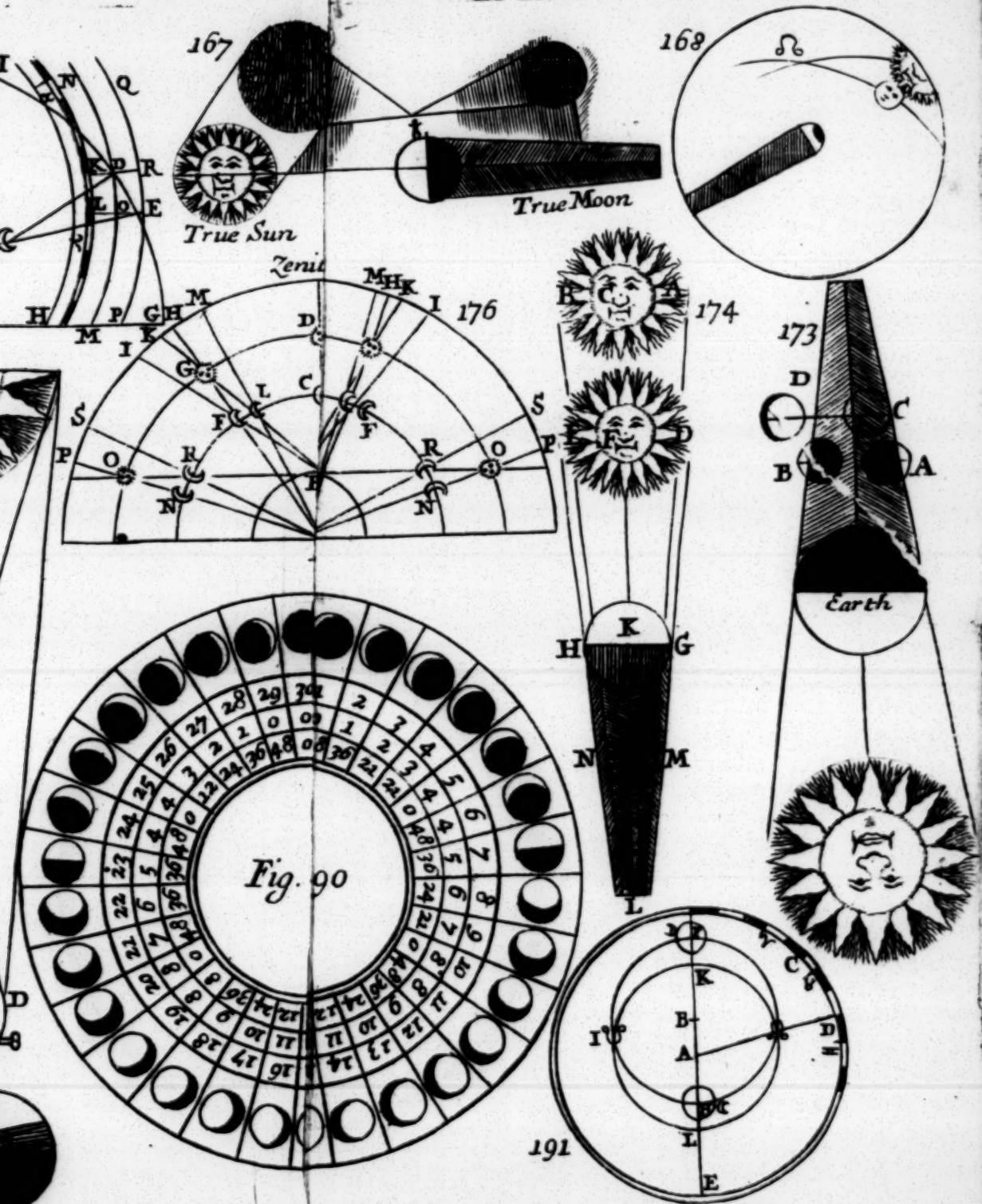
11. *The fifth Position of the Spbere* is of them, of which the Zenith resteth in the Polar Circle it self. Fig. 180.

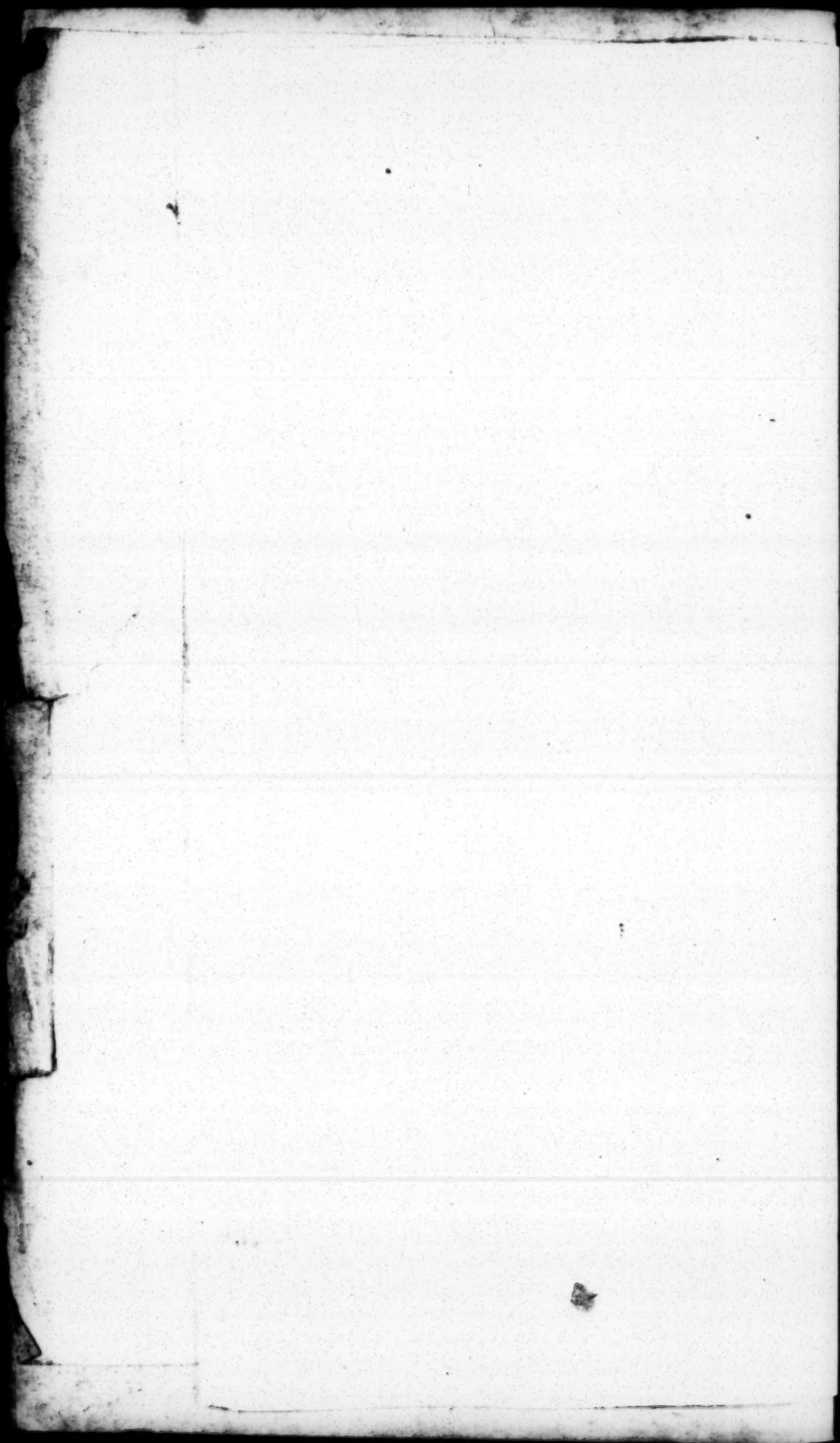
The Accidents of these are

1. All Stars, whose declination is greater than the Obliquity of the Ecliptick, either set not, if they are Northern; or rise not if they are Southern.
2. The longest day is of 24 Hours, the Sun being conversant in the North; the shortest Night either a Moment or nothing; and the contrary, if the Sun be conversant in the South, the longest Night is of 24 Hours, and the shortest day is either a Moment or nothing; and in this place the division of the Natural day ceases into an Artificial Day and Night, by consuming the whole Natural day, by Day or Night.
3. Therefore the Center of the Sun touches or shaves the Horizon twice every Year, either by rising or setting; to wit, in the Tropical Points, which touch the Horizon.
4. The



Place this at page 393





4. The Sun at Noon Time is always beheld to the South, only in the Tropick of *Cancer*, the Sun descending to the Circle of Midnight, may be beheld also in the North; therefore here the *Heteroscij* leave off, and the *Periscij* begin, which nevertheless have a Meridional Shadow only once in a Year, and is cast *ad infinitum*, by the Sun, as is said, occupying the Tropick of *Cancer*, and descending to the Circle of Midnight.

5. They have one Summer and one Winter, but the Winter so cold, as their Summer is warm, which Live under the Tropick; but the Summer not more warm, than is the Winter of the same.

12. The sixth Position is of them, whose Zenith consists between the Polar and the Pole. Fig. 199.

In these Places.

1. Few Stars rise and set; to wit, which decline not much from the Equator; the rest are either always conversant above the Horizon, or always beneath the same.

2. They have an Equinox, as other places of the Earth, by the Sun being in γ and π ; but in two degrees of them, of which the Declination North is equal to the distance of the Pole of the Equator from the Zenith, the Artificial day is of 24 Hours, leaving only such a Moment to Night; thence the day is perpetual, as long as the Sun is conversant in the remaining Arch: The same happens to the Night, in the opposite Signs.

3. Therefore the Sun consisting in that point, as well Northern as Southern, the declination of which is equal to the distance of the Pole and Zenith, four times in every Year it lightly touches the Horizon, in the Section of it with the Meridian, besides by the East or West.

4. The

4. The Sun being constitute above the Horizon, in the upper half of the Meridian, is beheld in the South, but in the neither or lower half, she is beheld some days in the North; whence they are *Periscij*, casting the Shadow into all Coasts, yet having the Southern longer than the Northern.

5. They have one Summer and one Winter, but the Summer like to our Winter; whence those places always rage with Cold; that the Snows and Ice permit access to the Sea, by resolute Sailors, only 2 Months.

13. *The Seventh and last Position of the Sphere*, is the Parallel Sphere it self, of them who inhabit under the North Pole, and have it for their Vertical Point. Fig. 181.

The Accidents of these are

1. Of the Fixed Stars, none arise nor set, but one half of the exercised Stars are always seen in the Night, the other always lye hid, as well in the day time as the Night.

2. The Artificial day is extended an whole half Year, as is also the Night; and the Sun arising in the Vernal Equinox, constitutes a motion, not Diurnal but Proper, and sets when it descends to the Autumnal Equinox.

3. To these the days are turned about in the Horizon, to all the rest they are turned about either above or beneath the Horizon.

(Hence it appears, that the Prophets and the Evangelists in describing the Holy Scripture, respected the Inhabitants not otherwise, but only by their Countries: for if we would refer the Descriptions of them to all parts of the World, in these places, from the 3 days of the Interment of Christ, would be made 3 Years, and from the 7 days of the Creation, would be made 7 Years.)

4. For

4. For as much as among those who Live under the Polar Circles, neither any Star by a Diurnal Motion, either rises or sets, nor does the Pole incline more to any part of the Horizon than to another; therefore neither is any thing Eastern nor Western, nor Northern nor Southern, unless, with respect to the Equator, the opposite Hemisphere of them that are Northern, may be said to be Southern: Hence also they cast Shadows alike into all parts of the World, by reason of the Suns being carried round about; and they are half of *Periscij*, having no *Periscij*: *Antacij* are made *Antipodes*, and they retain their Properties for the most part.

5. They have no Summer, but always Winter; yet some few days they perceive only some small slackness of Cold.

(For no Beams of the Sun shines on them in a whole half Year; in the time of the other half Year they are very Oblique; neither do they heat any thing: But also the cold Winds blowing from the places round about them, do not a little hinder this heat of the Sun, as often as the Winds blow; and so an hard long Winter hath almost no remedy, although those long Nights of Darknes admit some Remedies: The which are)

1. The Sun Beams rise above the Horizon, although the Center of the Sun lye hid as yet 15 Minutes beneath; which cause, adds 16 days time, in which the Sun may be seen.

2. *Refractions* in the Air, shewing the present Sun, although indeed as yet it keeps at a distance.

3. *Twilights*, which dure the greatest part of the Night.

4. The Moon, in the opposite Arch of the Sun, always shining with a full Orb.

5. The North Pole, by reason of the Eccentricity of the Sun, gains 8 days before the South Pole, which adds to the length of their day.

6. In which almost continual burning Gulfs appear.
(And these things concerning the divers Positions of a Northern Sphere, which, if they are ascribed to the opposite Signs, then presently are had the properties of the same, in like Positions of a Southern Sphere.)

CHAP. XX.

Of the Longitude and Latitude of Places.

1. **T**He Meridians, and the Equator, with its Parallels, shew the Measure, and Terms or Limits, of the Longitude and Latitude of places. Fig. 133.

2. *The Longitude of a Place, is an Arch of the Equator, or of a Parallel of it, numbered from the first Meridian, to the Meridian of the place, from the West into the East.*

3. *The first Meridian, according to Old Time, was that which was drawn through the Fortunate Islands, but at this time, 'tis that which is drawn through the Islands of Corvo, or of the Green Promontory.*

None is first in Nature, as there is neither beginning nor end of a Globe; but the Meridian passing through the Fortunate or Canary Islands, was accounted for the first, by Ancient Institution, partly because Nature it self had made the beginning there, of Africa and Europe, and partly because that places more Western were unknown to the Ancients.

Therefore after that, at this day, the vast Country of America being found out, it was noted as it were a new Orb, and the Modern Geographers have transferred the first Meridian, in places more Westerly; and they begin them from those Islands, in which the Magnetick Needle bends or turns precisely to the Pole of the World, which are called the Islands of Corvo, or the Green Promontory; and are 10 Degrees more Westerly than the Canaries.

But we, according to the Ancient manner used by Ptolomy; in this Tract, make the beginning of numbering of Longitude from the Fortunate Islands.

4. The

4. *The Latitude of a Place*, is an Arch of the Meridian, intercepted between the Equator, and the Parallel passing through the Vertex of the place.

Therefore the Parallel of a place being given, the Latitude of the place is also given; and the Latitude of a place is always equal to the elevation of the Pole, as hath been said before.

5. And the Latitude is two fold, *Northern*, which is numbred towards the North; and *Southern*, which is numbred towards the South.

6. And both, as well Longitude as Latitude, is either of the whole Earth, or of the known Earth, or of a certain Country, or of some particular place, numbred either from the first Meridian, or from some particular place.

7. Whence the difference of two places arises, as well of Longitude or of Meridians, as of Latitude, or of Parallels of the Equator, which may be known if you substract the lesser Longitude or Latitude from the greater.

Thus far of the I. Part of Geography.

P A R T II.
O F

G E O G R A P H Y.

Of the Dimension of the Terrene Globe.

I N Measuring the Earth, three things are considered.

1. That which is to be Measured.
2. The Measure.
3. The manner of Measuring.

C H A P. I.

Of Measuring.

There are many things, the Dimension of which is sought in this Geographical Science.

1. *The Perimeter*, or the Ambit of the greatest Circle in the Terrene Globe.
2. *The Diameter*, and *Semidiameter* of the same.
3. *The Area*, of any greatest Circle.
4. *The Perimeter*, *Diameter*, and *Semidiameter* of the *Parallels* of the *Equator*, and of the lesser Circles.
5. *The Convex Superficies* of the whole Terrene Globe.
6. *The Solidity* or *Grossness* of the same, consisting of three Dimensions.
7. *The Amplitude* or *Largeness* of the *Zones* and *Climates*.
8. *The Longitude* and *Latitude* of any *Region* or *Place*.
9. *The Distance* of two places, as well in *Degrees* as in *Miles*, &c.

C H A P. II.

Of Measures used in Measuring of the Terrene Globe.

1. THE Measures with which we Measure the Terrene Globe, are.

1. *Arches of Circles*, every one of which in like manner as in *Astronomy*, is divided into 360 *Degrees*, and one *Degree* into 60 first *Minutes* or *Scruples*, and each first *Minute* contains the same number, or 60 second *Minutes*, - and so on.

2. *Schwaben German Miles*, are determined or limited by a *Journey* of two *Hours*.

3. *Italian Miles*, or *Ancient Roman Miles*, 8 of which constitute one *German Mile*, witness *Lipsius*.

4. *Stadiums*

4. *Stadiums or Furlongs*, 8 of which make a Roman Mile, and 64 a German one.

5. A *Pace* in going, or 5 Foot, of which 25 are contained in one *Stadium*, 100 in a Roman Mile.

6. *Feet*, five of which make a *Pace*.

7. *Pershes*, consisting of some Feet, according to the Institution of any People, whence they are denominated, Ten Feet, Twelve Feet, Sixteen Feet, Sixteen Feet and an half, &c.

8. *Inches*, or a Fingers breadth and an half, 12 of which make one Foot.

II. Concerning these Measures, we must note, that if we would Measure them by Right Lines, they are Lines having only length; but if we would Measure *Superficies* or *Areas*, they are Square or Quadrate Superficies, stretched out equally into Length and Breadth: Lastly, if we would Measure their *Bodies*, they are Cubical Bodies, of equal length, breadth, and profundity or depth.

CH A P. III.

Of the manner of Measuring.

THE Method of Measuring the Terrene Globe, from the Foundations laid before in Arithmetick and Geometry, we will handle in the usual Mathematical manner, by Problems.

P R O B L E M I.

To turn Degrees of the Equator, and of any great Circle into German Miles.

1. Each Degree of any great Circle, either Celestial or Terrestrial, answers to 15 German Miles, or 69 English Statute Miles, commonly appointed by Artificers; that, or from thence it appears, because if we come nearer to the North 15 German Miles, we find the North Pole elevated one Degree higher than before, above the Horizon.

Q q 3

2. And

2. And since oftentimes, not only whole Miles come in the way to be handled, but also parts of Miles, you must know that we divide a Mile into 60 Scruples or Minutes, which may easily be exchanged into parts, commonly used; for 30 Scruples make half a Mile, or the Journey of one hour, 15 a fourth part of it, of half an hours Journey.

3. Therefore in turning Degrees and Scruples of the Equator into Miles and Sexagesimes of Miles.

1. We Multiply the Degrees and Scruples by—15.

2. We divide the Scruples by———60.

And we shall have what we seek, if we add the Quotient of Scruples to the Miles.

Example.

Degrees of the Equator———17° : 43'.

How many Miles do they make? Answer.

Draw 17 into 15, they make Miles———255.

And 43 multiplied by 15, make Scruples of Miles 645.

Which being divided by 60, the Quotient will be, Miles 10, and the residue 45'.

The Sum therefore of the Miles is———265 : 45'

Or $\frac{1}{4}$ of a Mile.

Shorter thus.

Draw the Degrees into 15, the Scruples divide by 4.

N O T E.

If when Degrees and Scruples of the Equator come to be Multiplied into Miles and Scruples of Miles, the Ratio is almost the same, as of Multiplying Degrees and Scruples into one another.

For

If we draw of } Degrees } into } Miles } They } Miles }
the Equator } Degrees } 1 Scrup. } produce } Prim } of miles
 } 1 Scrup. } 1 Scrup. } } Seco. }
The

The which, the Apexes being added, shew.

Example.

There are to be drawn ————— 5° : 17°
 Into Miles ————— 6 : 27.
 They produce Miles ————— 34° : 04° : $39''$.

PROBLEM II.

To find the Perimeter or Ambit of the Terrene Globe in Miles.

Since the Perimeter is 360° , say
 If Degrees. make Miles. what makes
 1. 15. 360° . *Ans.* 5400 miles.

PROBLEM III.

To find the Diameter and Semidiameter of the Terrene Globe in Miles.

As } To { So Miles } To the Diameter of the Earth
 $22\frac{7}{8}$ } { 5400 } { 1718 $\frac{2}{3}$.

Half of which is $859\frac{1}{4}$, the Semidiameter of it.
 But to shun the Fractions, for roundness }
 sake, we usually take for the Diameter 1720 } Miles.
 For the Semidiameter ————— 860 }

PROBLEM IV.

To find the Area of a great Circle in the Terrene Globe.

From the Circumference ————— 5400.
 Half the Circumference is ————— 2700.
 Drawn into the Semidiameter ————— 860.
 Produces the Area ————— 2322000.
 Of square German Miles.

PROBLEM V.

Given the distance of any lesser Circle from the Equator, or from a Parallel of the same Equator, to know the Semidiameter of it, and from the Semidiameter, the Circumference, in Degrees of which the Equator hath 360°.

For from the Equator towards each Pole, innumerable Parallels may be conceived, the distance of which from the Equator, makes known either the Latitude of the place, or the Altitude of the Pole, which agree in Quantity.

I. Therefore that we may know the Semidiameter of such a Parallel.

1. From the Quadrant of a Circle ——— 89 : 60.
Take away the given Parallel, for Example of — 48 : 34.
There remains ————— 41 : 26.

2. Seek the Sine of this remainer, it is — 66174.
Which is the Semidiameter of the Parallel sought, in parts of which the whole Sine is ————— 100000.

II. If we would know what Degrees of the Equator such a Parallel contains in its circumference.

Make it

As the whole Sine } To Degrees { So the Sine given } To Degrees
100000 } 360 { 66174 } 238° : 13'

For by drawing 66174 into 360, is made 23822640, and dividing by 100000, comes forth 238, and there remains 22640, which draw into 60, and again divide by 100000, and it gives Scruples 13, belonging to the Degrees.

Problem

PROBLEM VI.

To turn the Circumference of Degrees of such a Parallel into Miles.

I Way.

The Degrees and Scruples of the Circumference found by the foregoing Problem, multiply by, or draw into 15, and it produces the same Circumference in Miles: So $238^{\circ} : 15^{\circ}$ drawn into 15, produces Miles 3573 : 15'.

II Way.

Make it as the whole Sine } To Miles } So the Sine given } To Miles
100000. } 5400. } 66174 } 3573:23

PROBLEM VII.

To know how many Miles one Degree of such a Parallel makes.

As Degrees } To Miles } So Degrees } To Miles
360. } 3573:23 } 1. } 9:55.

For by dividing 3573 by 360, it produces in the Quotient 9, and 333 remains, which being Multiplied by 60, and again divided by 360, produces 55' or 56', belonging to the whole Miles.

From this Foundation is made the *Geographical Table*, which is annexed at the end of the Tables, shewing how many Miles agrees to one Degree in any one Parallel; which Parallels indeed are numbered only in whole Degrees: But if to any Parallel there should belong Scruples, the number of Miles nevertheless may be had from that Table answering to one Degree, by using the part Proportional; the way of finding which, I will shew by Example.

Example

Example.

In a Parallel Vertical to us ————— 48 : 34.
 How many Miles answer to one Degree?

Miles.

Answer, The Parallel 48th gives to one Degree 10 : 2;

The Parallel 49 gives ————— 9 : 50'

The Difference 1° or 60' gives ————— 0 : 12.

Say then.

If Minutes } Decrease } How much doth.

60. } 12. } 34.

The Operation being made, it produces 6' : 38" of Miles.

Miles

Which being taken away from 10 : 2

And so to one Degree in this Parallel, agrees Miles.

9 : 56'.

PROBLEM VIII

To change the Semidiameter of a Parallel into Miles.

As the whole Sine } To Miles } So the Semidiameter } To Miles.
 100000. } 860. } 66174. } 559, 77.

PROBLEM IX.

To conclude the Convex Superficie of the whole Terrene Globe in Miles.

The Area of the greatest Circle, is of Miles — 2322000

Draw them into ————— 4

And it produces the Convex Superficie, } 9288000
 of Miles —————

PROBLEM X.

To find the whole Grossness or Solidity of the Terrene Globe in Cubick Miles.

1. The

1. The Area of the greatest Circle ——— 2322000.
 Draw into the Diameter ——— 1720.
 And it makes ——— 3993840000.
2. This number divide by ——— 3.
 And the Quotient is ——— 1331280000.
3. The double of this, to wit, ——— 2662560000.
 is the Solidity of the whole Terrene Globe, in Cubick German Miles.

P R O B L E M X I.

To determine the Amplitude or Largeness of the Zones, in Miles.

1. Of the Torrid Zone.

We will assume for roundness sake, the distance of the Tropicks from the Equator, according to Kepler, which he hath appointed to be ——— $23^{\circ} : 30'$.

Therefore the Tropicks are distant }
 from one another ——— 47 : 00.

Which drawn into or multiplyed by — 15.

Make Miles ——— 705.

Which is the Latitude or Breadth of the Torrid Zone.

2. Of the Frigid Zone.

The distance of the Polar Circle from the Pole of the World, is the same as the distance of the Tropicks from the Equator, viz. $23^{\circ} : 30'$, which multiply by 15.

It makes German Miles ——— $352 \frac{1}{2}$.

That is, the Latitude or Breadth of one of the Frigid Zones, the subduple of the Torrid Zone.

3. Of the Temperate Zone.

1. From a Quadrant of a Circle ——— $89^{\circ} : 60'$.

Take away the distance of the Polar, }
 and Pole ——— 23 : 30.

There is left ——— 66 : 30.

2. Hence

2. Hence subtract the distance of the
 Tropick from the Equator— $\left. \begin{array}{l} 23^{\circ} : 30'. \\ \end{array} \right\}$
 There remains— $\text{---}43 : 00.$
 Which multiply by— $\text{---}15.$
 It makes the Latitude or Breadth
 of the Temperate Zone, Miles— $\left. \begin{array}{l} 645. \end{array} \right\}$

P R O B L E M XII.

To know, according to the Opinion of the Ancients, in what Climate any place is situate, from the quantity of the longest day being given.

- From the given Quantity of the day, for
 Example, of Hours— $\left. \begin{array}{l} 16. \\ \end{array} \right\}$
 Take away or subtract the Equinoctial
 day, or Hours— $\left. \begin{array}{l} 12. \\ \end{array} \right\}$
 The excess or difference, which is, Hours— $\text{---}04.$
 Double, and they make— $\text{---}08.$
 From this take away Unity, there is left— $\text{---}07.$
 Which is the Climate sought.

P R O B L E M XIII.

From a Climate given, to find the middle Parallel of it.

1. The Climate given, for Example, 7,
 double, and it makes— $\left. \begin{array}{l} 14. \\ \end{array} \right\}$
 2. To this add 3, and it makes— $\text{---}17.$
 Which is the middle Parallel of the Climate 7.

From which middle Parallel, the first and the third are always distant by Unity, and the Parallels of the Seventh Climate are 16. 17. 18.

P R O B L E M XIV.

In like manner, from a middle Parallel given, whose number is always odd, to find the Climate.

- From the middle Parallel, for Example— $\text{---}17^{\circ}.$
 Take away— $\text{---}03.$
 There remains— $\text{---}14.$
 Half the remainder is the Climate sought— $\text{---}07.$

P R O B L E M XV.

From any Parallel given of whatsoever Climate, to find the quantity of the longest day under that Parallel.

From

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From the Parallel, for Example 22° , take away Unity, 'tis 21° .
 This residue 21 divide by 4, and the Quotient will be— $5\frac{1}{4}$.
 To this Quotient add Hours— 12 .
 And you will produce Hours— $17\frac{1}{4}$.
 For the longest day under the Parallel of— 22° .

P R O B L E M XVI.

From the obliquity of the Ecliptick given, and from the certain quantity of the longest day, in whole Hours, half Hours, or Quarter Hours, (according as Climates are wont to be divided) to find the Altitude of the Pole, to which the length of the given day agrees.

1. Assume the obliquity of the Ecliptick— $23 : 31$.
 And the Tangent of it— 43516 .
2. Let the given longest day be, Hours— $16 : 30$.
 And the time of the Semidiurnal }
 Arch, Hours— $8 : 15$.
 And the half of the Equinoctial day, }
 Hours— $6 : 0$.
 Which subtracted, leaves the excess, }
 of Hours— $2 : 15$.
3. This excess changed into time of the }
 Equator, is— $33^{\circ} : 45'$.
 Which is the Ascensional difference of that day.
 The Sine of which is— 55557 .

Say then.

As the Tangent of $23^{\circ} : 31'$ } To the Sine } So the whole Sine
 43516 . } 55557 . } 100000 .

To the Tangent 127670 , the Arch of which $51^{\circ} 56'$, is the Altitude of the Pole of the place, in which the longest day is Hours $16 : 30$.

P R O B L E M XVII.

From the Altitude of the Pole given, at the beginning and end of the Climate, to find the Amplitude or Largeness of it.

For Example.

In the eighth Climate, the Altitude of the Pole is.
 Under

Under the third Parallel, at the end of it— $53^{\circ} : 18'$.

Under the first, in the beginning of the Climate— $30 : 25$.

The difference of which is— $02 : 53$.

Which makes Miles— $43 \frac{1}{2}$.

That is, the Amplitude or Largeness of the eighth Climate.

PROBLEM XVIII.

To find the Longitude of any place from the first Meridian, or of two places, from one another.

1. In the Terrestrial Globe.

Bring the given place under to the Meridian, and the point of Section of the Meridian and the Equator, will shew the Longitude of your place.

2. In Geographical Maps.

1. Which have the Equator, compute or reckon what Meridian passes through your place, which in the point in which it cuts the Equator, will also shew the Longitude of your place.

2. But if there be no Equator on the Map, lay a Ruler on your place, and withal, on two equal numbers, in the Columns from East to West, representing the Equator; which will shew the Longitude of the same.

But if you desire to know the Longitude of one place from another, or what Longitude two places have between themselves, take away the lesser from the greater, and the remainder will give what you seek.

3. From the Catalogue of Places.

For this end also are made Catalogues of the Longitude of places, by *Astronomers* and *Geographers*; in which either the Longitude of the places themselves are numbred, or the difference of time of the same; or from the first Meridian, or from some other certain place, from which difference of time, the Longitude of the place it self may also be had, by the following Problem.

Problem

P R O B L E M XIX.

From the Longitude of Places, to know the difference of time, or the contrary.

1. Multiply as well the Degrees as the Scruples of Longitude, by 4' Minutes of Hours, and you will have from the Scruples, second Minutes; from the Degrees, first Minutes of Hours; being changed by Sexagenary Division into the former Species or Kind, to wit, first Minutes and Hours.

For Example, $34^{\circ} : 37'$ of the Equator, how many Hours and Minutes do they make?

Answer, Multiply both Kinds or Species by 4, and you will have $136'$ and $148''$, that is, in dividing by 60; Hours $2 : 18' : 28''$.

2. Multiply as well the Hours as Minutes of Time, by 15° Degrees of the Equator, and you will have from the Hours, Degrees; from the Minutes of Hours, first Scruples of Degrees, by resolving them into Hours by Sexagenary Division. For Example.

Hours 7, Minutes 34, how many Degrees and Scruples are they of the Equator? Answer, from the Multiplication by 15° , they make Degrees of the Equator 105; first Scruples $510'$, that is, $113^{\circ} : 30'$.

And how that difference of Time, and consequently also the difference of Longitude, or of Meridians, may be had from Heaven it self, is shewn in the 36 Problem of Sphericals, at Page 474.

P R O B L E M XX.

To find the Latitude of any place from the Equator; or the difference of Latitude of two places.

1. In the Terrestrial Globe.

Bring your given place to the Meridian, and you will find the Latitude of it in the Meridian it self, numbring from the Equator.

2. In

2. In Geographical Tables.

Look what Parallel of the Equator passes through your place, which will shew the Latitude of your place, in Arches or Lines on the sides.

3. From Catalogues of Places.

For in these also the Latitudes of the most famous places, being noted, are found.

4. From Heaven.

The Latitude also of a place may be had, if you seek the altitude of the Pole, by the foregoing Astronomical Problems; for the Altitude of the Pole and the Latitude of the place, although they are divers, as hath been often said before, yet they wholly agree in quantity: and the difference of the Latitudes of two places is had, if you subtract the lesser from the greater.

P R O B L E M XIX.

To determine the Latitude or Breadth of any Country, either in Degrees of the Meridian, or Miles.

1. Take the two Parallels of the Equator, passing through the Extrems of the given Country.

2. Subtract the lesser from the greater, and you will have the Latitude of the Country in Degrees of the Meridian.

3. Multiply this by 15° , and it produces likewise the Latitude or Breadth of the Country in Miles.

For Example.

Germany is situate between the Parallels of the Equator 55° and 45° , the difference of which is 10° , which being Multiplied by 15, produces the Latitude or Breadth of Germany to be 150 German Miles, or 600 English Miles.

P R O B L E M XX.

To find the Longitude or Length of any Country.

1. Assume the two Meridians between which the Country lies, and subtract the lesser number from the

the greater, that you may have the Longitude or Length of the Country in Degrees.

2. Inquire also the Parallel passing through the middle of the Country, which you may have, if you add the two Extreame, and take half of the Sum.

3. In this middle Parallel seek how many Miles answer to one Degree, by Problems the 5th. 6th. and 7th. of this II. Part, at Page 600 and 601 and 602.

4. Multiply this number of Miles by the Degrees of Longitude found, and you will produce the Longitude or Length of the Country in Miles.

For Example.

1. Germany is situate between the Meridians 46 and 23, the difference of which is—23.

2. The Parallel passing through the middle of Germany is 50° , in which there is of Germany Miles 9 and 38' answering to one Degree, as you may see in the Table at the end of the Book, at Page 783.

3. Draw then these Miles into—23°.
And it produces Miles—221 : 34'.
The Length of Germany.

PROBLEM XXIII.

To find out the Journeying distance of two places in Miles, on the Globe or Geographical Chart.

1. Extend the Compasses from one place to the other, and take the distance of the same.

2. Apply this to the Equator, or to that which represents it, being depicted on the Chart, and Multiply the Degrees of this distance by 15.

3. Or if the Chart have a Scale of Miles ascribed

to it, apply the same distance to this Scale, which will make known the distance in Miles.

PROBLEM XXIV.

From the given Longitudes and Latitudes of Places, to find the Itinerary or Journeying distance in Miles, by calculation.

Divers Cases occur or happen about this Problem, as well General as Special; of the General there are three.

I. General Case is.

Of the places of them which are under the same Meridian.

I. If each place be under one Quadrant of the Meridian.

1. Take away the lesser Latitude from the greater.

2. Draw the remainder into 15, and you will have the distance in Miles.

Example.

The Latitude of Uranburg is ——— 55° : 55°

But of Rome ——— 42 : 02.

The difference ——— 13 : 53.

Which Multiplied by ——— 15.

Produces the distance of Miles ——— 208 $\frac{1}{2}$.

2. If the two places are under divers Quadrants of the Meridian, and yet in one half of it.

1. Add both the Latitudes.

2. Multiply the Sum by 15.

Example.

Riga, the chief City of Livonia, is North Latitude ——— 56° : 49°

The Cape of Good Hope, is South Latitude ——— 30 : 49°

The

The Sum of both is ————— $87 : 30$
 Being Multiplied by 15, gives Miles — $1312 \frac{1}{2}$.
 The distance of Riga and the Cape of Good Hope.

3. If they be in divers halves of the Meridian, and also in divers Quadrants, but joining together.

1. Take the Complements of the Latitudes of both.

2. Join them together in one Sum.

3. And multiply it by 15.

Example.

Berga in Norway is North Latitude — $60^{\circ} : 30'$

The Island of St. Peter in America, North Latitude $12 : 00$.

The Complement of the former is — $29 : 30$.

The Complement of the latter is — $78 : 00$.

The Sum of the Complements is — $107 : 30$.

Being drawn into 15, they make Miles $1612 \frac{1}{2}$.

4. If they are in divers halves, and divers Quadrants of the Meridian, and not contiguous.

1. Subtract the lesser Latitude from the greater.

2. The remainder subtract from a semicircle or 180° .

3. And multiply the residue by 15.

Example.

The Latitude of Rome is Northern — $42^{\circ} : 2'$

The Island Tuberon in America, its Latitude is Southern — $16 : 0$.

The difference — $26 : 2$.

Being taken from a Semicircle, leaves — $153 : 58$

Which multiplied by 15, makes Miles — $2399 : \frac{1}{2}$

That is, the distance of the two places given.

R 1 2

II General

II General Case is.

Of the places of them which are under the same Parallel.

1. If both are under the Equator, and have the difference of Longitude less than a Semicircle, subtract the lesser Longitude from the greater.

The remainder being less than a Semicircle, multiply by 15.

Example.

Andrelles hath Longitude ————— 128.

Amaræ, a Mountain of Ethiopia ————— 56.

The difference ————— 72.

Being Multiplied by 15, makes Miles — 1080.

The distance of the two places given.

2. If both are under the Equator, and they have the difference of Longitude greater than a Semicircle.

Subtract the lesser Longitude from the greater.

The residue being greater than a Semicircle, take it away from 360° .

Multiply this last remainder by 15.

Example.

The Lake of Parina, its Longitude is ————— 310.

The Island of St. Thomas, its Longitude is — 30.

The difference ————— 280.

Being taken from a Circle or 360° , leaves — 80.

Which multiplied by 15, makes Miles — 1200.

The true distance of the two places, because it cannot be that any place can be farther distant from the other than half the Ambit of the Terrene Globe.

3. If two places are without the Equator, under the same Parallel, and so agree in Latitude, and only

ly differ in Longitude; there are many ways of finding the distance of these.

I Way.

1. Subtract the lesser Longitude from the greater, that you may have the difference of them.

(Note, it is also to be Subtracted from a Circle, if it increase beyond a Semicircle.)

2. Inquire also the Latitude, or Parallel under which both places are conversant.

3. Seek how many Miles answer to one Degree in this Parallel, at Page 783.

4. Multiply the difference of Longitude into these Miles.

The number produced, will make known the distance of the places.

Example.

The Longitude of Thyatira of Jonia is— $58^{\circ} : 55'$.

Of Larissa of Macedonia, the Longitude is $50 : 35$.

The difference is ———— $8 : 20$.

The common Latitude serving both is— $40 : 0$.

Under which there answers to one Degree, Miles— $11 : 29$.

This drawn into the difference ———— $08 : 20$.

It makes the distance of Miles— $95 : 41$.

II Way shewn by the same Example.

The common Latitude— $40^{\circ} : 0$.

The Complement of it— $50 : 0$. Its Sine 76604.

The Longitude of Thyatira— $58 : 55$.

The Longitude of Larissa— $50 : 35$.

The difference— $8 : 20$.

The half is ———— $4 : 10$. Its Sine 7266.

Say then

As the whole Sine } To Sine compl. of Latit. } So the Sine of the half.
109000. } 76604. } 7266.

To the Sine of half the distance ———— 5556.

R r 3

11707

Whose Arch is ————— $3^{\circ} : 11'$.
 The double of it is ————— $6 : 22$.
 Which drawn into ————— 15 .
 Makes Miles ————— $95 : 30'$.

The true distance of these places, from a double Arch, not numbring in a Parallel, but in a great Circle.

III Way by Prosthaphæresis.

The Longit. of Thyatira $58^{\circ} : 55'$.
 Of Larissa ————— $50 : 35$.
 The difference ————— $08 : 20$.
 Half of it ————— $04 : 10$. the same $4^{\circ} : 10$.
 Its complement ————— $85 : 50$.
 The Latit. of the compl. $40 : 00$. the same $40 : 0$.
 The Sum of the two ————— $125 : 50$. the Sum $44 : 10$. Its Sine 69675
 The Excess * ————— $35 : 50$ ————— Its Sine 58543
 Subtract ————— The difference ————— 11132
 Half of it ————— 5566

Of this half, the Arch is ————— $3^{\circ} : 11'$.
 The double is ————— $6 : 22$.
 Multiplied by ————— 15 .
 Produces the distance of Miles ————— $95 : 30$.

Note at the Sign *.

1. If this Sum viz. $125^{\circ} : 50'$ should be less than a Quadrant, instead of the Excess above a Quadrant, we must take the complement of it to a Quadrant; and instead of the difference, the Sum or Aggregate of these Sines, and with the half of this Aggregate seek the Arch, the double of which multiplied by 15, shews the distance.

2. If the difference of Longitude be a Quadrant, to which in the Equator answers 1350 Miles, the shortest

shortest way is, if you assume the Sine of the Complement of the common Latitude, and say, if 100000 give 90° , or 1350, what doth the Sine of the Complement of the common Latitude give? The Operation being made, you will produce either Degrees, to be resolved into Miles, by drawing them into 15, or the Miles themselves of the distances of the places sought.

3. The same Method may also be observed in other places, differing only in Longitude, if you resolve the difference of Longitude into Miles and Scruples of Miles; as in the former Example, the Complement of the Latitude is 76604: The difference of Longitude is $8^\circ : 20'$ which under the Equator, makes in Miles 125, and of Scruples of Miles 7500', (which are produced if you multiply $8^\circ : 20'$ by 15, and again draw them into whole Miles by 60, and add the produced Scruples of Miles) Then if you say, if 100000 give 7500', Scruples of Miles, what doth 76604 give? you will produce Scruples of Miles 5745', that is, Miles 95, Scruples 45'.

III General Case is

Of the places of them which are situate under divers Parallels and Meridians, and so also they differ in Longitude and Latitude.

1. If one place be situate under the Equator, and the other without it, and the difference of Longitude be exactly 90° , draw 15 into 90, all the Calculation is performed, and you will produce 1350 Miles, by which such two places are distant between themselves.

2. If one place be under the Equator, and the other without the Equator, and the difference of Longitude less than a Quadrant.

The Process is thus.

| | | | | |
|--------------------------------|-------------------|---|------------------------|--------------------|
| <i>The Longitude of Essena</i> | $70^{\circ} : 3'$ | } | <i>Latit. of Tyrus</i> | $33^{\circ} : 20'$ |
| <i>The Longitude of Tyrus</i> | $67 : 0$ | | <i>Its Complement</i> | $56 : 40$ |
| <i>The difference</i> | $03 : 3$ | | <i>Its Sine</i> | 83548 |
| <i>The Complement</i> | $86 : 57$ | | | |
| <i>Its Sine</i> | 99858 | | | |

Say then

| | | |
|------------------|---|------------------------------|
| <i>As Radius</i> | <i>To Sine Compl. of diff. of Longit.</i> | <i>So Sine Compl. Latit.</i> |
| 100000 | 99858 | 83548 |

To 83429 , the Arch of which is $56^{\circ} : 32'$.
 And the Complement of it $33 : 28$.
 Which drawn into 15 .
 Shews the distance of Miles 502 .

Or

| | | | | |
|------------------------------|-------------------|---|-----------------|------------------|
| <i>The Longit. of Essena</i> | $70^{\circ} : 3'$ | } | <i>The same</i> | $3^{\circ} : 3'$ |
| <i>The Longit. of Tyrus</i> | $67 : 0$ | | <i>Compl.</i> | $56 : 40$ |
| <i>The difference</i> | $03 : 3$ | | <i>The Sum</i> | $59 : 43$ |
| <i>The Latit. of Tyrus</i> | $33 : 20$ | | <i>Its Sine</i> | 86354 |
| <i>The Complement</i> | $53 : 37$ | | <i>Its Sine</i> | 80507 |

The Sum 166861 .
 Half of it is 83430 .

Whose Arch is $56^{\circ} : 33'$.
 Its Complement $33 : 27$.
 Drawn into 15 .
 Makes the distance of Miles $501 : 35'$.

3. If

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3. If one place be under the Equator, and the other without the same, and the difference of Longitude be greater than a Quadrant.

Proceed thus.

The Long. of Calipolis is $164^{\circ} : 20'$.
 The Long. of Tyrus is $67 : 00$. Its Latitude $33^{\circ} : 20'$.
 The difference is $97 : 20$. Compl. Lat. $56 : 40$. Its sine 83548 .
 The Excess $07 : 20$. Its Sine 12764 .

Say then

As Radius } To Sine Compl. Latit. } So Sine of excess of differ. of Lon-
 100000 } 83548 } 12764 .

To 10664 , the Arch of which is $6 : 7$.
 To which a Quadrant being added, makes $96 : 7$.
 Which multiplied by 15 , makes Miles $1441 : 45$.

Or:

The Long. of Calipolis is $164 : 20$.
 The Long. of Tyrus $67 : 00$.
 The difference $97 : 20$.
 Compl. to a Semicircle } $82 : 40$. } Compl. of this $7^{\circ} : 20'$.
 Latitude of Tyrus $33 : 20$. } The same $33 : 20$.
 The Sum $116 : 00$. The Sum $40 : 40$. Its Sine 65166 .
 The excess of it above } $26 : 00$. } Its Sine 43837 .
 a Quadrant $26 : 00$.
 Subtract, and the differ. is 21329 .
 Half of it is 10664 .

The Arch of this half is $6 : 7$.
 To which a Quadrant being added 'tis $96 : 7$.
 That is Miles of distance $1441 : 45$, as before

4. If

4. If both places be situate without the Equator, towards the same Coast, and the difference of Longitude be less than a Quadrant.

The way of computing distances of this kind I will shew by Example, which the Studious in Geography may imitate.

The Longitude of Tubing is $33^{\circ} : 40'$.
 The Longit. of Antwerp is $28^{\circ} : 40'$.
 The difference is $5 : 00$. The Radius 100000.
 Its Complement $85 : 00$. Its Sine 99619 . Subtract
 The Versed sine of it 381 .

The Lat. of Antwerp is $51^{\circ} : 12'$. Its Compl. $38^{\circ} : 48'$.
 Latit. of Tubing is $48 : 34$. Its Compl. $41 : 26$ the Sum $80 : 14$.
 The difference $2 : 38$.
 Its Complement is $87 : 22$. Its sine 99894 .
 The Compl. of the sum is $9 : 45$. Its sine 16964 . Subtract
 The difference is 82930 .
 Half of it is 41465 .

Which being multiplied by the Versed Sine of Longitude 381 .
 And divided by the Radius, makes 158 .
 Then from the Sine Compl. of the Latitude 99894 .
 Subtract this Quotient 158 .
 And you will have 99736 .
 The Arch of which is $85^{\circ} : 50'$.
 And its Complement is $4 : 10$.
 Which drawn into 15, makes the distance of Miles $62 \frac{1}{2}$.

5. If both places be situate towards the same Coast, without the Equator, and the difference of Longitude be a Quadrant.

The Procefs is thus.

The Longit. of Daroacana is $118^{\circ} : 45'$.
 The Longitude of Spira is $28^{\circ} : 45'$.
 The difference is $90 : 00$.
 The Latitude of Spira is $49 : 20$. Its Compl. $40^{\circ} : 40'$. } The Sum
 The Latit. of Daroacana is $34 : 45$. Its Compl. $55 : 15$. } $95^{\circ} : 55'$
 The difference is $14 : 35$.
 Its Complement is $75 : 25$. Its sine is 96778.
 The excess of the sum above 90 is $5 : 55$. Its sine is 10308. subtract it

The difference is 86470 .
 Half of it is 43235 . tis
 The Sine of the Arch $25^{\circ} : 37'$.
 The Compl. of which $64 : 23$.
 Drawn into 15, makes Miles $965 : 45'$ or $\frac{1}{2}$.

6. If both places be without the Equator, towards the same Coast, having the difference of Longitude greater than a Quadrant.

In this Case proceed thus.

The Longitude of Carticardamna is $136^{\circ} : 50'$.
 The Longitude of Heidelberg is $30 : 45$.
 The difference is $106 : 05$. the radius 100000
 The excess of the diff. above a Quadr. is $16 : 05$. Its Sine 27704 add
 The Versed Sine 127704 .
 The Latit. of Heidelberg is $49 : 35$. Its compl. $40 : 25$. } The Sum
 Of Carticardamna is $12 : 40$. Its compl. $77 : 20$. } $117^{\circ} : 45'$

The

The difference is ———— $36 : 55$.
 Its Complement is ———— $53 : 05$. Its Sine is 79951.
 The excess of the Sum is ———— $27 : 45$. Its Sine is 46361. add

The Sum is ———— 126512.

Half of it ———— 63256.

Draw into the Versed Sine of the Latit. 127704.

And being divided by the Radius, it makes 80780.

Whence subtract the sine Compl. of the Lat. 79951.

It makes ———— 829.

The Sine of the Arch ———— $0^{\circ} : 28' : 30''$.

To which add a Quadrant — 90.

It makes the distance ———— $90 : 28 : 30$.

Which drawn into 15, makes Miles 1357.

7. If both places be without the Equator, but one situated towards the North, the other towards the South, and the difference of Longitude greater than a Quadrant, proceed thus: as the Example of *Bassa*, in the Island of *Taprobana*, and of *Heidelberg*, taken from *Mestlin*,

The Longitude of *Bassa* is $126 : 0$.

Of *Heidelberg* is ———— $29 : 15$. according to *Mestlin*.

The difference is ———— $96 : 45$. the Radius 100000.

The Excess is ———— $6 : 45$. Its sine ———— 11754. subtract

It makes the Versed sine of the diff. of Longitude 88246.

The Lat. of *Heidelberg* is $49 : 22$. Its Compl. north $40 : 38$ The sum

Of *Bassa* ———— $06 : 30$. Its compl. south $83 : 30$ $124^{\circ} : 8'$

The difference is ———— $42 : 52$.

Its Complement is ———— $47 : 08$. Its sine ———— 73294.

The excess of the sum is ———— $34 : 08$. Its sine ———— 56112. add

The Aggregate is ———— 129406.

Half of it is ———— 64703.

Which

Which drawn into the Versed Sine of the
Longitude ————— } 88246.

And divided by the Radius, makes ————— 57097.

Which subtracted from the Sine of the
Complements of Latitude ————— } 73294.

Leaves ————— 16197.

The Arch of which is ————— $9^{\circ} : 19'$

Add a Quadrant ————— $90 : 00$.

They make ————— $99 : 19$.

Which being drawn into 15, } 1489 : $45'$ or $\frac{1}{2}$.
makes the distance of Miles —

8. But if the difference of Longitude in this case, be less than a Quadrant, we must take the Complement of the difference of Longitude, and add the Sine of it to the Radius, that we may have the Versed Sine of the Longitude.

Example.

The Longit. of Bassa is $126 : 0$.

The Longit. of Stockholm $42 : 38$.

The difference is ————— $83 : 22$. The Radius is ————— 100000.

Its complement is ————— $06 : 38$. Its sine is ————— 11552. add

The Versed sine ————— 111552.

The Latit. of Stockholm $60 : 30$. Its compl. is $29^{\circ} : 30'$ The sum

Of Bassa is ————— $06 : 30$. Its compl. is $83 : 30$ } $113^{\circ} : 0'$

The difference is ————— $54 : 00$.

Its Complement is ————— $36 : 00$. Its sine is ————— 58779.

The excess of the sum ————— $23 : 00$. Its sine is ————— 39023.

The Aggregate ————— 97802.

Half of it is ————— 48901.

Which multiplied by the Versed Sine, and divided
by Radius, makes 54550.

Which

Which being subtracted from the sine Complement of Latitude,
Leaves—4229.

Which is the Arch of— $2^{\circ} : 25'$.

Add a Quadrant—90 : 00.

And it makes—92 : 25.

Which multiplied by 15, produces the distance of
Miles 1386.

9. If it should happen in this case, that the Sine of the Complement of Latitude be so small, that the Quotient arising, from the multiplying the half into the Versed Sine of the Longitude, and dividing the Product by the Radius, it cannot be taken away from the Sine of the Complement of Latitude; the contrary must be done, and the Sine of the Complement of the Latitude must be taken from the Quotient, and a Quadrant not added to the remaining Arch, but we must take the Complement of it to a Quadrant, and convert it into Miles.

Example.

The Long. of the Island Thyle is $33^{\circ} : 0'$.

Of the Island of St. Thomas— $27^{\circ} : 20'$.

The difference is ———— 5 : 40. The Radius is 100000.

Its Complement is ———— 84 : 20. Its Sine 99511. add

And it makes the Versed sine 199511.

The Latitude of Thyle is— $63^{\circ} : 0'$. Its compl. $27^{\circ} : 0'$. The sum

The Latit. of St. Thomas is— $16^{\circ} : 0'$. Its compl. $74^{\circ} : 0'$. 101.

The difference is ———— 47 : 0.

Its Complement ———— 43 : 0. Its sine 68200.

The excess of the sum ———— 11 : 0. Its sine 19081. add

The Aggregate—87281.

Half of it is—43640.

Which drawn into the Versed Sine of Lon- }
gitude, and divided by the Radius, makes— } 87066.

From this take away the Sine of the Com- }
plement of the difference of Latitude— } 68200.

There remains ———— 18866.

The Arch of which is $10^{\circ} : 52'$.

And its Complement—79 : 08

Which

Which Multiplied by 15, makes the distance of Miles 1187.

10. If the difference of Longitude be chiefly 90° , the thing is done briefly, if you take half of the difference between the Sine of the Complement of difference of Latitude, and the excess of the Sum, and to this add an arch of a Quadrant.

Example.

The Longitude of Bassa is— $126^\circ : 0'$.
 Of Camarina in Italy is— $36^\circ : 0'$.
 The difference is— $90^\circ : 0'$.
 The Latitude of Camarina— $43^\circ : 0'$. Its Compl. $47^\circ : 0'$. } The sum
 The Latitude of Bassa is— $06^\circ : 30'$. Its Compl. $83^\circ : 30'$ } $130^\circ 30'$
 The difference is— $36^\circ : 30'$.
 Its Complement is— $53^\circ : 30'$. Its sine is 80386.
 The excess of the sum— $40^\circ : 30'$. Its sine—64945. subtract
 The difference—15441.
 Half of it is—7720.
 To the Arch of which— $4^\circ : 26'$.
 Add a Quadrant— $90^\circ : 00'$.
 And they make— $94^\circ : 26'$.
 Which multiplied by 15, makes the distance of Miles 1416 : $30'$.

Another way.

Of inquiring the distances of Places, from the Longitude and Latitude of differences.

Some in this computation of places, use the 47th. Proposition of the 1. Book of Euclid, which Ratio, although 'tis less exquisite, especially when two places are far distant, because they misuse the Arches of Parallels for right Lines; notwithstanding we would not altogether neglect it,

And

And 'tis thus.

1. Take the difference, as well of the Longitudes, as of the Latitudes of the places.
2. Multiply the difference of Latitude by 15, that you may have it in Miles.
3. Half of the same difference add to the lesser Latitude, that you may have the mean Latitude or Parallel.
4. See how many Miles answer to one Degree of this mean Parallel, at Page 783.
5. Multiply this into the difference of Longitude.
6. Take the Square of the Product, by resolving it first, into first Minutes.
7. To this Square, add also the Square of the difference of Latitude, turned into Miles, in like manner by resolving them into Minutes.
8. From the Sum of these two Squares, extract the Square Root, which will be the distance of the places being reduced again to Miles, in dividing by 60.

Example.

The Longit. of Tubing $33^{\circ} : 40'$. } The Latit. of Antwerp $51^{\circ} : 12'$
 Of Antwerp is ———— $28 : 40$. } Of Tubing the Latit. $48 : 34$.
 The difference is ———— $05 : 00$. } The difference is ———— $02 : 38$.

Which makes Miles $39 : 30'$, or 2340'.

Half the difference of Latitude ———— $01 : 19$.

Being added to the lesser Latitude ———— $48 : 34$.

Exhibits the mean Parallel ———— $49 : 53$.

In which, one Degree contains Miles $9 : 39$.

Which drawn into the diff. of Long. ———— $5 : 00$.

Makes Miles ———— $48 : 15$.

Or ———— 2895'.

The Square of 2895 is ———— 8381025.

The Square of 2340 is ———— 5475600.

The

The Sum of the Squares ————— 13856625
 The Square Root of which is ————— 3722
 Being divided by ————— 60
 Produces miles ————— 62 : 2
 The distance of the given places.

NOTES.

Concerning two places, one of which is of North Latitude, the other of South.

1. In this case the difference of Latitude is not taken by Subtracting the lesser from the greater, but by the addition of both Latitudes.

2. If the Latitude of either place hath the same or equal quantity, either of them may be taken for the mean Parallel, with which the miles are to be inquired agreeing to one degree of the Parallel : But if these Latitudes are unequal, then join the half of the greater Latitude to the lesser, and it will produce the mean Parallel, of which the miles are to be inquired agreeing to one Degree.

PROBLEM XXV.

The Longitude and Latitude of Places being known, to ascribe the same into Geographical Tables.

1. Assume the Chart at Pleasure ; whose Right-hand part you have for the East Coast, the Left-hand part for the West, the upper part for the North, and the lower for the South.

2. Divide the Extrems of the Chart or Paper into certain degrees many or few (according to the Longitude and Latitude of the Country which you desire to describe), observing this, that if you desire to make the Ordination of Places distinct and exact, you must allow so much space to each degree, that at least every tenth Minute also may be noted.

SS

Apply

3. Apply a Thread, or draw an occult Line by a Rule, to the Degrees and Minutes of Longitude known in the upper and lower Part, numbring from the West into the East.

4. In the same manner also number the Latitude from the South into the North, and sign it also by a Thread or an occult Line; and where the Threads or Lines cut each other mutually at right Angles, there ascribe your Place; act in the same manner also with others.

P R O B L E M XXVI.

To inquire the Sites and Distances of Places seen from one another, and to describe the whole Country seen, on a Chart or Paper.

1. You must prepare an Orb, and in it a Circle divided according to the 4 Coasts of the World into 4 Quadrants, and each into 90 Degrees, and so the whole Circle will be divided into 360 Degrees, and it must be furnish'd with an Index with sights to look through.

2. With this ascend some Tower or any high Place, from which many Places may be seen lying round about, and direct your Orb to the Coast of the World by help of the Compass or magnetic Needle, to the place from which you desire the Prospect, and begin to Number from the next Coast.

And for Example, let the first place of our Station be Herrenberg a City of Wirtemberg, in the high Arch of which I look towards the South.

3. Then by the Orb remaining unmoved in this Site, I direct the Index with the Sights to the place next to the Line of East, and note the Distance of it from the East; or the Angle of Vision, which also I ascribe into the Chart prepared for the purpose, being shewn by a Circle described on my Chart, from the center of which, I draw a Line through
the

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the Degree of Distance or of the Angle of Vision of the Place observed : And let this first place observed by us be the Tower of *Rosbeck*, distant from the East towards the South by the Angle of Vision 43° by a gross Calculation which suffices instead of an Example.

4. But from hence I direct the Index with the Sights in order, to the Second, Third and Fourth, Fifth, &c. Place, and I observe all the Angles of Vision, and in the same manner I design them in my Chart by drawing a right Line of Vision from the Center.

The Places are these in our Example, Fig. 204.

| | | | | | | |
|----------------------|---------------------------------|---|-------------------|--------------------------|---|------------------------------|
| <i>Wurmling</i> | } Distant from the Line of East | { | $56^{\circ} : 0'$ | } From the Meridian Line | { | $2^{\circ} : 0'$
$20 : 0$ |
| <i>Old Rotemburg</i> | | | $76 : 0$ | | | |
| <i>Thailfing</i> | | | $92 : 0$ | | | |
| <i>Bondorf</i> | | | $110 : 0$ | | | |

5. This being done I dispose my self to some other place now to be observed, and there I direct my Orb or Instrument, so to the Coasts or parts of the World, that the Degrees of this place observed in the Circle, directly respect the former place of Station, and be opposit to it Diametrically ; and let the place of this second Station to us be the Tower of *Rosbeck*.

6. Then from this second place, I inquire again, anew, the Angles of Vision of all the aforesaid Places, and also of others which I can attain to by Sight.

These Places in our Example are.

Wurmling distant from the Line of East 90° , and therefore directly beholding the South.

| | | | | | | |
|------------------------|-----------------------------|---|-------------------|-----------------------|---|-------------------------------|
| <i>Old Rottenburg.</i> | } Distant from the Meridian | { | $26^{\circ} : 0'$ | } From the I. of West | { | $10^{\circ} : 0'$
$36 : 0$ |
| <i>Bondorf.</i> | | | $80 : 0$ | | | |
| <i>Thailfing.</i> | | | $100 : 0$ | | | |
| <i>Herrenberg.</i> | | | $126 : 0$ | | | |

SS 2

7. Which

7. Which places being thus observed from my second Place of Station, I reassume my Chart, or Draught, and in it I seek the Line of Vision deduced or drawn from the first into this second Station; in which I assume the point continued, distant from the point of the former Place as much as you please; And by how much remoter the Point is assumed, the Chart to be described will be the greater, and the measures will be made the more exact.

8. That accepted Point will represent the second place of Station, from which I describe another Circle, and in it I number the Angles of Vision observed in the second Station, and again I draw from the Center, the Lines of Vision of the afore-said Places as before, *See Fig. 204. being a Scheme of the description of the Places, in which.*

The great Letters denote the Sites of the Places: But the small Letters the Degrees of the Instrument, through which the visive Lines pass.

| | |
|------------------|---------------------|
| H h. Herrenberg. | A a. Old Rotenburg. |
| R r. Roseck. | B b. Bondorf. |
| W u. Wurmling. | T t. Thailfing. |

9. But if you please to extend the Chart further, and to design some whole Country in it, dispose your self to a third Station, and direct your Instrument to the second Station, in the same manner as before you did it in the Second with respect to the First.

10. And the Towns and Cities being described in course, you may easily also insert, Rivers, Woods and Mountains, according as they are further from or nearer to these Places.

11. Lastly, if after you have described the Chart, you are willing to measure the distances of all the Places described in it from one another, you must

A TABLE of the

| | | | | | | | | |
|----------------------------------|---|-------------------------------------|---|--|---|---|---|--|
| I.
Islands,
to wit, | { | Of Great, 9 | { | Brittannia.
Japonia.
Islandia.
Sicilia.
Hibernia.
Hispaniola.
Cuba. | { | Canadensis.
Sumatra.
Madagascar.
Java major.
Celebes.
Creta.
Luconia. | { | Borneo.
Novazembla.
California.
Sardinia.
Frisslandia.
Newland about new F.
Mindano, Ceilanum. |
| | | Of Mean, 13 | | | | | | |
| | { | Of Small, 9 | { | Gilolo,
Corfica,
Seeland of Denmark, and
The most famous, being solitary or alone, are F.
Maderas, &c. | { | Amboina,
Majorica,
Ciprus, from Negropo
Jamaica, &c. in the G.
The most famous, being solitary or alone, are F.
Maderas, &c. | { | Timor, they lye betw
Ciprus, from Negropo
Jamaica, &c. in the G.
The most famous, being solitary or alone, are F.
Maderas, &c. |
| | | Of Least, of which we consider | { | A Pile, viz. those which in a great number lye in the Sea, very near together, as the | { | Canaries,
Flemmish,
Hesperides,
The Gulf of Mexico
Maldiva, | { | Africa it self,
Peleponesus, to wit
Chersonesus Tauric
Cambaja. |
| II.
Peninsula's,
which are | { | Of Oblongs, of which there are many | { | Chersonesus aurea, to wit, Malacca, adjoining to
Cimbrica, to wit, Jutlandia, adjoining to Hol
Covea, adjoining to Tartary.
The North and South parts of America. | | | | |
| | | Adjoining Peninsula's are | { | Italy, Greece, Achaia, in particular
Hispania, Asia the lesser, Arabia,
Norway, with Sweden and Lappia, | { | Beach, the
The Kingdo
New Britan
Monomotap | | |
| | | | | | | | | |
| III.
Isthmus,
to wit, | { | | { | Between Egypt and Arabia, viz. between Africa
Of Corinth, between Peleponesus and Achaia.
Of Pannamensis, viz. belonging to America, the l
Between utlandia and Hollatia.
Between Malacca and India. | | | | |
| | | | | | | | | |
| IV.
Continents
to wit, | { | The Old Orb, whose parts are | { | Europe.
Asia.
Africa. | { | The bounds of these Continents are. | { | On the North, th
On the East, the
On the South, th
On the West, th |
| | | The new Orb America | { | Northr.
Southr. | { | The Bounds of this are | { | On the North, th
On the East, the
On the South, the
On the West, the |
| | | | | | | | | |

The Land of the North Polar, viz. Greenland, compass
The Land of the South Polar, viz. Magellanica, unknow

the Earth.

t new France.
anum.

ve between the Indian Islands.

Negropont, they lye into the Mediterranean Sea.

n the Gulf of Mexicane.

e, are Rhodes, Malta, Lemnos, St. Hellens, St. Thomas, The
ras, &c.

| | |
|---|---|
| About Madagafcar.
Molucca and Bandana.
Philippina.
Japponia. | The Islands of the
Robbers.
Of the Aegean Sea.
About Britania.
The Islands of Salomon,
&c. |
|---|---|

f,
to wit, the Morea, Chersonefus of Greece,
Taurica, viz. Tartaria Precopenfis,

oining to India.
to Holfatia.

| |
|---|
| Jucatan.
Chersonefus of Thracia.
New France.
Jonia, Cnidensis.
Myndensis. |
|---|

ch, the Country of Magellanica, and new Guinea,
Kingdom of Indoftan, Conchinchina,
w Britania in America,
nomotapa, Camboja, &c.

Africa and Afia.
haja.

ra, the longest of all.

allest on the Land.

orth, the North Sea, the Frozen and the Tartarian Sea.

ast, the Pacifick and the Indian Sea.

uth, the South Sea.

est, the Atlantick Sea.

orth, the Strais of Davis.

st, the Atlantick Sea.

th, the Magellanick Sea.

est, the Pacifick Sea.

ompassed about every where by the Sea and Straits.

unknown or not found.

the Ocean, and called the Sea. { The Atlantick or North Sea, which divides— } The New And Old } Orb, and
 { The Pacifick Sea, or Mare Delzur, between the } in some places beholding the
 { Northern or } Far Northern. }
 { Southern, viz. The Magellanick, presenting

Gulfs, which are either { oblong } 1. The Mediterranean Sea, viz. the Gulf of Africa and the Countries of Europe to the lesser Asia, whose Parts
 { six of which are } 2. The Baltick Sea, viz. the Gulf of Codan, whence the Secondar Gulfs.
 { } 3. The Arabian Gulf, between Africa
 { } 4. The Persian Gulf, between Arabia
 { } 5. The Californian Gulf, between California
 { } 6. The Corcensian Gulf, between Corcoran of Tartaria and China.

Which is

or { 1. The Mexican Gulf, between Mexico
 { broad, } 2. The Gangetick Gulf, viz. the Gulf of the Shoars of Idostan and
 { which } 3. The Gulf between Malacca
 { are } 4. The White Sea, on the North
 { six, as } 5. The Sea Lantchidol, between the Magellanick Land.
 { } 6. The Sea of Hudson, between Hudson arising from the North Ocean.

The narrow Seas or Straits, to wit, of

{ Magellanica, which comes from the narrow Seas or Straits.
 { Le Maire, nigh to Magellanica, a narrow Strait, about new Zembla, the Strait of Anian, which tends from the Strait of Davis, of Forbischer, in which, The Straits of Gadirana, in which, The Straits of the Arabian Sea, The Straits of the Persian Sea, in the Hellespont and Bosphorus, in which, Danicum or Orescundicum, through

The Caspian Sea is like to a Lake, but that it is Salt to be ascribed to the sides of the Gulfs of the Ocean.

T A B L E of the Sea.

Orb, and about this } The Ethiopick, } Ocean.
 } The Gullick, }
 } The Brittanick, }
 between the extreame of Asia and the India Islands, and looks upon the West of America.
 the place also nigh the Tartarian Ocean,
 adding the North Land.
 presenting it self about the Magellanick Land; part of which is called the Indian Sea.

viz. the inner, between } The Iberick or Spanish Sea, the Ligurn, the Siculum,
 of Europe, running down } The Creticum, the Aegean Sea, &c.
 e Parts are, } The Secondary } The Adriatick, viz. the Gulf of
 } Gulfs } Venice.
 } } The Black Sea, Propontis, or Mard
 } } Marmora.

e Gulf of } Livonicus, } depending } Corinthiacus, } And many
 secondary } Botnicus, } upon } Saronicus, } other Gulfs.
 } Finnicus, } } Argolicus, }
 } } Termaicus, }

een Africa and Arabia. }
 en Arabia and Persia. }
 between California and new Granada. } To be looked
 between Corea and the utmost Shoars } upon or
 } beheld.

between America North and South. }
 viz. the Gulf of Bengal, between }
 a and Malacca. }
 Malacca and Camboia. } may be
 the North Ocean, between Lappia } beheld.
 Moscovia. }
 between Beach and new Guinea, of }
 between new Francia and Canada, }
 Ocean. }

from the Atlantick or Aethiopian Sea into the Pacifick, it is the longest of all

anica, and the same use.

ubla, through which it passes from the North into the Tartarian Ocean.

the Tartatian to the Pacifick Sea, although at this time 'tis called in doubt.

which, is away from the Atlantick into the Tartarian or Pacifick Sea.

in which they pass from the Atlantick Ocean into the Mediterranean Sea.

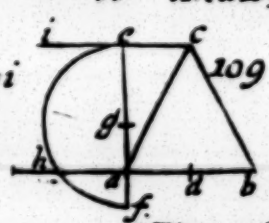
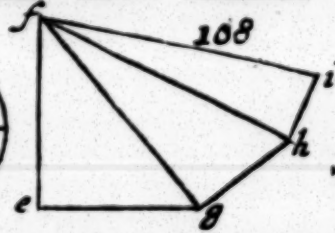
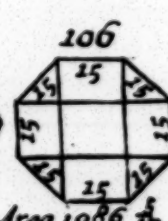
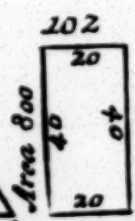
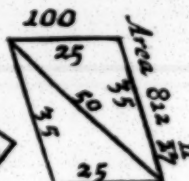
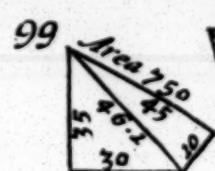
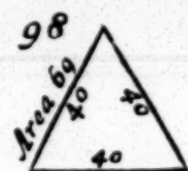
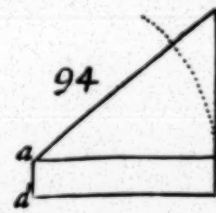
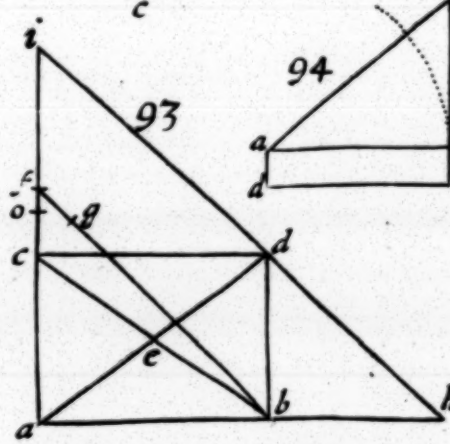
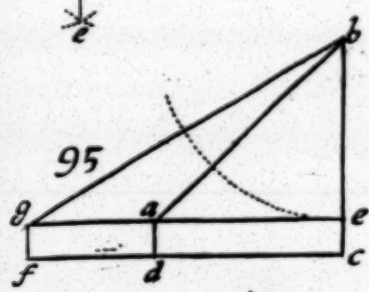
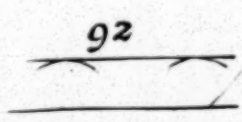
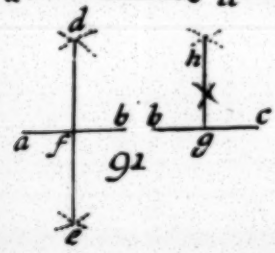
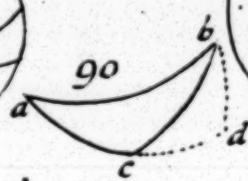
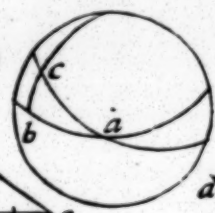
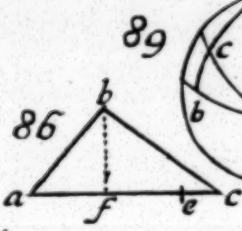
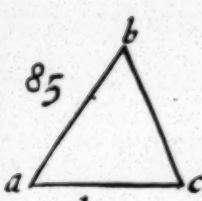
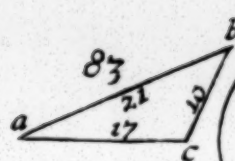
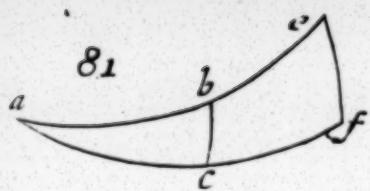
in Sea, in which they pass into the Arabian Gulf.

Sea, in which they pass into the Persian Gulf.

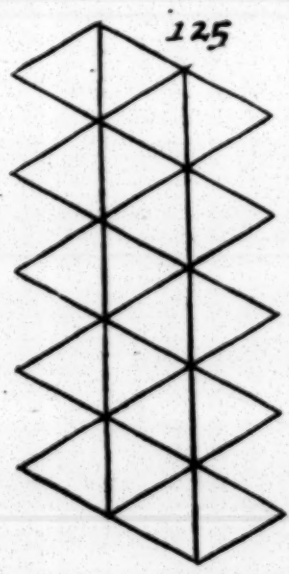
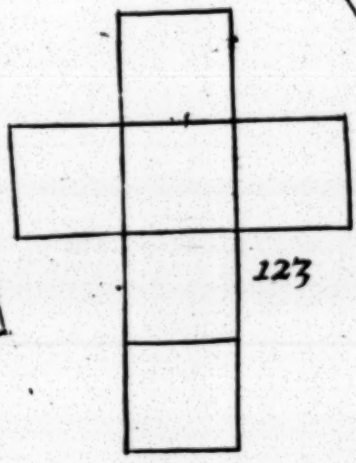
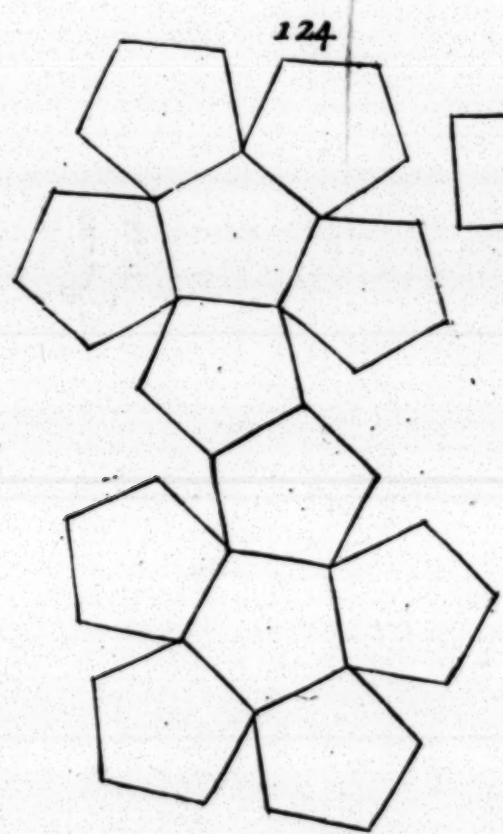
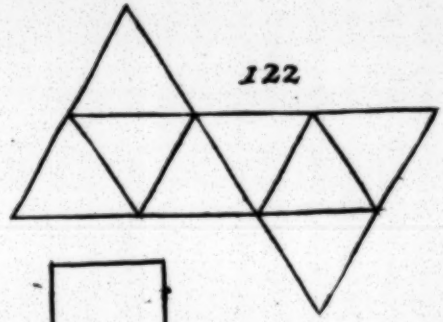
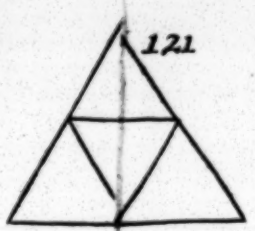
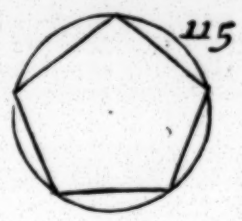
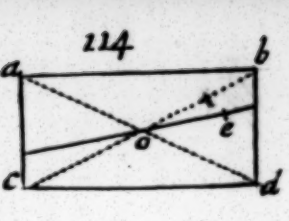
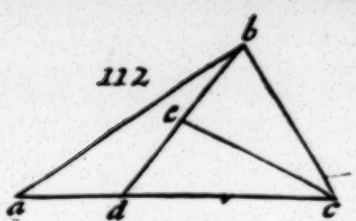
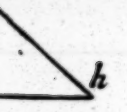
in which they pass from the Aegean into the Black Sea.

n, through which they pass into the Baltick Sea, between Scandia and Zealand.

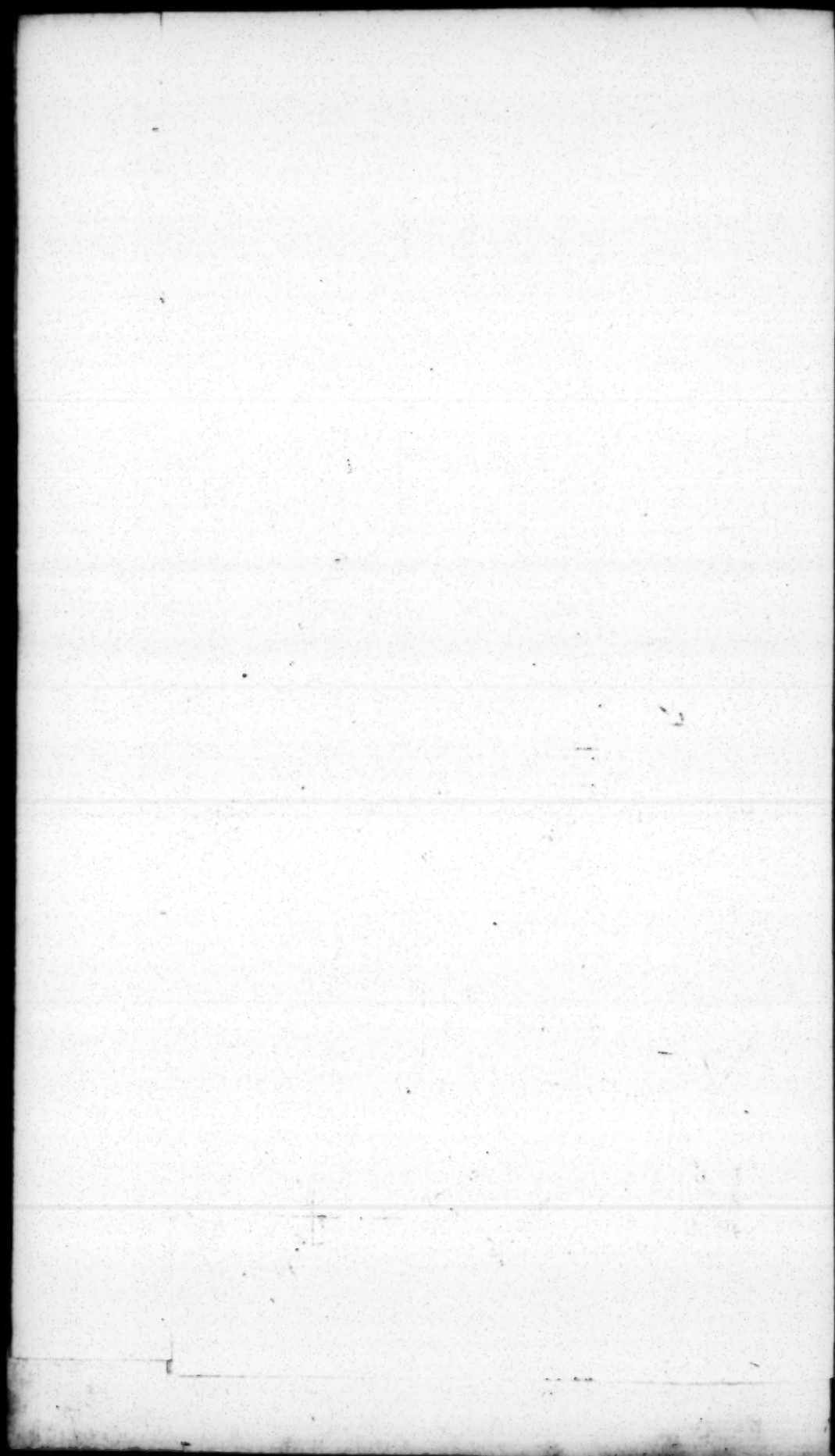
it is Salt, and concerning it, whether it be peculiar, or whether or no it ought
 the Ocean, with which it joins by a Subterranean way, is as yet a doubt.



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must inquire the Itinerary distance of two places in Miles, or which is more safe, in hours.

Then take the Line of distance of the same places, which they have in your Chart, and divide that Line into so many parts as the Distance of places contains true Miles: So you will have a Scale of Miles, by the help whereof you may measure all the places inscribed on your Chart or Map.

The End of GEOGRAPHY.

AN EPILOGUE of OPTICS.

The PREFACE.

1. **O**ptics, which they also call the Instrument of the Sight, and Perspective, is a Science which considers the differences, the properties, the affections, and accidents of Visible Species, or of things perceivable by the Eyes, and the Nature of Vision it self, and detects and corrects the Errors, which are often wont to arise from Vision or Seeing.

2. 'Tis usual to constitute three Parts of it.

1. The Optics, so called in Particular, which acts concerning the simple Vision of Species, running in directly through one Medium or mean into the Eyes.

2. Catoptrics, concerning the Vision of outward Forms or Representations reflected and beaten back by other things.

3. Mesoptics or Dioptrics, concerning the Vision of outward Forms or Species refracted in divers Mediums or Means.

Part I. of OPTICS.

CHAP. I.

Concerning the Eye, the Instrument of Vision.

1. **T**he proper Organ and Instrument of Vision or Seeing, is the Eye: As being the chiefest thing with

the rest of living Creatures, so first of all to the use of Men; by a wonderful Artifice made by the Divine Wisdom, to the end that we may see, that is, that we may understand and perceive the Species and Figures of things at a Distance.

And tis a matter of very great concern, that Men and other living Creatures, know also those things which are far remote or at a distance; not only that they may prosecute the Good, and fly timely from the Evil; but chiefly, that Men in Contemplating this whole Machine of the World, may know the Invisible God, from the Visible Things, and Praise and Magnifie him as the most Wise, the most Excellent, the great Creator and Governor of all things.

And as the other Senses without the External and Corporal Instrument, percieve nothing; so also without the Eye a living Creature sees not: And as the Sight is the most excellent Sense of all, leading us to the knowledge of many things, which the other Senses can never reach to: So the Structure of the Eye is most Artificial and Admirable, which from the Physical Science, or natural Philosophy, is briefly to be repeated.

2. *The Parts of the Eye are threefold, the Skins, the Humors, the Sinews or Nerves.*

Understand this of the essential and chief Parts; for there are many of the secondary and accidental, having also notwithstanding their excellent Use; concerning which we shall say somewhat in that which follows.

3. *Any Eye hath three Coats or Skins: The Horny; the first that covers the Optic Nerves, and the Tunicle like a Net.*

This in like manner is to be understood, concerning the principal Tunicles common to the whole Eye; besides which, there are others also proper to certain Parts; so that some are wont to number eight Tunicles in all: But Tunicles in this place are called Covers, which encompass each other mutually, and involve and constitute the Eye in the whole or the greatest part of it.

4. The

4. The Horny, is that outer part which encompasses or covers all the remaining part of the Eye.

The outer, to wit, with respect to the other principal parts: For also from the Pericranium is begat some very thin and pellucid Membranes, said to be related on a kin, which also covers this horny Tunicle; and retains it as it were by a band in its place: And, tis called Horny, because its foremost Part which sticks out somewhat, is like a shining polish'd Horn, although the hinder part without the White, is inwardly dark, thick, opacous, and nigh to gristly.

5. The Optic Nerve Tunicle, is that which is withdrawn of the Horny, and also encompasses the Eye, but is not whole as the Horny, but is perforated by a hole, which is called the *Apple or Sight of the Eye*.

And this is distinguished into two Parts; the first and outer is most black, drawing to a brownish Colour, like to the huske of a Grape Stone, of which the Stalk is slipped of: But the hinder part is more Azure or Blewish, and is called *Chorion*: And that hole which we call the *Sight of the Eye*, in Darknes and in Night, is almost three times wider than in the Light, and encompasses the wide Circle, shining through the Horny, which is, called *Iris*, and is of divers Colours, Dark, Brown, Fallow or Yellowish, &c. according to the variety of Temperament, and also according to the changeable Age of a Man.

6. The Tunicle, in fashion of a Net, is the inmost, and is resembling the Substance of the Brain, yet more Snively and Reddish, arising from the *Serings* or Nerves dilating or enlarging themselves in the hollow Hemisphere of the Eye.

The Nets are like to the round end in a Sack (whence tis called *Retina*, *Reticularis*, that is like a Net)

Net) or more like to a Tunnel or Funnel; to this on the forepart round about, sticks or adheres that which is called the process, belonging to the Eyelid, arising from the Optic or Sinewy Tunicle, and distinguished like the Teeth of a Combe; and from thence they receive their Name, because they are made of Hairs from the Optic Sinewy Tunicle, in the manner of the Eyelids; proceeding as it were from casting forth bright Beams, and are like to the Hairs of the Eyelids.

7. The Humors of the Eye are three; distinguished by their Seat or Place on every side, to wit, Watery, Christalline, and Glassey.

8. The Watery possesses the foremost Sea of the Eye, between the Horny Tunicle and the Christalline Humor.

Tis most thin, shining, fluid like Water, whence also it receives its Name; yet clammy as the white of an Egg: And strangled in the middle, returning into the neck from the Optic Nerve, which flows within and without the hole of the sight of the Eye.

9. The Christalline Humor, is in the middle between the Watery and the Glassey Humors, folded in by a pellucid Skin or Tunicle, which therefore is called Chrystalline, also Arachmoides or the Web of a Spider, by reason of its thinness.

For it contains the Chrystalline Humor in the middle of the Process of the Eyelids, like the hanging up of a Spiders Web: And the Chrystalline Humor is transparent and shineth like to a Glass made of Christal; nor yet is it hard, nor flewid, but like to Wax yet somewhat Firmer.

10. The Glassy Humor resides in the hinder part of the Eye, wrapped up also by a most thin and shining little Skin called Hyaloides Tunica.

Tis shining, alike as Christal, but softer, yet fixed and like to melted Glass, in which the Chrystalline

line

line Humor resides in its latter seat, as a precious stone or gemm in a Ring.

11. Of the Nerves or Strings of the Eyes, *some are Movers or Stirrers, others are Instruments of Sight or Optics.*

Those motions of the Eyes, do Service in a six-fold manner, by lifting up, by depressing, by two being transverse or crosswise, and all of them being circular.

And the Optics or Instruments of Sight are one to each Eye, which notwithstanding meet or convene both in one, which the Brain sustains, and is the vehicle Spirit or life of Sight.

And these things are to be foreknown concerning the structure of the Eye in Optics, which the most wise Maker or Builder hath placed on high, as the Windows of the Mind or Understanding, that the Sight may extend to places further remote, as from high Mirrors or looking Glasses: And he includes them in hollow places to secure them, and for the sake of a more expedite motion hath made them Spherical, and in movable Orbs: Briefly, he hath likewise directed all things which seem very little, to most excellent Uses; concerning which some thing shall be said in the Procedure.

And in this place also we must note, that

12. The Eyes of Men are either *Myopes, or Presbytas, or Oydetesoi.*

1. *Myopes, are they, which see distinctly near at Hand, but more confused afar of; as he that acts a sedentary Life, applies himself to Learning, or other subtil Artifices, and accustoms his Eyes to things near at hand, but to things remote he sees but little.*

2. *Presbytas, are they, which see remote things distinctly, and things near at hand confusedly, as old Men for the most part; whence is derived the Etymology: Or because*

because by a Child, the Eyes are accustomed more to remote things, than to things nigh at hand.

Lastly, Oydetesoi are those which see neither one nor the other, but all things confusedly, as being dim-sighted; or they behold both distinctly, as those which are endued with sound healthful Eyes.

Heber also are referred Nictalpes, which like Cats see better by Night than in the Day-time: See the Scheme of the Eye at Fig. 205. where.

A A, is the protuberance or swelling out of the Horny Tunicle.

K K, is the opacous latter part of it.

C C, is the foremost part of the Optic Sinewy Tunicle.

N N, the latter or hinder part.

B, the hole, the sight of it, with the ambient Iris, or Circle with divers Colours, which is between the white, and the apple or sight of the Eye.

H, The netty Tunicle, to which they adjoin or cleave on the forepart.

H H, The Process belonging to the Eye lids.

L L, The growing Tunicle.

E E, The Chryselline Humor, included by that like a Cobweb.

F, The Glassy Humor, covered by the glassy green Colour **M M**.

D D, The watery Humor.

G, The Optic Nerve.

CHAP. II.

Concerning the Object seen, and the Kindes of Visible Things.

T He object seen, is observed or Visible: which is so, either by Act, or also by Power.

For

For whatsoever seems or appears, that likewise may be seen : But not every thing which may be seen, does also appear : And there are many visible Things, which not only incur to the Eyes of one or another, nor to the Eyes of all Men, nor perchance will ever incur in this Life.

2. Tis from visible Power or Possibility, that altho^{ugh} by reason of Impediments coming between, tis not seen, nevertheless it may be seen if the Impediments are removed.

So in darkness many things are in possibility Visible, which nevertheless by reason of Darkness intervening are not seen ; but they may be seen ; and by light arriving they are actually seen.

3. And Visible Action is, that which grants or supposes to all things the necessary requisits to Seeing, being perceived by the Eyes themselves.

There are many such requisits which are required either to plainly Seeing, or to See well ; of which one or another being wanting, the Vision or Seeing, becomes either none at all, or confused and erroneous.

And for this end the Optic discipline is invented, which explains all these requisits, and detects and corrects the Errors of the one and the other Springing up.

4. Moreover some visible things are Visible by themselves, others by accidents.

5. A thing is Visible by it self, which is bereft by Sense, that is perceived only by Sight, excluding the help of other Sences.

6. And tis either Primary, which is seen by it self at first Sight, also in darkness, as all lucid or shining Things : Or secondarily, which in Darkness is not seen, but only is made by clear light ; as Colours.

These Visible things may be considered two-fold ; 1. Absolutely, even as they are in themselves and

and in their own Nature; 2. with relation to the Eye, to wit, how they have each thing in themselves by the agreeing Operation of it: This latter consideration is Optics, the former is Physics or natural Philosophy; notwithstanding some things also are to be assayed, before we proceed in Optics with an inoffensive Foot.

7. Visible by Accident, (*which also are visible by another thing, and are called common Visible things*) are they which offer not themselves to sight at the first and by themselves: But either are Visible by being Privative alone, or absent by themselves; or not firm alone, but they are commonly perceived by the help also of other Senses, or by reasoning.

Brightness, Darknes and Shadows are beheld Privative, for these are perceived by the absence of Light, but that by the absence of Colour.

But those things which are perceived also by another Sense, and by the help of Reasoning are many; which yet may seem to be reduced to the following Kinds: As they are,

1. Magnitude, under which is contained Continuity or whole length, and Separation; Smoothness and Roughness: Thickness and Thinness.

2. Place, under which is contained various Distances and Sites; to the Right-hand, to the Left-hand: Upwards, downwards; before, behind.

3. Number, under which, Identity or Sameness, and Diversity.

4. Figure, under which is contained Fairness and Deformity; Likeness and Unlikeness.

5. Motion, under which is contained the opposite to rest.

CHAP. III.

Concerning the Primary Object of Sight, viz. Light, and bright Things.

1. **T** Herefore the Primary visible Objects of the Eyes, are Lucid things, not Colours, as are commonly supposed. For

For Colours are not rendered Visible but by Light and in the Light; Lucid or Shining things are seen of themselves, also in Darkness, which is from the reason of the Object.

2. Lucid things, are denominated from Light, which is either Natural, or arises from the first.

3. Original Light, viz. Created by God the first Day of the World, is not an Accident but a Substance.

For 1. There was not any thing Created with it, but it was a Primary Creature; 2. The fountain and original of all Light in the World; 3. The containiner of Quantity: To which God Attributes the Praise of Goodness before other things.

4. Nor yet is it a Substance of it self made of any Matter, but without Matter, or Immaterialized.

For Materiated, is that which is made from matter, or drawn out from the Power of it, but Light is not Educued from matter, but is Introduced into it; not existing as Partaking of it, but that it might render to it a Partnership or Partaking: For Matter in the beginning was drowned in Darkness, *Genesis* the 1. Therefore Light could not be stretched out of it self: Indeed God commanded the Light to Shine out of Darkness as in the 2 of *Corinthians* and the 4, but this is declared concerning Place, not concerning the matter of Light; read also the 2 of *Cor.* and the 6, where he saith, *and what Communion hath Light with Darkness.*

Lastly, every Materiated thing is compounded of many Parts, and divided into the same by real Separation, and is subject to many other Mutations and Changes, which cannot be said of Light.

5. Light, is a Substance, endued with three essential Faculties, to wit, with the Virtues of Enlighning, of Heating, and of Moving.

1. The Vertue of Enlighning, is without doubt, that which dispels Darkness, detects obicure things, renders or yields visible Colours; Exhilerates and makes glad the Creatures, and performs
innumerable

innumerable other things, which God dispenses to his Creatures by Light, as it were by a Channel.

2. *The Vertue of Heating*, Experience also shews; for Light hath always Heat conjoined as an unseparable Companion; as we find this in many other things but chiefly in the Sun, which as it is the fountain and original of Light, so it is also of Heat: And as Cold follows after Darknes, so Heat follows after Light; Thus the Earth is most Dark and most Cold.

3. Not less evident also is the *Power of moving*, in Light, and is adjoined to it by Heat: And where Heat is, there also Motion follows; and Gross and cold things, as Water Frozen, the Earth, and Stones, want Motion of themselves: But from Fire which looses Vapors into Smoak, or being hastned and incited in another manner by Heat, they assume Motion, and communicate Motion to things, in which they are inclosed between: That now shall I not say, that the Sun, in which Light and Heat are found in the greatest Degree, and moves by a most swift Motion, is understood also to turn about other movables.

Therefore *Light* is the original Substance, constituted by three distinct Faculties, yet making one Essence, viz. furnished with the, *Power of Illuminating, of Heating, of Moving*; from which all Light, Heat, and Motion which is in the World, after *God*, is produced in the first place, and by it self.

And although according to the number of these three Faculties, it ought to be called by three Names, yet commonly tis only called Light, from the faculty of Enlightning, as the most potent, and more known and manifest; the consideration of which alone belongs to this Science of Optics.

6. *The Nature of Light, is of it self Spiritual and consisting in a point, penetrating most solid Bodies, if they are Pellucid.*

For

For every Immateriated thing is Spiritual, and the least or smallest Spark of Light which may be seen, makes glad by its Light, not less than a burning Mountain or the most vast Body of the Sun: Since therefore it hath no determinate and definite Quantity, it must needs be, that the Essence and Nature of it consists in a point.

7. *The most simple Light considered without all Matter, is Invisible: But being Immersed with any matter, and incorporated in it, and so not any more Light, simply existing, but is made a Lucid Body, and is seen by it self, and Illustrating or Enlightning other things, causes them also to be seen.*

Therefore here the ambiguity of the word *Light* is well to be noted: For tis taken either for simple Light, considered absolutely in it self, as when *God said let there be Light, Genesis the 1.* Or for Light existing in some certain matter; in which manner it is not any longer simple Light, but becomes a Lucid Body: Although therefore simple Light without matter is no longer in the World, nor perchance also never hath been; (for *Moses* testified that not before, but after matter, and in matter Light is enclosed) but only abstracted from Matter by Cogitation as other forms, although they continue always in Matter, yet by the Mind they are wont to be abstracted from it: Yet notwithstanding by the common speaking, they are called also *Light*, all those *Lucid Bodies* which are endued plentifully with Light, they have the Power of enlightning others: Such are the Sun, the Stars, and all Fires under the Heavens: And in this signification also we hereafter shall use the word *Light*: Which Light in matter, at length becomes *Visible*, since in its simplicity it cannot be comprehended by Eyes: For without matter, Light is a Point, which
in

indeed (unless as often as the extream of quantity is considered) is not only so: And because tis not a material point of no force in it self and dead; but being formal, is most efficacious and lively, diffusive of it self, capable of Quantity and Magnitude (even as we see the least spark may be increased into so great a Magnitude that it will enlighten a large Space) therefore from this diffusion of it self and from the assuming of quantity, tis also rendered Visible. Which cannot be, unless it can Communicate it self into fit Matter, in which to dilate it self; least that by dilating it self into nothing, it should undertake a disappointed worke, which is contrary to Nature.

And that quantity of Light is not *Material*, which is made by the increase of many Points, to a certain distance continued or joined together; but *Formal*, being made by the diffusion of it self into Matter.

8. *The immersed Matter of Light agrees to any point by Radiation, that is by the Efflux or Ejaculation of it into a distant Place, and if nothing hinder, it radiates Spherically and orbicularly; that is into every opposit Point to which a Right-line may be drawn.*

For the Office of Light, is to communicate it self to other things, and to Illustrate them, that they may be seen; which since it cannot be done by the accession of the fountain it self to things: It remaineth that it may be done by the local enterance and efflux or flowing from its original Point: Which point is central, diffusing or spreading it self into the orb, which that it may enlighten all round, as because a point is every where alike, having no cause why it should spread it self into one Coast or Part more than into another, tis necessary likewise that this Efflux or flowing should agree.

9. *And*

9. *And to this Efflux a middle space is required, through which the passage of Light is given to distant things.*

For this Efflux is of Local Motion, which necessarily requires space or place in which it should be made: And since this space cannot be emptiness, which nature doth not admit, therefore it must needs be a Body, and not opacous or shadowy, but perspicuous, or pellucid, and passable to Light.

10. *For Light only passeth manifestly through perspicuous or Pellucid Bodies; but through Opacous, either not at all, or insensibly.*

Pellucid from the reason of the Name, is that which some Light shines through, which also is called Perspicuous, as often as any thing is beheld through it: Those Bodies are such which are,
 1. Chiefly One, that is being made distinctly, and not uneven by any variety of Superficies; but constituted by a most equal course of all the inner Parts, terminating mutually in themselves: 2. Composed by no Thickness, or by an indifferent one. 3. Moreover Tinged with none or a very little Colour: For, Colours do not altogether hinder Shineing through, but then only, when either by Reason of the gross plenty of the Colours they exceed the mean, or by some Impediment intervening, they introduce obscurity: Especially when they are very much Black and very much White, they are not passable to Light, and therefore neither Pellucid: And Pellucids differ by degrees: The Sky holds the first Place among all, and clear Water the second; Crystal succeeds these, next Glass, then Ice, Alom, Horn, and some Gems: *Pellucid* is opposite to *Opacous*, which Light passes not through; and *Opacous* is uneven by many Superficies, and obtains much of Thickness and of Colours: Although perchance of themselves no bodies are so Opacous, which
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by reason of some thinness, permit not some passage to Light, which nevertheless by reason of the Weakness of our Eyes is not perceived.

11. *Every Point of Light flows from many Lines.*

For the Spherical, from which this flowing of Light frames the enlightning all the Orb round about, hath many Lines extending from its Center to the Superficies.

And hence arises another Distinction which some make a dispute, between *Light* as the Fountain, and *Light* as it were a stream; for they call that Light, which is in a Lucid Body, as in the Sun: but that a light Body, which is produced from Light and flows every where: Nevertheless this distinction is not perpetual, for very often these two Words are confounded by some writers, and one used for the other.

12. *Light being apt to proceed very far from it self, is bounded by the Terms either of deficient matter, or not convenient to receive the Light.*

For the nature of Light consists in a Point, from which flow innumerable others to the constituting the greatness; which greatness as it consists in a Point, so also it ends in a Point: And since all Points are of the same Nature, therefore the Powers are the same of the Last as of the First, neither ever doth this flux stay, as long as receiving matter is at hand; without which, Light, as it acts not in vain, so never intermits its Operation between those things that are proper.

13. *The motion of Light is not made in time, but in a moment.*

Experience proves this, for if some Light be hid against, and the covering suddenly taken away, it doth not enter by little and little, but in a moment; also it enlightens all the remote Places, to which the Power of it extends: For since Light of it self wants matter, neither hath it any thing which re-

sists it; nor weight, by which the powers or forces of it are measured.

14. *Light in it's Efflux proceeds by right Lines, which are called Rays or Beams.*

For it affects the Spherical Figure, whose generation consists in equality of Intervals, through which Point in the middle, 'tis divided in the Superficies by right Lines.

15. *And it acquires by this, right Motion, or by the right-line Efflux, two Dimensions, viz. Length and Breadth.*

For from all the Points of Light, flowing from right Line Motion, as well in Length as in Breadth, a Superficie consisting of Length and Breadth must needs arise from thence.

16. *A Diminishing to Light in it's departing from the Center, happens in the Breadth but not in the Length.*

For the Length of Light depends on a continued casting forth of Points of Light; of all which, since the Nature and Substance is the same, and the Momentary strong Caster forth the same; the same mixture of all the Rays or Beams between themselves, as well in the beginning or Origine, as in the distance removed from the same, no-diminishing hath place; which is otherwise considered in Light, from the Ratio of Latitude, where the Rays are joyned at the Center, and they mixed by it wider between themselves, by how much they are remoter from the Original.

17. *Light falling on the more Thick Superficies of an Opacous Body, is reflected or beaten back, into the opposite part from whence it came.*

For the Motion of Light, is Action, consisting between contraries; which contraries in this Place are *Perfussion* or *Smiting*; because Light carries the Superficies towards one part, and *Repercussion* or *beating back*, into the other part.

But hence two kinds of Light arise, of which the chief is, the Efflux it self of Light from it's Fountain; as if the Sun should shine through the Window into the Chamber; but the other and second Light, is the *Repercussed* or *Reflected*, which is procured by Reflection from that first or Chief; such as is in the remains of darkned Angles of Houses much differing in Degrees, because one may procure the other, since Repercussion agrees to each.

18. *Light falling on the Superficie of a Thicker or Thinner mean, than that is which it enters first, is refracted or diminished; so that one right-lined Ray departs into two Contiguous ones.*

For because Motion is proper to light, this property of Motion also happens to it, which is the Impediment from the thicker Superficie; and this is the Third kind of Light which from the Optics is called *Refracted*.

19. *Light either passing Coloured things or near to them, and recoiling again, is imbrued and incompassed by the same.*

Hence Colours enlightned and repercussed, shine as Light it self, and this is seen in Light passing through Windows, which conveys the Colours of the Window to the Objected Bodies: And passing by or over green Meadows, it seems green, from the Light being imbrued by the green.

CHAP IV.

Of Colours.

1. *Colours are not reflected by the Power of Primary Objects, but of Secondary Objects of sight.*

For Light is seen by it self, although in darkness, which is from the Reason of the Object; but Colours

Colours are not seen in darkness, although they also exist in darkness, but are only enlightned by Light: Therefore although Colours are also visible by themselves, in respect of them which are seen by accident, or by another only; yet they have not the Reason of the Primary Object, but of the Secondary only, as often as they are allied to Light: Therefore as of Light, so also of Colours, the Reason and Nature ought to be understood aforehand in some Measure.

2. *The Principles of Colours are Light and Darknes.*

For in these are the Radixes of all Colours, or if we may speak Vulgarly, the chief Colours are Lightish and Darkish; which notwithstanding is not real Colour, but only the Privation of Light in matter; Whence those things which are said positively concerning Darknes, are to be understood of matter, not as it is matter, but as being deprived from Light. 10

And that from Darknes and Light Colours are begot, the Colours of the Rain-bow may be an argument; Likewise of Clouds, of Crystals, of Windows which are enlightned by the Sun; which Colours are begot, from the mixture of Light and Darkish matter: So also Ruddy or Red Colour is Compounded from Lightish and Black, as appears in Prunes, in which from a mixtion from Fire and Black Coals redness is begot; the flame of green Wood makes a Light Red or Yellowish; because the brightness of the Fire is mixt with Black Smoak.

Also the Sun, although being White and shining, being beheld thro' Mists or Smoak, it appears of a Light Red or Yellowish Colour.

Therefore since all Colours are from the same kind, why should we not appoint the same Origine to be of all, to wit, from the Commixture of Light and Darknes, which may be infinite, and by no Man fully to be comprehended. T t 3 3. There-

3. *Therefore all Colours are nothing else than divers commixtures arising from engrafted Light, and Matter of Darknes.*

The Causes and Essence of them are so controverted and obscure, that *Scaliger* says, How is it manifest that they are Colours? which is true, if you understand it concerning the degrees of Mixture of every one; the variety of which seems to be above humane understanding; as also from the degrees of difference of all other Mixtures: but if we be stained by no preconceived false Opinion, but love the truth, we may easily arrive at, though not the perfect, yet some knowledge of Colours, to wit, if we begin from the first Colours of Light and Darknes (for these are not Coloured) and look into the Nature of them accurately; From which it will easily appear that all the rest are no other than divers mixtures of Light, and of Matter belonging to Darknes, and of a Degree lying between Infinite extrems.

4. *Therefore the Extrems of all Colours are Light and Darknes.*

For these as they are beginnings, so are they also extrems; and as it were Limits or bounds, which no Colour may transcend.

5. *But of the Beginnings of these Colours, the Extrems are Whiteness and Blackness; of which, that cometh nearest to Light, this to Darknes.*

Therefore *Whiteness* is a Colour, for the most part of Light, having little of Darknes; on the contrary, *Blackness*, having much of Darknes; and little of Light: *Light*, is living Whiteness; *Whiteness* is as it were dead Light, which pours not forth it self immediately: White Colour receives all other Colours; and therefore, in like manner as light, by it's power is all Colours: But Black receives no other: for tis only a Privation of Colours, and whatsoever it hath of Positive,

Positive, it hath all that not of it self but from Matter.

6. *The rise of other middle Colours between Whiteness and Blackness; those have least Light, which most depart from Whiteness; those have most of Darknes, which come nighest to Blackness.*

The middle or mean between *Whiteness* and *Blackness* seems *Red* or *Ruddy*, mixt of equal Portions of Light and of Darknes: From hence if you would ascend to *Whiteness*, this seems to be the order; *Red*, *Green*, a *Bright Yellow*, *White*: If from the same *Red*, you would descend to *Blackness*; this order may seem to be constituted; *Red*, *Blue*, *Brown*, *Black*.

And between every one also may be constituted innumerable differences, according to the degrees of mixture of them: Therefore that commixture of Colours from Light and Darknes is certain, although the way of mixture, and the degrees of every one, is past finding out by Human Weakness; Which also comes in use in other mixtures; For who shall tell me how many varieties of mixtures may be made from one Vessel of Wine and of Water, if all the degrees ought to be observed? in which things, some Image of unmeasurableness lies hid.

7. *Lastly, tis to be noted, that some Colours are true, others apparent.*

The true, are those which arise from the proper and ingrafted Light, and remain always in it's Subject, even in Darknes: and *Apparent*, are those which are begot only from accidental Light, which departing again, they vanish: such are the Colours which are beheld in Looking-Glasses, Windows enlightned by the Light of the Sun, in the Rainbow, in Clouds, &c. which are not inseparable accidents, as the Colours in Cloath; If so be they

appear not all in the same Place : Whither also, by some are referred the Wings and Plumes of Feathers of Peacocks, of Doves, and of other Birds ; But these seem more to be true and inherent Colours, and to have only this peculiar, that each Superficie is of Divers Colours : Whence it comes to pass that they appear sometimes of a Golden, sometimes of a Green, sometimes of a Blue Colour : That which Art also imitates often in the Weaving of Silken Garments.

C H A P. V.

Concerning Visible things by accident.

1. **B**etween many visible things by accident, which are beheld privately, there deserves a peculiar Consideration, viz. of Pellucidity, or Shining, of Darkness, of Shadow.

For the others rehears'd before, which are known by the help of other Senses, or by the Benefit of Reasoning ; as *Magnitude, Place, Number, Figure, Motion*, and those things which are comprehended under these, are known, not only from Mathematical and Physical Discipline, but also by daily use and experience to all.

2. *Pellucidity or Brightness, is the quality of Bodies which are conspicuous either from no Colour, or from another, more than from the proper Colour.*

Therefore all brightness is of the Sky, which indeed hath not the least of Colour which can be perceived : The Argument of which thing is, that it transmits even to us, clearly and undefiled, the smallest of the Stars, with the difference of Colours, which would be tinged from the Colour of the Sky if there were any: Therefore if it were lawful by Physics, the whole space of the Sky may be accounted for Vacuum

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um or Emptiness: Next to this clearness of the Sky, is the clearness of the Air, which also of it self hath no Colour, altho' being illuminated cross-wise by the Sun-beams, and beheld a great way, it acquires a Blewish Colour, commonly call'd Sky Colour, and is thought to begin in Heaven: When nevertheless this Colour is inherent to the Air, which appears from this, because very remote mountainous Places, by the most pure Air Shining through them, paint themselves upon Paper with a Blueish Colour, by so much the more sad and obscure as the Interval or Space is longer.

To these succeeds the Pellucidity or clearness of fair Water, of Crystal, of Glass, of Ice, &c. which indeed have Colour, but little and thin, and they easily assume the Colours of neighbouring things, by which they are made conspicuous.

3. *A Shadow is the diminishing of Light, by an opacous Body interposed.*

For when light casts it self on some Body which denies passage to the Lucid Rays, then a shadow is begot or generated, which is behind the Opacous Body: Notwithstanding a thick Pellucid also may cast a Shadow in a thinner Mean: So a Watery Globe in the Air opposite to the Sun, casts a Shadow although thin: Yea also the Air it self makes a Shadow, but only perceivable in the Sky, where the Moon is deprived from its Light.

4. *Darkness is the total privation of all Light.*

Such only was in the Abyss, before Light was Created, and perchance will be in the Infernal Dungeon of the Damned, which therefore is also called utter Darkness: But at this day, after the rising of the Sun, properly so Speaking, there is none: But in Speaking improperly or in comparison, most thick Shadows also are called Darkness; such is in most
deep

deep Prisons, and Places inaccessible to all Light: Likewise under Day, by Night, by a new Moon, and from the Heaven cover'd by Clouds: For else Night is no other than the Shadow of the Earth, which if we could elevate above the Sky, the opacous Body projected from the Earth would be discerned in the most clear Light of the Sun.

C H A P. VI.

Concerning Radiation or Shining and Visible Beams, and the Angles constituted by them.

From the Radiation of Light, even as from a primary Visible, are had also other Visibles, which Radiate or Shine; that is, they pour out their Species through a pellucid Medium.

For as other Sensibles, Sounds and Smells, send forth their Species: So also Visibles always spread abroad somewhat: For because to Communicate themselves afar of, that is, being look'd upon they ought to appear; therefore also by the Diffusion of their Species, from the imitation of Light, this communication must needs be made. And this Diffusion is called in the Optics, *Radiation or Shining*, which notwithstanding all other things besides the Light, they have it not from themselves but from Light, which Communicates it self not Intrinsick only, but also Extrinsick to all things to which it may extend, of which also it puts on the Colour and Figure.

Therefore although all things should have their Light within themselves, whereby they are Coloured and Figured; yet this Light is as it were bound in Matter, neither can it go forth: But when Light is also Communicated to Extrinsec things, and Illustrates them, then at length by means of this Communication, and by the Illustrated Light, and by putting on the Colours and Figures of things, they also spread abroad and Communicate themselves, and so as the Optics speak, they may Radiate or Shine.

2. Therefore

2. *Therefore Radiation or Shining, or being Radiated ; is no other, than of Light, either Primary from its fountain, or Communicated by other things, and tinged by the Colour and Figure of them, by the Ejaculation proceeding from the Superficie of them,*

This communicated Light, although also reflected and reverberated by things, to which 'tis Communicated : There is nevertheless some difference between this Communicated, and that which the Optics at another time call Reflected or Reverberated, of which hereafter : For light Reverberated is notwithstanding the Light of that thing, from whence this flowing or falling upon is laid open or appears : But this communicated Light now becomes after a sort the Light of its Superficie, to which it was communicated, and which Illustrates it.

3. *And this Radiation also is made according to right Lines, which are called Radii or Beams.*

Therefore a Ray is no other than a right Line, whereby a visible Species is spread abroad, to wit, the shortest way ; which Nature where it can be done, always insists on : This Rectitude may also be discerned by the Eyes, in the extreams of Shadows terminated by a lucid Ray, which are always right Lines by reason of the direct Rays : The Vulgar, think Rays to be the diffused Light it self, being deceived, because the Optics for the sake of Teaching often times so speak, as if they were the Lucid Lines of Rays, and they represent the Radiation of Lucids by right Lines, but 'tis a mistake, for they are only the motion of Light : As therefore other while, right Motion is a right Line, by which a moveable Body goes : So also the motion of Light is a right Line. And as a right Line of Motion of
any

any Body belongs not to the Body it self, so also the Ray or Line of Motion of Light, is nothing of the Light it self.

4. *And every visible Radiates or Shines; 1. From each of its Points; 2. By a diaphanous Medium, viz. that may be seen or shone thorow; 3. In every opposite Point whereby a Finite right Line may be drawn; and that 4. most swiftly and continually, and 5. yet distinctly.*

1. Indeed neither one nor other Point only, but every visible Radiates by its whole Superficie together: But for the sake of Teaching, the Optics assume certain Points, from which they draw Lines as it were Rays, that they may Demonstrate all things more exquisitely in a Mathematical manner: And that not from one only, but from all the visible Points of a thing, and therefore the whole Superficie of its Species being spread abroad, is made manifest.

1. In the Sun: *From the extream Superficie of which also the Rays descend, Illuminating the Earth, when the Centre of it, and more than half of the Superficie is hid by the intervening of the Moon in Eclipses.*

2. *From Looking-glasses or Speculums in which the whole visible Image appears together.*

3. *From hence, that a great many Eyes may see the whole visible Object together.*

4. *Lastly, If from the Center only, or from any Point, one Ray only flow; no Shadow is terminated, but all are included either by equidistant Rays, or by Rays always diverging or parting.*

2. And it is manifest from the aforesaid, that a Diaphanous Medium is required to Radiation, for where a Medium is not, there can neither be Local Motion, of which sort is the Motion of Light: And Opacous, transmits not the Rays of Light, but bends them other ways.

3. Because

3. Because in every opposite Point, by which a Finite right Line may be drawn, Radiation is made; also from thence it is evident, that Light, the most excellent of Creatures, always affects a most excellent Figure; this is Spherical, the Generation of which is understood from Lines going forth from the Center to the Superficies in all parts: If therefore we thus place a Lucid Point in Perspicuity, it will send round about into all parts, infinite Perspicuous Rays: But if a Point be Radiating in an opacous Superficie, and cannot Radiate every where, yet it Radiates Hemispherically, unless hindred by another opacous Intervening: And if it be acute angled at the top, it Radiates more than Hemispherically.

But if any one should Object, that in Looking-glasses or Speculums; of one Point, one Image only is beheld, and therefore not every visible Radiates in every opposite Point of the Mirror or Looking-glass; he must know, that indeed every point Radiates into infinite Points of the Mirror or Speculum; but from the Eye there cannot be discerned but one: Because by the Laws of reflection the other Rays are reflected another way; but only that which is reflected at equal Angles, occurs to the Eye and may be seen, concerning which we shall say more hereafter.

4. That Radiation is most swift, the fixed Stars are an Experiment, which being removed very far from us, are seen at the same moment that the nearest Objects are: And the cause is, the Immateriality and Spirituality of visible Species: For since as they want Matter, so also they want Weight, nor do they require sensible Time to Motion, which should be proportionate to themselves, but they can transfer themselves in a moment, what way is given.

5. The continuity of Radiation may be proved from this, that the Efflux is so joined or knit to its Fountaina,

Fountain, that it can never and in no place be separated from it ; even as is seen in Lucids, which being taken away, Illumination is taken away ; being placed, Illumination is posited, depending from that Efflux : Whence it is not necessary indeed that it should cease one moment, nor even as Light it self, can ever be separated from its Fountain.

There are some Men who seek or ask, where that Light goes away, which is expelled by shutting the Doors or Window-shuts of some Chamber ; or in some candle being extinguished, as we are wont to speak.

And some indeed appoint or conclude, that the Light of the Sun and of other Lucids, go forth from its Fountain, not remaining the same, but always to perish, and the former being abolished, a new one succeeds thereupon : Even as a Shadow follows not the Opacous Body, but by the vanishing of the former another new one is begat ; or as Water belonging to a River is not always the same, other Water always occupying the place of the elapsed or slipped away, and succeeding it.

But we shall believe this afterwards, if any one will prove to us firmly, the Annihilation of Natural things, especially of Substantial : For this Axiom seems to be unmoveable, As by Nature, from nothing, nothing can be made, so by the same also, somewhat cannot be made nothing ; but as Creation, so also Annihilation is reserved to God only.

Of Shadows indeed 'tis no wonder that they go into nothing, since also, nothing, is but the Privation of Light : But real Light is a Substance, whence by Nature it cannot go away into nothing : Flowing or River-Water, we know which way it goes away : In like manner the place may be shewn, whither Light goes away, that is shut out of a Chamber, or in a Candle, when it seems extinct ? It cannot

not be said otherwise, than that since by its Origine it remains always and inseparably connected or joined together, and when it seems Extinguish'd to us, it retreats it self in a moment and recurs to its Fountain : As also in a moment it diffuses it self what way is given.

6. Lastly, that Rays and Species of Visible things, penetrate through a Medium distinctly and unmixt, may be proved by some Examples : For the Light of Candles extinct or put out, they go through, by some one hole in which they are most closely United, being sent by the intimate change into themselves, they appear distinctly on the objected Wall, and directly opposite to those Candles : In the same manner it wholly happens to Colours also : For being Visible, every one by his Colours altho, divers, seems distinctly : So also Solar Rays passing through a glass Window of divers Colours, every one is beheld distinctly by its Colours on the objected Superficie.

5. *Moreover a Ray is either right or oblique.*

Indeed every Ray or Radius considered absolutely and in it self, is right, neither can any Visible, radiate by crooked Lines ; But in speaking comparatively, in respect of Incidence, to wit, which way a Ray falls on an opposite Superficie, that which is made either right or oblique, is called also a right or oblique Ray.

6. *And that is a right Ray which is Perpendicular to the Eye, and passes through the Center of it ; An oblique Ray is the contrary.*

And with this difference also, every Visible is said to fall either directly or obliquely to the Sight, according as its Rays constitute either right or oblique Angles with the Sight.

7. *But also hence an Optic Angle springs or arises which is the concurrence of Rays in an opposite Superficie.*

For

For as from Geometrical Lines, when they meet or concur, or diverge or part again from the point of concurrence, Angles are made: In the same manner also things being compared, it is with Rays of Light or of the Sight, which in their Superficie by some concurrence and divarication they constitute Optic Angles.

8. *And an Optic Angle is either Plain or Solid.*

Spherical, as in Geometry, here are none given, since all the Rays are right Lines, neither do they make other than right-lined Angles.

9. *'Tis a plain Angle which is begat from a plain Figure, and a Solid from a Solid Figure.*

That consists of two Lines, and is begat with only the length of a Line beheld: But this hath many Lines, and is made when a Body and some Figure is beheld.

CHAP. VII.

Of Optic Figures.

1. **V**isibles, Radiate, and are Irradiated according to the Optic Figure, from the Base into an acute Point.

For since Radiation is made from any visible point, into any objected Point, according to a right Line; and from many right Lines, from more than two concurring, a Figure is constituted: Hence also from Optic Rays, Optic Figures must needs be made: Of which, by reason of the great variety of Visible things, the variety also is very great, that scarcely it can otherwise be described, than that they arise from any Base whatsoever into a Point.

2. *And an Optic Figure is either of Radiation or Illumination.*

3. *A Figure of Radiation, is that whose Base exists in a visible Superficie, but the Vertex in the Eye, or another opposite Superficie.*

Therefore

Therefore this constitutes Figure, when any visible whatsoever radiates from many Points into the Eye, or into any opposit Point, in which all those Rayes concur.

4. *Tis a Figure of Illumination which hath the Vertex in the point of Radiance and the Base in the Superficie of the thing Irradiated or Illuminated.*

This begets Figures, when from one point of Radiance many Rays go forth, and are understood to be carried to the Sight of the Eye, or to any opposit Superficies.

5. *By this reason are made innumerable Optic Figures between Radiating and Irradiating.*

For because a Visible radiates from every Point into every Opposit, and therefore the Species of one part of the Visible is in what part you will of the other: And since in whatsoever great Figure given, innumerable lesser may be conceiv'd; tis certain that all the same Figures also flow from hence thither: Which will be made evident, if from two Parallel Lines, on each side the middle Point of that to this, and of this to that, you describe many Triangles intersecting one another mutually.

6. *Points Radiate according to a right Line; Lines according to Triangles; Superficies and Bodies according to Pyramids or Cones.*

For Lines are terminated by two Points; from which, if two Rays concur or meet in one opposit Point, tis manifest, that this concurrence constitutes a Triangle, whose Base is in the visive Line, and the Vertex in the opposit Point.

But when a Superficies radiates by all its extremes into the opposit Point, it constitutes in this manner, either an Optic Pyramid, if the Superficies seen be of Angles; or a Cone, if it be round, although in common speaking, this difference for the most Part is neglected; And every Optic Figure, whether proceeding from an angled Superficies, or from a round Superficies is usually called an Optic Pyramid.

7. In these Optic Figures the Axis also is consider'd, which is the Ray from the Visible to the Sight.

And twofold, Optic and Common: That is a Ray which passes from the visible Point through the Centers of the several parts of the Eye, to the middle of the Circle of the Optic Nerve, and there are two of those to each Eye: But this, which is one and Immutable, is the Ray falling Perpendicularly, from the concurrence of the Optic Axes, into the visible Point in the Diameter of the Eyes; or the right Line connecting the Centers of the Eyes, See Fig. 206.

As if *e* be the visible Point, *df* both the Eyes: the Optic Axes are the Rays *ed* and *ef*, but the common Axis is the Ray *ec*.

CHAP. VIII.

Of Vision it self, and the Manner and Species of it.

1. **A**S all Sence, so also Vision, is some Passion of the Eye, like as of the Instrument of Sight brought in from the Visible.

For we see not always by a second Act, but also by Power; which Power is traduced into act, when the Eye is moved from the Visible, which motion is some alteration and Passion; therefore tis evident, that the Eye sometimes is hurt by a more excellent Visible.

2. Therefore Vision is not made without the contact of a visible thing, and of the seeing Eye.

For all Passion is made by the contact of things acting in themselves mutually: therefore since Vision is also Passion, tis necessary that some Contact intercede between the Eye and the visible Object.

3. This contact is of the Rays of Light in Specie, that is, tinged by the Colour and Figure of the visible thing, revibrated from the visible Object to the Eyes.

For the Eyes and the visible Object neither can

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nor ought to touch one another mutually in the Ratio of Substance, because it is necessary that a thing should be seen afar off, and if the Substantial Contact should be made of the Object and of the Eyes, the Eyes would be much hurt by many things: Therefore it behoves that some other thing should come between these two, which either may go forth from the Eye to the Visible, or may enter into the Eyes from the visible Object, and be received by them.

Hence arises that noble Question, debated between those two chief Philosophers *Plato* and *Aristotle*, of which he contends in his *Timæus*.

That Vision is made by the Emission of certain Rays from the Eye to the thing seen. And this same Vision he would have made by the Immission and Reception of visible Species within the Sight of the Eye.

Between these, *Galen* being willing to become a Mediator, contends that Vision is made partly by emitting or sending forth, and partly by receiving: That is, from the Eye goe forth visive Spirits, and least they should be weakned by too much elongation, being abroad, they presently Unite; that is, Co-radiating with the visible Species, sticking in the Illuminated Air; but both being united, they run together to the Eye, and there are received and judged.

But what that Ray or Spirit of Sight sent, is; and how to be comprehended going forth from the Eyes to the Visible things being oft times far distant, such as are Stars in the highest Heavens, likewise what the Species of a visible thing is, and how it goes out from the Visible, and enters into the Eye, none of the said Philosophers could clearly explain, as is to be seen in *Macrobius*, Book 7 Saturn: and *Zarabella* in his Book of Vision: Therefore the Modern Philosophers think with *Aristotle*.

1. *That Vision is made by the Reception of visible Species into the Eye.*
2. *And they say that Visible Species, are Rays of Light, tinged by the Colour and Figure of the outward Object, and casting themselves to the Eyes.*

For since nothing can be seen but that which is Lucid or Clear, or illustrated by Light : And Light is reverberated from every Point of the thing Illustrated, to every Point between which, right Lines may be drawn, and indeed Tinged and Embrewed by the Species, Colour, and Figure of the Visible thing ; what is more easie, than from this to gather, that Light it self communicated to things, and tinged by the Colour and Figure of them, is a visible Species, by which is made the contact of the visible Object and of the Eye, and therefore the Vision it self : And like as what's said before concerning Rays, that although there be nothing of Light it self, but only the way by which Light is scattered : yet the Optics very often speak concerning them, as if they were real Lucid Lines, because they cannot otherwise describe and represent the Motion of Light than by Lines : So also they speak sometimes concerning Vision, as if it were made by the Emission of Rays from the Eyes, or in imitation of the Ancients, which truly believed the Emission, either because tis described with Vision or sight, the reason is the same, or entering from the Visible or thing seen to the Eye ; nevertheless in true and accurate speaking and perceiving, always the reception of Rays into the Eye, and the Effluxion from the visible thing, is understood to be the same : But these things do not as yet suffice the declaring the way to Vision, we must also speak to what is made by those Species into the Eye, which way or method is owing to the most Ingenious Mathematician *John Kepler*, according to whose Opinion.

4. *Vision is made in the Eye it self, and almost in the manner which things existing in the Light abroad, appear and are described in a close Chamber, by a small hole, on an opposit white Wall.*

For if in a close or shut Chamber, the other Walls being covered over by a black Veil, the Light being let in through a small hole, it will represent that on the opposit white Wall, whatsoever it is that is abroad without the Chamber and over against it in a clear Light, either of the Sun or of the Day, or of a

Linke

Linke lighted is also carried; to which truly the Prospect through the hole appears, only the Site is turned upside down, so that the upper part is exchanged to the lower part, and the right Hand to the left.

5. And so also is Vision made, when Light from all that part of the world which is before the Eyes, and in this also of the thing which we behold directly, tinged by the Species and Image flowing into the Eyes, by or through the pellucid horny Tunicle adjoining the watery Humor, and the Little hole of the Sight in the ChrySTALLINE Humor, and from this into the glassy Humor, being refracted in these divers Means or Mediums by the Optic Laws once or twice, and at last into the hollow and the White; but by the somewhat Redish or somewhat Blew or Azure Surface of the netlike Tunicle, whereby the Optic Nerve receives the same; to wit, all the Rays being dispersed most strongly, and as it were gathered into one Point, is described; yet so, that those things that abroad are on the right Hand, are conveyed to the left side of the netlike Tunicle, those on the left Hand to the right Side; those above to the lower part, and those below to the upper part of the same.

Nevertheless this inversion begets no Error: For as in a Mirror or Looking-glass, that which is on the right Hand is made the left to the Image or thing represented, nor yet is any one deceived in Judgment between the Right and Left; so neither here need we fear that the inverted Vision follows this inversion.

6. From the aforesaid things, as it were the Luminous Picture of the Spirit of the Eye or Instrument of Sight, which resides in the Optic Nerves, they are disposed and changed, by which the Judgment Seat is retained within, in the cavern of the Brain, to the Mind or to the faculty of the Instrument of Sight, where from Judgment made of the thing seen, at last Vision is finished.

For Vision is not absolute and perfect, only infer'd by Passion from the Object; but tis necessary that moreover action be added, in what manner the

Judgment is produced concerning the thing seen, in which the Nature of the Object is gathered from other things discerned, and what is good or ill in it is discerned: For if Vision should be determined alone by Passion infer'd from the Object without Judgment, it would not be an operation of the Mind, but only an objected action External, and also Looking-glasses and other things would see, in which likewise the Visible Species are Imprinted: And also that Judgment may be made, concerning the thing seen; the Point of the net-like Tunicle, which terminates the Image of the thing seen, even the very same is the Center and extreame of the Optic Nerve; which in this place connects the Netlike Tunicle, in which resides the Optic Spirit; which may also suffer, and be changed by the Images and Colours is evident from this, that the Species of strongest Colours remain in the Sight after the looking upon them is done, and they are mingled by the Colours impressed by a new looking on, that from both a confused Colour is made.

7. Therefore Vision (as hitherto briefly hath been said, we comprehend) is the visible Species received within the Eye, and painted on the inner part of the netlike Tunicle, and being conveyed through the Optic Spirits residing in the Optic Nerves, to the Sensitive Soul to Judge, See Fig. 207.

This method of Vision the Excellent *William Schickhard* hath delineated in Fig. 207. In which the Visible is AB, the Eye CDEFLM; the two extreame Points of the Visible or thing seen AB (that we may shun confusion) they Radiate in the pellucid Protuberance or sticking out of the Horny Tunicle CDEF, which because tis thicker than Air, and of equal thickness with the contiguous Watery Humor, therefore the Rays got in, are refracted, and assemble, that they may enter by the small Sight of the Eye GH into the Chrystalline Humour IK: In which Superficies although it be thicker, they are not refracted, or surely but little, because the same is almost of the convexity with the Horny Tunicle and therefore falling in Perpendicularly, they indeed pass through unchanged; but they go out from the hinder Hyperbolical Superficie of it IK into the thin Medium, viz. into the glassy Humour, and are refracted there again; And they meet together at
last

last in the wall of the netlike Tunicle L M, and are Painted there most strongly (because all the dispersed Rays are gather'd into one Point) but in an Inverted manner; The right hand Point A is painted on the left side M; in the like manner, B in L: But because the netlike Tunicle is full of visive Spirit, therefore it Transmits the Image through the Orifice of the Optic nerve which is in the same place at the Brain, and the common Sense or Judge; not any more by right lines but by crooked windings of the Nerves: And this is briefly that wonderful and much spoken of Method of Vision.

8. *Moreover Vision is sometimes true, other times false,*

9. *True Vision is made, when we see a thing truly such as it is, sited in its Place, endued with just Magnitude, Tinged by its native Colour, in very deed moving or resting: In brief, when between the Vision and the verity of the thing seen there is no diversity.*

10. *And this true Vision is again twofold, Confused and distinct.*

11. *Distinct Vision is, when the visible Species is apprehended distinctly according to the whole and the parts, and rightly Judged concerning them.*

12. *And Confused Vision is, when we comprehend the whole together Confusedly without accurate discerning.*

13. *Lastly False Vision is, when we see a thing Sited, Coloured, or howsoever affected, in another manner than indeed it is.*

As when a Finger and a Mountain appear of the same Magnitude; or when high Mountains seen, are thought to be of a red Colour, when in truth they are Green; when they appear great, and are small, crooked when streight, when things seem a far off that are near at hand, or the contrary.

Such deceivings, as about other things, so especially about the Sense of Vision, innumerable Occur, by reason of many causes; which, as much as may

done, to detect and shew the Errors, and to Correct them, is the Office of the Optics.

C H A P. IX.

Of the Requisites necessary to all true and distinct Vision.

I. On the Visible part.

1. **T**hat any thing be of it self Visible, either by Act or Power, tis necessary, that it be material or made of stuff, and endued with just Quantity or Magnitude.

1. For *Immaterialized* things, such as are points and spiritual Creatures, fall under no Sence of themselves, or *per se*, and therefore neither do they fall under Sight: For whatsoever is seen, that is presented to us by some kind of quantity; yea our understanding scarcely perceives any Thing, but together also how much is it overshadowed.

2. But also a Visible ought to be of just Quantity.

For things of the least Quantity also escape the Sight of the Eyes, as are Solar Atomes, the Thousandth part of a Thousand, the Members of a Mite in Cheese, and some other things; notwithstanding the smallest of which may be gathered by some made Art, the Artificers of which must be sharp Sighted, or otherwise to frame them than by the help of the Eyes, to wit, by the help of Optic Artifice, which may be done, so that the least thing shall appear great, and may be discerned clearly: And although every Visible is needful to be *Quantum* or so great or so much, yet every *Quantum* is not visible; as the Skie and the Air next to us, which although they are most vast Bodies, yet they cannot be perceived by the Eyes: And therefore,

3. Every Visible ought to have some Thickness whereby the Sight may be terminated.

For the Skie and the Air are therefore not seen, because by reason all the Rays are transmitted through

through the thinness of the light ; nor do they fix any where, that they may be reflected to the Eye ; therefore there is need of thickness whereby the Raies are stopt, that it suffers not a passage to them, or surely not to all, but being driven back again they are brought to the Eye.

3. *Yet of Bodies, we behold only the Superficies of things ; nor all together, unless seldom, which things therefore tis necessary should be laid before the Eyes.*

For if the Eyes be turned against, either to the point of sharp bodies, or the sharp sides of them joined together ; the Body is not rightly seen ; of which neither can we penetrate the Inner part by sight, to which neither doth Light pass (unless they are Pellucid, which are here excepted) that therefore Superficies only ought to be beheld, or rather the Species of Superficies: And we seldom discern all the Superficies of a whole Body together ; nor but in Pyramids or Cones, of which the Diameter of the Bases being turned against the Eyes, is a lesser distance than of the two Eyes from one another ; For then on every side an entrance lies open to the visive Rayes of sight, that all the Superficies of it may be seen although but weakly, by Reason of too much obliquity.

4. *The Superficies of Visible things ought to be distained by Colour, either proper, or Alien and accidental.*

For Prime Light itself that it may be seen, tis necessary that it assume not only quantity in matter, but also Colour, by a certain coming close together; Nor Second Light which flows from the First, and slides by the Superficies unless streightned, it acquires colour that it may be seen ; But only the colour of Superficies to the end that the tinged Light may revibrate into the Eyes ; But the Sky, because it hath indeed no colour, neither also can any thing of it be seen ; as also the Air, unless so long as the Heaven is clear, being illuminated crossways by the Sun Beams, and assumes a Sky colour.

5. *Nor yet is the Colour of Visibles sufficient to Vision ; but if it be not Light it self, or somewhat Lucid ; it also behoves to be illustrated by external Ligbr.*

For

For in Darkness nothing is beheld ; for the proper Object being absent, no Sense is made ; therefore since Light is the proper Object of Sight, if it be wanting, also no Vision can be made manifest : Although Colours also are the Object of Sight, and in Darkness, *inhare* to things not less, than in Light, yet they are not seen in Darkness, because they are not the Principal Object ; neither have they the diffusive force of it *per se* or of themselves, and therefore neither can they cast themselves to the Eyes, neither have they any thing of themselves whereby they can change the Spirit of Light unless they are received into another, which shall have the power of changing, and by that they change or alter : But such a thing is Light alone, which Illustrates Colours, and tinges them, it agitates our Spirit residing in the Optic Nerves by Illumination, and changes it ; from which changing, is made also Vision, and the Judgment concerning Visibles.

6. *Lastly to be Visible also, is not to be too much shining or Splendid.*

For as in another manner, the excelling the sensible part destroys the sensitive Instrument ; even as the sharp sound of great Guns dulls the Ears, and sowre or bitter things weaken the Taste, more fragrant things the smelling : So also too much brightness hurts the Eyes, and hinders Vision or seeing, if you look on it long : The Reason is, because the Rayes from a Splendid thing reflecting in abundance into the Eye, they enter in together at one Point, and there scorch the Optic Nerve, and Dry up its humours.

So *Democritus* is said to have put out his Eyes, by the reflexion of the Suns Rayes on a Brazen shield placed against it, that he might inquire more nicely into the secrets of Nature : And the *Souldiers* of *Zenophon*, from the too much glittering of Snow contracted great harms.

II. *On the Part of the Beholder, or of the Eyes.*

1. *The Sharpness of seeing is not the same of all Eyes, yea indeed not the same always of one Man.*

For some see the same thing more sharp than others ; others see remote things better than things nigh at hand, and the contrary ; others see nothing distinctly, but all things confusedly : also some have a sharp Eye in their Youth, and in old Age much weakned.

2. *The Temperament of Eyes ought to be Waterish.*

Plato, would have them to be fiery, but experience testifies from the Eye being rubbed or scratched that it flows with Water, which is a sign of a Waterish Temperament ; the same being necessary, that the Eyes may receive the Images of things the better.

3. *Tis also necessary to the distinct making of Vision, that the Eyes be sound and whole.*

That is, not hurt nor distorted, nor any part impeded ; also embrued by no Alien colours, or by vitiated coloured humors : For the Eye having hurts, distorts, or in whatsoever manner impeded, sees nothing, or but little, nor distinctly : So of whom the Eye is imbrued by an Alien colour, as it happens to those which behold splendid things strongly illuminated ; or coloured things more earnestly and a long time, or having the Eyes infected by vicious coloured humors, they see not distinctly : For as to a Tongue infected with Yellow Choller all things are bitter ; so if the Eye be embrued or distained by Alien or strange Colours, also the things which ought to be seen, are corrupted or discoloured with the same, nor do they shew their genuine Species on kinds.

4. *That Vision be made distinctly, is necessary that the eyes be moved through all and each part, not only of the visible thing, but of the whole mundane Hemisphere placed before them.*

For

For if it should be without this Motion, distinct vision would not be made, unless in the middle or medium; and those things which cast themselves from the sides, by reason of obliquity, would be seen confusedly: Therefore it behoves that the Motion of the Eyes be indeed successive but most swift; for which cause also the Eyes are made Spherical that they may move most swiftly upon their Center.

5. *By one Eye indeed we see not more acutely, but yet more directly than by both.*

There are some which think a thing may be seen by one Eye more acutely than by two; because that the visive vertue removes into that one, and there becomes more strongly united than when dispersed.

But Experience thwarts this opinion; the cause added, holds good only with those to which one of their Eyes are put out, but not to them who voluntarily shut one: nevertheless tis true, that by one Eye, we see more directly than by both; for from both Eyes a nigh visible is not seen in its place, but in divers, by reason of the distance of the Eyes, and the intervening Parallax; concerning which many things shall be spoken to hereafter, in the Chapter concerning vision of Place: Therefore to shun this Parallax, if we would see any thing directly in its place, tis necessary to shut one Eye; Astronomers do so, to constitute an Angle between two Stars, in which business there is need only of one Center of them, at which the Angle is estimated; wherefore they shut one Eye; imitating Artificers who in trying the rectitude or straitness of a Ruler shut one Eye: They do it also who direct or shoot Arrows, and also Guns at a Mark, for all these indeed for divers causes, both Eyes being used, hinders; concerning which thing also *Aristotle* treats in his *Problems* at Sect. 31 *Questions* the 2 and 21.

III. On

III. *On the Part of him that sees and beholds.*

1. *That any thing may be seen distinctly, it behoves him that looks, to be quiet.*

For if he be agitated much, and cannot have his Eyes fixt on the thing seen, and also disturbs the Spirit; Those things seem to move together, which notwithstanding are at rest.

2. *Moreover, unless he that sees, be intent to see, and adds over and above a right Imagination and Ratiocination of Vision, the Vision cannot be made distinctly.*

For he that is full of Thoughts, and attent on other things, often times sees not those things which are observed by his Eyes: But if the Imagination also be corrupt, or the mind embrued with a preconceived false Opinion, a false Ratiocination or reason also follows, and a false Judgment, which also effects or makes a false Vision.

IV. *On the part of the Rays and Angles of the things seen.*

1. *Whatsoever is seen, is seen under an Angle.*

For since Vision is effected by the Radiation of VISIBLES into the Eyes, and those are belonging to Angles, going together from the Base to one Point of the Eye: tis evident all things are seen under an Angle.

2. *Under a right Angle, or greater than a right, the Eye being unmoved, nothing is seen: since the greatest Angle of a thing seen is somewhat less than a right Angle, and therefore the greatest of all acute Angles.*

The cause is, because the Diameter of the Sight of the Eye is so small, that the Lines educed from the Center of the ChrySTALLINE Humor, cannot contain a right Angle, although by night when that Diameter is enlarged.

So by day we cannot constitute an whole Quadrant of a great Celestial Circle, suppose of the Meridian or of the Horizon, to comprehend together by Sight, in as much, as how much also the Eyes are enlarged, the Sight also of them is strained.

3. *Under*

3. *Under an Angle so much acute, that it escapes the Sight, nothing is seen.*

Therefore although Mathematicians divide Angles infinitely, yet because the Subject of Optics is not pure, it cannot descend in the least beyond Sense.

4. *A right Angled Ray is always stronger than an Oblique, and an oblique angled Ray, than another that is more Oblique.*

Of it self, and by its Nature, no Ray is more excellent than another; because a Visible, equally radiates in an Orb to all the differences of Position, but by accident one Ray may excel another, while the case of them being right Angled or Oblique, strengthen or weaken them in the objected Body: So the Rays of the Sun, although they always convey their heat with them, yet they warm more, by how much they fall more rightly on the Subjected Earth, and the less, by falling oblique: Hence they warm more with us in Summer than in Winter, not so much by reason of the greater nearness of the Sun, as that they are cast more rightly on the Earth in the Summer, and more oblique in the Winter.

And the Reason which is of Heat, is likewise the same of Illumination, as well in Lucids as in other visible Rays.

And the cause of this Diversity is; 1, because in general, all natural Motions being right, are stronger than oblique, as if the back of any one be smitten or struck by a club or stick perpendicularly, he feels that blow more forceably, than if it had been struck according to the length of the back; therefore what is done in other Motions, must be imagined also to be done in the Illumination of Rays: Especially, 2, because also right Rays are shorter, and strike nearer at hand, of which also the Force of Angles by Equality in both, are united and gathered together, more united.

5. *Vision*

5. *Vision is most certainly made by the Axis.*

For this, since tis always right and Perpendicular to the Eye, it offers the thing effectually to the Sight; by the other Rays, Vision is made more certain, by how much they are nearer to the Axis; and it is less certain, by how much the more they are remote from the Axis, and Obliquier: Hence, when there is only one Axis which conveys the thing most effectually to the Sight, tis necessary that the Eyes turn about so long, until all the Parts of the Visible Object be Perpendicular to them, and so tis known more surely and distinctly.

V. *On the Part of the Medium or Middle.*

1. *That any thing be seen, not only the thing seen is to be placed before the Eyes, but also between the Eye and the thing seen, tis necessary that a middle Space intervene.*

For since tis light which conveys the Species of things to the Eyes by right Lines; therefore neither those things which are on the back or behind, can be seen, (for although Light likewise falls on them, and starts back, yet it offers not to the Eye, and therefore excites no Vision) neither can those things be discerned, which are placed before the Eyes without a Medium coming between, since neither Light falls in on them, nor can be reflected from them into the Eye: Nevertheless some things also are beheld without an outward Medium, as are Sparkles of the Eyes, inhering Species in Spirits after the removal of the Visible thing, the Impurity of the Humors of the Eyes after Pains of the Head, &c. therefore it will be a Rule to be accepted with some exception.

2. *That this Medium is a Solid Body, nothing hinders; but tis necessary that it be Diaphanous, Perspicuous, or Pellucid.*

This

This is the division of Opinions between *Democritus* and *Aristotle*: *Democritus* thought if it were an empty Space, an Ant might be discovered creeping in the Sky, from whose Opinion *John Kepler* seems not to disagree, who opined, if there were intirely no Medium of Bodies, Vision would be accomplished better; because by how much thicker that is, it the more hinders and exchanges the transient Rays, whence consequently it seems, by how much thinner, even to nothing, to wit, by it the Rays would pass more expeditely: But *Aristotle* contends, that through *Vacuum* or *Emptiness*, if that should be, nothing could be seen plainly; whose Opinion seems the truer; for Light conveyed through *Vacuum* is not seen, whose Office is to Illustrate: And if it should illustrate *Vacuum*, it would Illustrate nothing, which is contrary to Nature; yet in this place we must not be Ignorant, that there is an Instrument invented by Modern Philosophers, in which Space is shewn; concerning which tis certain, that not only some thick Body, but neither Water, nor Air, nor Sky is contained there: Either then that Space is *Vacuum*, or if Nature admits not this, without another Body intervening, Light by it self can fill Space, the Nature of which is, so long to diffuse or pour out it self, until it be repelled from the objected opacous Body.

For tis necessary otherwile that there should be a perspicuous Medium, that the conveyed Light may receive the visible Species, and may return it to the opposit Eyes, for the Superficies of opacous Bodies permit not only the passing of Light to the Eye, but they turn it from the Eyes, and revibrate or strike it back into the opposit.

If this Medium be Pellucid, it hinders not, although it be Solid and also hardned for as *Kepler* teaches, Light, although also it should have quantity,



ty, it would not be hindred by the Solidity of Bodies, to the end that it might less pass through them: For whatsoever is hindred, 'tis hindred or expelled by that which is of the same kind, as a Body by a Body: A Solid then hath three Dimensions as 'tis a Solid, but Light only contains two Dimensions, viz. Length and Breadth, therefore Light, and the Rays of it, suffer nothing from Solids, for as much as they are Solids, neither do they affect one another mutually as to Solidity.

3. *'Tis not necessary to Seeing, that the Medium Space between the Eye and the thing seen, should always be Illustrated; and when it is necessary, that it be not too much Illustrated.*

To the Seeing of Lucids 'tis not necessary: Since we see in dark Mediums, not only burning Forms, but also rotten Wood, and the shining Scales of Fishes; also 'tis certain that Stars may be seen in the day time from deep Wells, from Caves, and from Dens inaccessible to Light: And those things which are not Lucid of themselves, want indeed the presence and Illustration of External Light, but that Illustration must not be too much; for if either the mean, or the thing seen, be too much Illustrated, the Vision or Seeing is rather hindred than helpt: Hence, to the inspecting of any thing accurately, we lift up the Hand to the Eye, or we behold through a small hole, that too much plenty of Light may be hindered, and the weakning of the Eyes taken away: Hence also, they see better, whose Eyes are deep in; and for this reason Nature hath given the Eye-brows themselves, that from the Hairs, the Light may be somewhat refracted.

4. *The Medium Illustrated, is besprinkled of foreign Colour.*

For Light passing by or through coloured Things, is affected by the Colours of them, from which also the visible Species is stained; neither of them being conveyed without mixture to the Eye.

VI. *From the Reason of Circumstances of Place and of Time.*

1. *The Sight apprehends nothing distinctly, unless presented by Proportional distance.*

XX

For

For Vision, since it requires a Medium, requires also distance, which is necessary to be indifferent and proportionate with the thing to be seen, neither too much remote, nor yet too near : For Visibles applied near to the Sight, appear confused, and of a blackish Colour, and almost intirely without Figure: Even as therefore from longer Radiation, the Species of Visible Points are dispersed, and mingled with the middle or Medium Shadows ; so shorter Radiation conveys the visible Species, thick, muddy, and something confused, which being diffused in the Air a little farther, they are made thin, they are cleansed, and brought back in order ; not otherwise, than the Vapours of Flowers and odoriferous Things, from a shorter Interval, convey much Grossness and Feculency with them to the Nose, which render the Vapours of the same unpleasant ; but by flying from a farther Space, they are made more pure, by cutting off the Feculent and thick Parts, and are brought Sweet to the Nose : Tho' some see acutely things afar off, others nigh at Hand ; yet that is not to be ascribed to the Radiation of the visible Species, but to the different quickness of the Optic faculty, and to the disposition of the Eyes.

2. *Nevertheless the Term or bound of Vision cannot certainly be determined.*

There are two things chiefly, which Limit our Sight, that it be not carried further.

1. *The rising of the Earth*, by reason of which, the Sight scarcely extends more than five *German Miles* on plain Ground, or on the Sea, unless one be placed in an higher place, as on an Hill, or on the Mast of a Ship : For if there be given an Hill of one Mile high, on it, the Sight may extend it self more than 40 Miles : This cause is all alike with respect to all Men.

2. *The weakness of the Sight*, which in some is greater, in others less : Hence some see things far distant, others see but little those things which are nigh, so that certain Rules cannot be constituted.

3. *Yet we know Generals afar off and sooner, than we have seen before.*
So something being seen afar off, I understand it at first to be
a living

a living Creature, because it moves and goes forward: Afterwards, that 'tis a Man, because he Walks upright, by and by, a Male by his apparel, at last being near at hand I know him to be *John* or *Peter*.

4. *True Vision and Distinct is not made, but in sensible time.*

For if any one turns the Eyes in a mome[n]t, the Eyes do not distinctly but deceives the Sight; even as in the most swift turning about of any Point, the Point seems to be a Circle; for the Eyes ought to be carried to all the visible Parts; which Motion is not made in a Moment, but successively in time proportionated to Motion; which nevertheless because of the swift Sharpness of the Sight, which is as it were momentary, it distinguishes accurately, unless between like Species.

These things commonly are the Requisites necessary to true, certain, and distinct Vision, one of which or more, if they are either wanting, or depart from indifferency; forthwith will be made either none, or false, or uncertain and confused Vision.

And so far concerning Vision in general; concerning the Affections and Accidents of Vision, and of each visible, it follows in Particular:

C H A P. X.

Of the Vision of Light.

1. **T**He sight in seizing of stronger Light, is grieved and hurt.

1. That happens by looking on the Sun, and by receiving the splendor of reflected Light in a Looking-glass: But the cause is, because as all Sense, so also the visive Passion is some Instrument of Seeing, brought in from the Object, which when 'tis immoderate offends the Sense.

2. Also it may come to pass, that Light, the Mother of all heat, by Imprinting the Colour on the Sight, Changes the Temper of it, and emits vital Spirits, the Ministers of Vision, so that a making Blind may easily follow.

2. *The intense Images of Light remain in the Eye, although after the beholding of stronger Lights.*

Hence it comes to pass, that also by the Eyes being shut, the Image of Light is perceived from the precedent Aspect: And also such place less Illuminated, seems obscure and Darksom, until that Image vanish away.

3. *Stronger Light, bears down or oppresses the Weaker, and hides it and weakens it.*

So by the Light of the Sun or of the Day, somewhat, also of the Light of the Moon, of Stars, and of Candles, are not indeed extinguished, but weakened and hid.

4. *Light is more evident in Darknes.*

For contrary opposits between themselves, shine more clear: Moreover Light it self dilates larger and freer in Darknes it self, because of its proper Gift, which is to Illustrate obscure things, rejoycing to perform its Office: And hence Astronomers gather, as is said before, that the Moon wholly wants proper Light, otherwise it would be impossible, that in Total Eclipses, the whole should be parted with from the Eyes.

5. *Fiery Light, shining or radiating in flaming Matter, appears greater from afar off, than when near at hand.*

For the Sight being remote, distinguishes not between the Light effused from a Flame, and the near Illuminated Air, which is thought equally to Shine, by being beheld afar off: But in a place near at hand the Sight distinguishes better between these two: But yet we must understand here, the just and proportionate Distance; for a Candle also appears less from a greater Distance, than from a just Distance.

6. *Lighted Candles being seen by Night, with the Eye-brows a little depressed, seem to cast certain Lucid Rays like Pyramids.*

The cause of this is not the Light, which (in like Circumstances) Radiates uniformly every way, but in the Site of the Eye-brows, the hairs of which, may in the manner of Looking-glasses, cause that reverberated Glittering, so that at the Motion of them, also those Pyramid Rayes move and are inclined; are raised up and depressed.

7. *Vehement or fierce Light shews many Visibles, which*

which the Weak hides : On the contrary, weak Light also shews some things, which fierce or vehement Hides.

So weak Light Hides small Writings and Sculptures, stronger Light Illustrates and shews them : On the contrary, the vehement Light of the Sun in the day time hides the Stars : The same being dulated in Eclipses, sometimes shews the same Stars, that in the day time they may be beheld about Noon.

8. *Light gathered together is stronger than dispersed.*

Hence it comes to pass, that a burning Candle enlightens a small bed Chamber stronger and more sensibly than a large Room.

9. *A Luminous Body illuminates an opacous Body, so much the stronger as it is nearer.*

For as other things acting, they Act so much the stronger by how much they are nearer to the patient, so the same also happens in Illumination : For the Rays as they are nigher, are more press'd together, and therefore stronger : When remote, they are more dilated or spread, and therefore they go forth fewer and more remiss.

10. *The Rays of Light, especially of the Sun, being continued as far off, appear Parallels.*

For although they go forth from one Point, and separate always more and more, yet where there is none, or but very little proportion, between the distance of the Lucid, and the Plane on which the Rays fall, the difference of inclination is not perceived, and so are esteemed Parallels.

11. *Light, the further it falls through a small hole on an objected wall, it becomes the Larger.*

For the Rays come together in a Point of the hole, and intersect each other, and again immediately are spread wide from one another, and the further they are stretched forth, the more they depart from one another, and they scatter the Light, wider on the opposite Wall.

12. *Light having entered through an hole into an Opacous place, represents it self on some Opacous Object, and the further, it is the broader, yet by an inversed Site: And is indeed made perfectly round if it flow in Perpendicularly through a round hole, and 'tis more Imperfect, the obliquer it is: The same comes to pass in all Visibles illustrated from Light without.*

The cause of the Site being inverted, is the intersection of the Rays in the hole; which makes, that those things which are Inferior abroad or without, become Superior within on the opacous Body, and the contrary: In the same manner also other Visibles coloured, being emitted in the day time into an opacous or shady Room through an hole (especially Conical, viz. narrow within, and open a little wider abroad) yet they are discerned everted or turn'd upside down, if they are Illuminated by the Sun from abroad, and received into the Chamber on white Paper, and well illuminated; so that Birds flying, and Men, are turned up-side down, and are beheld as it were *Antipodes* Walking to and fro on the objected Table; and the more clear and evident, if a Chrystalline Convex optic Glass be put to the hole, if it can be had: To which, if another optic Glass of the same Convexity be adhibited, a little more distant from the former, than the Semidiameter of its Convexity; for then these Images will be everted again, that they will be beheld, not everted or turned up-side down, but rightly.

13. *Also Light entering through an angular or corner'd Hole, and far continued, at length becomes round.*

This Theorem wants Limitation; For 1, if as well the shining Superficies, as the Window or Hole through which it enters, have circular Figures, the Rays also falling Perpendicularly, will describe a perfect Circle on the Wall.

2. If the Window, and the Superficies of Light have divers Figures, and each one distant from the Wall by a like Number of their Diameters, it makes a confusion of Figures, and the Figure of the Rays will participate equally of both, as well of the Figure of the shining, as of the Window.

3. But

3. But if the Window be distant from the Wall by fewer of its Diameters, than the shining Superficies is distant from the same by its Diameters, then the Figure of the Rays, and the Site of it, comes nearer to the Figure of the Window.

4. Lastly, if the Window be distant from the Wall, by more of its Diameters, but the shining Superficies by fewer of its Diameters, the Figure of the Rays will more imitate the Figure of the shining Superficies in an everted Site; and by how much the more that is, by so much the more also will this be: And hence it is, that the Rays of the Sun being sent in through whatsoever small chink or cleft, falls on the objected Plane in the form of a Circle, in which the Eclipses of the Sun may be beheld fairly by unhurt Eyes: Concerning which you may see more in *Kepler's Optics Prop. 37. and the following.*

CHAP. XI.

Of the Vision of Colours.

1. **A**lso Colours strongly Illuminated, leave their Image in the Eye.

For if any one behold earnestly, Colour strongly Illuminated, he being turned to a Colour weakly Illuminated, will animadvert or think the former to be mixed with this.

2. *Light, acts strongly on Whiteness, and Illuminates it more than Blackness.*

Like as if Red Drops fall into Water, that is Humor into Humor, it spreads it self in it, but falling on its Heterogene, Dust, 'tis gathered into a little Globe: So Light acts stronger on Whiteness being akin to it, and more Homogene; than on Blackness which hath little or no affinity with Light: And hence also it comes to pass, that Light, for Example, falling on White Paper, Illuminates it further, than if it fall on any thing that is Black; to wit, because that Light also being cast about by White, by reason of likeness, therefore it makes it grow considerably, that the parts chose into the Number are most strongly Illustrat-

ted : And from this is begat an Opinion that *Whiteness separates, Blackness assembles or gathers together the visive Rays* ; but here the Action of Whiteness and Blackness is none, but tis a Passion, neither can any Superficies be effected by its Colour, that the Ray should fall in, in another manner than the optic Laws themselves compel to fall in, or light upon.

3. *Fair or bright Colour, darkens or shades the more obscure Colour next to it ; and in like manner, dark Colour Illustrates the light Colour next to it.*

This, Painters observe, and by the apt placing of Colours only, they know how to represent deep holes, on high, depressed, wide : Hence also if in an obscure place in the Heaven and wanting Light, a lucid or white Border encompasses it round about, it seems a deep hole in the middle, which they call a Gulf.

4. *In darkness no Colour is seen.*

Not that Night deprives a thing of its Colour, or by Light being absent, none of it is in it ; but because by the absence of Light the radiation of Colours is prohibited.

5. *Colour placed in Light, the nearer it comes to Whiteness, the clearer it seems, supposing all other Circumstances are the same.*

For Colour is light in Power, or Light buried in Matter, and there, as it were bound, and therefore the chief agreement or relation of Colour and of Light is, that they act mutually in one another, and the stronger Light arriving on the out-side, the weaker brightness lying hid within, fetches out with its force, as if it were a Spark out of a Flint ; or even as the heat in Ginger is not found by Act, but if it come unto a casual Humor, then at length it pricks, and begins to Communicate it self ; so accidental Light approaching on the out-side, pricks the potential Light of Colours, that it pours forth its Species through the mean or middle.

6. *One and the same Colour in the Eyes, may be varied in divers manners as to appearance.*

I. By

1. *By reason of Light*; for weak Light shews not Colour perfectly; so in the time of the Eclipse of the Sun, all things appear Yellowish; in the Evening, Plowed Lands seem Reddish or Purple Coloured, by the Sun Beams being struck lightly on them; many things having one appearance or face in the Light of the Sun, another in the Light of a Candle, &c.

2. *By the reason of Shadow*; for a Shadow falling on clear Colours, Darkens the same: Briefly, from the various Mixture of Light and of Colour, arise all the differences of Colours; as we see in Clouds.

3. *By the Reason of Site or Situation*; for from beholding divers Sites, also divers Colours are exhibited; so the necks of Swans and of Peacocks, according as they are beheld in divers manners, they bring divers Colours to the Eyes.

4. *By reason of the Medium*; for looking through coloured Glass, the Visible is thought to be tinged with the same Colour; in Walking in green Meadows, the Visible appears Green, viz, from the Green Species continually flowing from Herbs and Grass.

5. *By reason of the Interval*; hence, White, seen afar off, is thought Blackish, or somewhat Black; remote Mountains, of Azure Colour; although this Deception also may be from the Colour of the Air.

6. *By reason of the Eyes*; for the Rays of these are often defiled by vicious Humors, as when all things seem Yellow to those that have the Yellow Jaundise: if a thick and gross Humor possess the middle of the Sight of the Eye, all things appear bored or pierced thorow, if this Humor be dispersed and black, all things seems full of Flies; If the ChrySTALLINE Humor be impure, all things are thought to be seen through a Cloud, or if it be divided, all things appear divided.

CHAP. XII.

Of the Vision of Shadows and of Darkness.

1. *Darkness cannot be seen.*

D For it is wholly the privation or absence of Light:

Light : Which absence either of Light, or of any thing, cannot be perceived of Sight nor any other Sense ; but only by privation, (that is, by the removing, or by the denying of it to the thing of which it is the Privation) or absence : As therefore Silence is not otherwise known, than by the removing of Sound ; and in general, not a Being, otherwise not known but by the negation of a Being ; so Darkneſs cannot be apprehended otherwise than by the negation or removal of Light.

2. *A Shadow also may be seen from another, to wit, from the first Light being absent, but the Second present.*

For a Shadow is not the Privation of total Light, but only the lessening the same, by the Interjected opaque or shady Body ; by which, the first, flowing from the Lucid Body is turned away, but the second left ; or being reflected from other Bodies, which is weaker than the first Light, yet the presence of it seems the Shadow.

3. *As Light weakens and hides Light, so a greater Shadow weakens and hides a less.*

Therefore in the night the Moon being present, the Shadows of η or φ are scarcely or not seen, which notwithstanding may be seen the Moon being absent.

4. *A Shadow being added to a Shadow encreases it, and it becomes Darker.*

For as Light is, so is also a Shadow, either *Prime* or *First* ; which is begat by the Privation of the first Light ; or *Second*, that which is also procured of more, and also by the absence of the second Light ; whence that participates of more Light, this of less.

5. *Night, is a Shadow and Darkneſs.*

For God himself called the Night Darkneſs, *Genesis* 1. But Astronomers call this the Shadow of the Earth ; and both rightly : For if from Light which Lightens the whole World, we can behold the whole Earth, a part of it being turned from the
Sun

Sun and wrapt up by the Night, the Shadow would be perceivable by Sight: But because such aspect is denied to us, and we are wrapt up by the Shadow of the whole Earth, neither also can we discern Light at the Sides, even as otherwise is done from Shadows: Therefore Night, especially without the Moon; and the starry Heaven being cover'd by Clouds, is accounted meer Darknes.

6. *A Shadow moves not, although it seems to move; it being either from the Motion of the Light, or of the opacous Body.*

For 'tis only the Privation of the first Light, in which, Motion happens not: And from a Body being moved, either Opacous or Lucid, always one and another Shadow is produced, from the former vanishing; which the Eye, by reason of equality, thinks to be the same with the former: When therefore a Shadow is said to depart or proceed, tis to be understood according to common speaking, and not to the very truth of things: The thing is otherwise with Light, which moves together by the fountain being moved, and Coheres inseparably to its Fountain.

7. *A Shadow Imitates the Figure of the opacous Body, as far as it can.*

The more direct an opacous Wall and a Candle are sited to each other, the truer the Shadow Imitates the Figure of the opacous Body: On the contrary, the more Oblique, the more it deceives; so if the Shadow of a right lined Figure fall on a Sphere, it will seem crooked.

8. *A Shadow cast on a part opposit to Light, is terminated by the Lucid Rays.*

So the Sun in the East casts its Shadow into the West: And the Sun being in the South casts its Shadow into the North: The earth existing in the middle of the Zodiac, casts a Shadow into the place of the Zodiac, by the Sun being directly and diametrically opposit, as is seen in Eclipses of the Moon.

9. *The Extreem of a Shadow cannot be seen exactly.*

Which happens for two causes: For 1, contiguous Light which is apt to communicate and diffuse, insinuates it self to the neighbouring Shadow, and is mingled a little with it: 2, The
terminating

terminating Shadow of the Lucid Rays are many, flowing as one, from every Point of the Lucid Body, one of which lifts up or takes away another.

10. *We discern some things being placed in Darknefs; not Interchangeably.*

For means, or Mediums, are not necessary to Vision, but only the Illustration of Visibles is required, wherefore by Night, we behold the Stars from Darknefs; and in the day time, from most deep Wells or Pits, we discern the inaccessible Stars by the diurnal Light: So in *Egypt*, the *Egyptians* could see from the most thick Darknefs, the Land of *Gosen* being Illustrated, but the *Israelites* not back again what was acted among those.

11. *Also Pellucids may cast some Shadow, but in a thinner Medium, than the Pellucids themselves are.*

So a watery or chrySTALLINE Globe, in the Air opposit to the Sun, casts a Shadow, although thin; so the Air it self also makes a Shadow, but only perceivable in the Sky; which way the Moon being deprived of its Light, is the Shadow of the Air, not of the Earth, *Kepler* Teaches and Demonstrates.

12. *There are as many Shadows as Luminous Bodies, in respect of Opacous, in an indirect Site or Situation.*

So if you place only one opacous Body in a Transverse Site to three Candles, we shall see a three-fold Shadow of it: So in the night, the Heaven being serene, one opacous Body casts so many little Shadows, as there are lucid Stars Shining in the spacious ambit of Heaven: But these either are not visible to any one in open view; because that the blacker Shadows of the Moon and of ☿ surpass them; even as in like manner, the splendor of them exceeds the splendor of the other Stars and obscures them: But if luminous Bodies are sited directly, and the Opacous be put in the same right Line: They beget only one Shadow, because those many Luminous, are as it were one.

13. *The length of a Shadow varies, according to the diverse altitude of the Luminous Body.*

It is longer by how much the Light is lower, with respect to the vertical Circle; shorter, by how much higher; none at all, when 'tis Vertical.

So with us, the Noon Shadow is the shortest; but the Morning

ning and the Evening the longest; because the Sun there is higher, here lower: Nor it Matters not whether in respect of distance, it be nigher the Earth, or remoter from it: But in the Torrid Zone, to which the Sun becomes Vertical, those cast no Shadow from themselves.

14. *The Shadow of an opacous Spherical Body, being greater than of a Lucid Body, is a Chalathoeidis, See Fig. 208.*

So called from *Calathi*, or an inverted Pyramid, or the Figure of a right lined Top, and the lower part of it, is about the opacous Body very narrow; but by how much 'tis stretched out further, 'tis rendred larger and thicker: For the Rays of Light, touching the extreams of the Diameters of the opacous and luminous Bodies, the longer or further they are produced beyond the opacous Body, the more they depart from one another, and are further distant; whence the Shadow existing very little at the opacous Body, is spread out by little and little, even infinitely.

15. *The Shadow of an opacous spherical Body, being equal to the Luminous, is a Chylindroeidis.*

For it hath the form of a Cylinder, or of a round Column, and tis cast of equal thickness *ad infinitum*; for since the Diameter of the opacous Body is equal to the Diameter of the luminous Body, and therefore the Rays join the extreams of them, they become Parallels, and being infinitely produced will never meet, but always be distant by equal Spaces between themselves.

16. *The Shadow of an opacous spherical Body, being less than the Luminous, is a Conoicis.*

That is, it hath the Figure of a right Cone or of a Pyramid, and is thickest next the opacous Body, but the further it goes forth, the more it lessens, until at last it ends in a Point: For the extream Rays of the Lucid Body falling on the opacous Body, illuminate more than half of it, and being produced beyond the opacous Body, they meet, and at last are sharpened to a Point.

17. *Shadows*

17. *Shadows of unequal and parallel opacous Bodies, being terminated by the same Ray, or by divers of equal height, are proportionals to their altitudes, Fig. 209.*

For let the two opacous Parallels be de and bc terminated by the same Ray bda , I say that as the shorter ae , is to the length of its Body de : So also is the longer Shadow ac to the Altitude of its Body bc : Whence from the Altitude of Shadows of Bodies, may be gathered what we have said before in *Geometry* at pag. 216. and the following.

C H A P. XIII.

Of the Vision of Pellucidity or Shining through.

1. **A** *Pellucid is not seen, but perceived by the privation of Colour.*

So we see not Air in Air, Water in Water, Glass in Glass, and every Pellucid in an equal Pellucid; and because we know they are not coloured, we count them to be diaphanous, viz. that may be seen, or shone thorough.

2. *Yet by condensation or thickning in a thinner Medium, or by rarefaction in a thicker, they may sometimes be seen.*

In the former Case, 'tis as it were Opacous, by reason of thickness and condensation; and as often after that as no other solid Occurs, so often it may be discerned, although weakly: In this manner we discern Chrystal or Glass in the Air: So also that Air which is compassed about by the Lunar Body like to our Air, we may see, because it is in a thinner Medium, to wit, constituted in the Sky: The which the fixed Stars when they are nigh to the Moon testify, which being a little entered may yet be seen: So lastly we obtain by Sight, the remotest Air from a *Serene Heaven* by condensation, in the same manner as a Cloud beheld afar off, we behold with our Eye to be near, when nevertheless while we walk in it, we very hardly or not at all discern it. • We

We have an example of the latter case, when the Lucid Rays of the Sun are beheld entering through a Window, which although by reason of the congened heat, they rarefie the Air, yet because they draw plenty of Light with them, they also terminate the sight.

3. *In Pellucids, not only the Superficies, as in others; but the whole Solidity is seen.*

For in these, the Rays subsist not in the Superficie as in other opacous Bodies, but also they penetrate the inner Parts, and shew them to the Eye.

C H A P. XIV.

Of the Vision of Magnitude.

1. **H**ow much the Angles are to the Beholder, so much the thing appears, especially if the distance be unknown.

For Vision is made by Angles; therefore if the Angle of Vision is great, the thing also appears great; if small, it appears small: And if two things are seen under an equal Angle of Vision, they may be thought equal, although indeed they are otherwise in themselves: So the Hand or the Finger, if we move them nigh to the Eye, may seem greater than a Mountain or a whole City, because the Angles are such: So the two Luminaries ☉ and ☾ being seen almost under the same Angle, may be esteemed equal, because the Sight knows not the unequal distance of them: So lastly ♀ and the Dog Star seem to the unskilful to be of the same Quantity, for the same cause, because they know not the unequal Distance; and all the Stars may be thought to be fixt to the same Orb.

2. *Of Equal things, that which is nearest is thought greatest, if they Shine upon, or irradiate the Eyes directly.*

The reason is, because the Angles of things near at hand are the greater: So to a Traveller, a remote City seems little, but coming nearer to the same it seems greater, because the Angle of Vision is increased; the City remaining alike in it self.

3. *Yet the difference of Distances is greater, than of the Magnitudes of Angles.*

Hence

Hence it comes to pass, that equal Objects unequally remote, are seen, not under the same proportion of Angles of which the Distances themselves are, nor, for Example, being moved doubly nearer, do they therefore also appear doubly greater; so although the Moon in Apogæon should be doubly further from the Earth than in Perigæon, yet she is not therefore doubly greater in Perigæon in her apparent Diameter: Nevertheless the Star of Mars, because of the great variety of its Distance, also its Magnitude indeed not really, but to our Sight varies notably: For sometimes when 'tis at the highest 'tis scarce comparable with Stars of the second Magnitude: But sometimes when it comes nigh to the Earth, it encreases so much that 'tis equal to *Jupiter*, nor can it be discerned from it, unless by its Place in the Zodiack, and by its golden Colour: And by the Ignorant, is accounted for an extraordinary and prodigious Star.

So it came to pass in the Year of our Lord 1529 in which *Solyman* the Emperor of the *Turks* sitting down against *Vienna*, that being attested by *Philip Melancthon*, in Book 1 of *Physicks* in which he wrote of *Mars*.

It was also retrograde and of an unusual Magnitude, and its Body was beheld of an horrible red Colour, by reason of its nearness to the Earth, so that many being Ignorant of the Causes, thought it to be a new and prodigious Star in the Heaven, also some believed it to be a Comet, which also came to pass concerning it in the Year 1653.

5. The Eye sees less of a nearer Magnitude than of a Remoter.

The reason is because the capacity of the visive Angle hath a sure bound, nor doth it arise to a right Angle, much less to an Obtuse, Fig. 210.

Therefore although the Eye coming to a Visible, thinks to see it more; because nearer things are beheld under a greater Angle of Vision; yet in truth it sees less, because the Angle of Vision, although

though greatest of all, can no more include the whole visible, since tis become obtuse, under which nothing can be seen.

The reason is the same of Spherical Bodies, concerning which, the Eye being remote, sees more; but being nearer, sees less, as is seen in Fig. 210, in which the visive Raies of the Eye being remote, approach nearer to the Diameter of the Globe, than if the Rayes of the Eye had been nearer.

6. *Magnitude is varied by obliquity, by the Night, by Light, by Twilights, by the Eye, by the Medium, &c.*

1. For the more oblique the aspect of a thing is, the less the Magnitude of it appears, and by it all things are more curtailed and made shorter.

2. That Lucid things in Darkness seem greater, is said before: But on the contrary, because Light spreads and enlarges it self every where, it comes to pass, that contiguous Opaques are somewhat cut off and taken away: for if you appoint a Staff nigh at hand to be laid over against a candle, but you look on it afar off, it will appear, where the splendor passes, snagged on both sides, and as it were made hollow by notches and gaps.

3. Also according to the various Disposition of the Eyes, the same thing may be seen one while greater, another while less.

4. A Medium also that is thick, increases Magnitudes: and one that is thin, diminishes them, as shall be shewn hereafter.

CHAP. XV.

Concerning the Vision of Place.

1. **I**N a Spherical, the lowest place is the Center; the uppermost, is whatever departs from the Centre towards the Superficies.

For since the highest and lowest are distinct, and there is no reason, if we would seek these in a Superficie on every side alike to it self, why we should

call one point the highest, another the lowest; tis necessary that of these, viz. of the highest and lowest, we should assume one in the Center, the other in the Superficie: Because therefore the Superficies of the whole Spherical World is accounted by all for the upper part, it follows, that the Center of the World, and therefore also of other Sphericals is the lowest place.

2. *The Eye, in respect of the World, wheresoever placed, is always in the lower place, and they are Superior to it, whatsoever depart from the Eye.*

For every Eye wheresoever placed, by the imagination Frames a Sphere about it self, placing it self in the Centre: which if therefore the Center is the lowest place of the Sphere, the Eye always will be in the lowest place, all the rest are in the upper places: so if we Men are conversant in the Center of the World, all other places of the World will be above us, of which no one doubts: And if any one should conclude, because we account all things above us, we therefore are conversant in the Center of the World, he would argue amiss: for if beyond the Center of the World we should be elevated to Jupiter or Saturn, there also we may conceive a Sphere about us, and we should describe it by the Eye, and therefore we should be notwithstanding in the lowest place.

But because those things which are beneath our Feet, we call below; that is true in respect of the Center of the Earth, of our abode, some part of which we are; but elevated above the Center of the Earth: Therefore we truly say, all things even to the Center are beneath us, in which Center if we should be, those things would not be above, which now we say to be beneath us: Or if we should remove the whole Earth by Imagination, then we should be in the lower place, having still nothing beneath us: That also ought to be known from the

custom

custom of common speaking : for commonly those things are called rightly, Superior or above, to the seeing of which we lift up our Eyes, and those inferior or beneath, in the beholding of which we have need to depress or cast them down : and hence it follows, if we should inhabit in the lower Hemisphere of the Globe of Saturn, and we would behold our earth, it would be to us by lifting up the Eyes, and therefore the earth would seem to us to be sited in the Superior place.

3. *Such as is the Site of the Rayes, such the place seems.*

For those things which are beheld by Rays to the right hand, to the left hand, to the Superiour ; are seen on the right, on the left hand, or Superior, or Inferior, altho' in truth they have equal Site.

Of two things placed before us in the same right Line Parallel to the Horizon, that which is further seems higher, the nearer, seems the lower ; because that is discerned by a Superior Ray, and this by an inferior one : In the same manner, of two things placed in the same right Line Parallel to the Horizon on our right side ; that which is the furthest seems on the left side, and that which is nearest on the right ; because that is beheld by a Ray to the left side, but this by a Ray to the right side : From this deception of sight, an erroneous opinion is begat, whereby *the waters are thought higher than the Shores or Banks*, when in truth they are lower : For if standing on the Shore, you Measure by an Instrument the Level of the Waves, all which also the extream sight obtains, you will in no wise find the elevation in them, but always some small declivity or leaning downwards ; and it will appear, that the Waves are not above the Earth or Shores, but subside or remain beneath the same : Therefore the Sensitive estimation is deceived in this, that the extream parts of the Waves are beheld by the Eyes more elevated than the nearer parts ; whence also they think them to be higher than the nearer : yet nothing hinders, that is said, less concerning these things, than concerning others, not as they are indeed, but even as

they appear to our sight; the which not only happens in common speaking, but also oftentimes in holy Scripture it self; as when Christ commanded *Peter* the Fisher to Lanch into the Deép and to loose his Net to Catch, *Luke 5.* also hence it follows that,

4. *Things afar off are more like to contraries: that is,*

Being further continued on the right hand, they seem on the left, and the contrary: also things above, the further they are from sight, they are thought lower by it, and interchangeably: So to one standing on the Bank of a River, it may come to pass, that the River may seem to run not downwards but upwards, because the site of the Rays is in such manner.

5. *When the Image of a thing seen, is in the Eye, yet the sight judges divers ways concerning the distance of the thing it self.*

1. *From the Light being stronger or weaker.*

Light from afar, comes Languid and weak, but from near at hand it comes strong; which gives the discovery of distance.

2. *From things put between the Eye and the thing seen.*

For not only some thing is seen from afar, but also those things which are placed between that and the Eye, whence the *Ratio* or Reason of distance is gathered.

3. *From the Air being between.*

For this also is Represented to the Eye: for since it is transpicious; from every point of it, the Rays of Light recoil or are beaten back to the Eye: Therefore the parts of the Air next to the Eye, appear near, those that are distant, appear so; lastly the Mediums are Represented in the middle or mean: Even as also in a Looking-Glass, not only our countenance, but also the space placed between us and the Glass, is received into it, which is made less or larger, according as we approach to the Glass or depart from it.

4. *From the distance of both the Eyes.*

For two Eyes are given to living Creatures, not in compensation of one being lost, as is commonly supposed, since Nature designs nothing to be lost or damaged:

damaged : but for the knowing the distances of things, by the Geometrical Dimension of a Triangle.

For as in Geometry, from two Angles of a Triangle given, with the interjected side, the other sides are given : So by light, a Triangle is constituted, the two longer sides of which, are the Rays from the point seen, conveyed to both the Eyes, and the third lesser side, is the distance of the Eyes, which with the two longer sides, form two Angles in the Eyes themselves : Therefore in this Triangle, the lesser side, to wit, the distance of both Eyes, is known by the accustomed common sense : But the Angles at that distance become known, from the sense of wresting about of the Eyes ; for the more they are wrested about, the Angles are made the less, and they argue the thing to be the nearer ; And the less that the Eyes are wrested about, the Angles are made the greater, by the discovery of a greater distance, yet of having sensible proportion to the distance of the Eyes : For if the excess be very great, the Axes of the Eyes are in a manner directed Parallely, neither do they comprehend that distance otherwise, than any other greater by infinite parts.

Nevertheless this faculty of knowing distances, arose at first from this Society between the two Eyes, also afterwards it was derived into each eye ; since by the other affections of the Eye, as also by the motion of the head ; by which Motion, one Eye being moved, may be instead of two.

6. *From many things put between the Eye and the thing seen, it seems to be further off.*

Hence the Stars in the Horizon, or constituted nigh to it, appear remoter ; because many Tracts of earth are placed between : as Mountains, Vallies, Towered Cities, Rivers ; which in some Measure they shew the vision of a huge distance of the Heaven : On the contrary, Vertical Stars, or of the Meridian, are thought nearer, because while the Eyes are raised upwards, we behold nothing in the middle between (besides the meer dia-

phanous Heaven) from which we estimate the distance of the Stars : So Clouds, appear higher in Hilly places than in Planes, as in *Helvetia*, than in *Alsatia*, for the same cause.

7. *The same thing being near, seems in divers places, by Reason of the Parallax of the Eyes.*

Even as before in Astronomical things at Pag. 349. we have said concerning the Moon, which by Reason of the Parallax, if she be beheld from the Superficie of the Earth, appears in one place in the Heaven; if from the Center, in another : So the same happens to every thing seen, by Reason of the Parallax of the Eyes, for suppose in that Scheme Fig. 166. A and B to be the two Eyes of one Man, and C the thing seen, which will appear to the Eye B, almost under O, but to the Eye A, under D; The whole of this thing, and the palpable Example, a Man may take from himself; for hold the head unmoved, shut the Eyes by turns : being opened behold the Nose, if you look to the right Hand, you will behold the Nose to the left hand, if you look to the left you will see the same to the right hand part of the Pavement : And so as often as you change the Eyes, the Nose will seem to make a leap, although it rest : The same also happens if any thing be set far distant by beholding it with either Eye.

C H A P. XVI.

Of the Vision of Number.

1. *From two Eyes being well, and moved together, one thing seems.*

Indeed all Images of things seen, are painted in both Eyes : And that one thing seems one from two Eyes, the cause is, not the Union of the two Nerves of the Eye, as some think, for as much as by two Eyes also we may see two Images of one thing, notwithstanding the Nerves being conjoined ; but the true Cause is, the equal Site of the Axes in both Eyes, whence it is, that in like manner each Eye and the common Sense or Judge, accounts two like Images for one and the same, because the right Eye accepts not another Species, nor Sited in another manner, than the left, but altogether the same.

2. *Many small things seen afar off, may seem one contiguous thing.*

That happens in the Milky way, which is a Pile
of

of many very small Stars, which by Reason of their smalness, multitude and close coherence, no man can easily distinguish, unless by using an Optic Tube, or Prospect-Glass : So likewise it comes to pass in Letters Writ on Paper, which are seen from some interval, the whole appears of a Homogene and Ashey Colour ; or if any one make a hole through a Pot, and it being filled with live Coals, and being beheld afar off in the Night, the whole will seem burning : for the Sight by Reason of too much distance cannot discern so many small Particles.

3. *One Visible also may seem two.*

To wit, if the Eyes be distorted or set awry from their Natural Sight, and moreover if the Axes of the Eyes are not in the same manner to one another ; for then, not the same, but divers Images are imprinted on each. While therefore, the right Eye shews this, the left Eye shews another Image to common Sense, it comes to pass, that for one, two are apprehended : that which frequently is wont to happen to sleepy persons and Drunkards : The same comes to pass, if some thing be direct to one Eye, and you behold it obliquely by the other ; or if you Squeez one of the Eyes, or in any manner turn it awry : But the nearer the visible is, the more distant the two little Images will be from one another.

4. *And number is apprehended from the parting of Visibles.*

For from the Multitude of Visions, from the Magnitude of things, from the Motion, from the Site, from the place, and from other disagreeing known things of Visibles; the Multitude, and the number of Visible things are known.

CHAP. XVII.

Of the Vision of Figure.

1. **P**oints most swiftly turned about, seem Lines of the Circumference of a Circle.

The cause is from the swiftness of the Motion, and from the Similitude of space, by Reason of

which the Eye cannot apprehend the distinction of places.

2. *Of Parallel Lines or Equal Superficies, those seem less, that are furthest distant, and at last they are thought to meet.*

The Reason is, because of the Angle of the Beholder being directed to things equidistant, but being directed to things unequally distant, about nearer things they are made greater, about remote things they always are made more acute; and when at last they make a most sharp Angle, they are thought wholly to go into one another, and to depart from the Reason of Parallels.

3. *Circles seem to be right Lines; the Eye being placed either in the Center, or in the same Plane.*

For Points seen by high Rays appear high, and more remote from the Center, than those which are seen by lower Rays.

Hence Celestial Circles, of which the Center is our Eye, are accurately represented by right Lines; for example, the Meridian by the Meridian Line; for which cause also they are depicted only by right Lines on Sundials: For this cause also the Moon being bisected into Quadratures, appears not crooked as at other times, but terminated by a right Line.

4. *A Square Superficies seen obliquely, is thought to be an Oblong Trapezia: but being Circular, it appears an Ellipsis or like a Lentil, and the contrary, see Fig. 211.*

For all things in general, if they are beheld obliquely, they lose their true Figure, and the true Figure is only seen, if the Eye is perpendicularly direct to the Center of the Superficies.

5. *A Sphere beheld afar off seems a Circle.*

For since all Lines falling on the same Superficies, of a Spherical body, are seen equal; it is necessary that the Superficies of a Spherical body seem a Plane: So the Sun and the Moon which are Globous Bodies

Bodies, yet they seem to be plane Circles, just like Platters.

6. *Angular Figures being seen at a distance, appear round.*

For because every thing that is beheld hath a certain length of distance, which being filled up, is no longer seen; and therefore Angles, as the less parts, about which the breadth of the observed is less, they sooner vanish; and the Figure remains either round or nigh to a round: And therefore Square Towers being beheld afar off, are accounted for round ones.

7. *Of a Globe, the Diameter of which is equal to the distance of the Eyes, one half is seen: Of which 'tis greater, the lesser Segment is seen, of which 'tis less, the greater Segment is seen.*

For by drawing Lines from the Eyes to the objected Globe, there they touch in the extrems themselves of its Diameter: in that place on this side, but here they touch the Globe beyond the Diameter, whence:

8. *If the Eye come nigh to the Sphere, it indeed sees less of the Sphere, though it may fancy it sees more.*

For from afar off, the visive Angle is less, yet it comprehends more of the Sphere; but the Angle near at hand, is indeed greater, but comprehends less: Whence this part seems greater than that, by reason of the greater Angle, when in truth 'tis less by reason of the lesser comprehension.

C H A P. XVIII.

Of the Vision of Motion and of Rest.

1. **M**otion and the differences of it, since they are not proper objects of Sight, neither have they peculiar Sense, but are only judged by common Sense, from the comprehension of Visibles, under a certain difference of changing of Place and of Time: But Rest is seen under an unchangeable Site in sensible Time.

For

For if a Visible; shall have to an Eye constituted in the same Place, a Site at one time, differing to the Site of it at another time, we gather it hath been moved forward: If the Site be the same it hath rested: And from the Space through which the visible is moved, either by a crooked, or a right Line, the motion also is Judged to be either crooked or right.

2. *Things nigh at hand equally swift, either in the same, or on the contrary part, seem to rest.*

So if two Fishermen in the lower Necker, each of which rowing in his Skiff, nigh each other with an equal motion, one seems to the other to rest (supposing all other Circumstances are the same, that is, not having the reason of the Shoar, and other immovables of Seeing nigh at hand) because altho' they Sail or Row together, yet one Skiff to the other, appears Sited in the same manner, viz. to the right Hand or to the left Hand, and therefore no Motion seems to be.

The same happens in two contrary Motions, if they are alike Swift, that they abolish one another mutually, so if a barge Man in his Barge, shall Ascend, even so much against a River, as his Barge Descends according to the River, then being beheld cross-wise in a green Field he will always seem to remain in the same place, in respect of the Shoar, and therefore to rest: In this manner also, from the Hypothesis of *Epicycles*, the Planets in the Heaven seem to be Stationary or Retrograde, although they proceed by their perpetual Motion.

3. *The distance of VISIBLES being unequal from the Eye, but of equal Motion: The Motion of the more remote, appears the slower.*

For the Space of the more remote is discerned under a less Angle of Vision, but of the nearer, under a greater; whence that is thought less, this greater, although in truth they are equal.

4. *A moveable, being more remote and equal in Swiftness, sometimes also it will seem to go back.*

For when 'tis nearer it seems to move swifter, being remote, it will seem afterwards to relinquish it self; which relinquishing, where the Species of Motion is not discerned accurately, is thought a going back.

5. *An Orb in very Swift Motion, seems to rest.*

For because of the velocity of Motion, the Eye discerns not some and the other Parts, and when it returns most swiftly to the former Site, it seems altogether not to move, but to rest; so an artificial Wheel, turned swiftly through large Streets, is thought also not to be moved by a circular Motion about the proper Axis, but only to be moved by a right Motion: the same may be minded in a Top, and the Wheel of a Potter.

6. *Most slow Motion (either true or apparent) also seems to rest.*

So the fixed Stars seem to rest, neither should we know them to move, unless by the benefit of reasoning we gather it from this, because in a morning they are conversant in another place than in the Evening: The Cause is this, because although indeed they are moved swiftly, viz. according to the Hypothesis of the Ancients, in one Minute of an hour 75479 German Miles: Yet to us who comprehend this huge Space by a small Angle of Vision, they seem to move most slowly; which Motion in a small time cannot be perceived: So also any Index of a Clock or Watch, we cannot discern it to go forward, but we gather the Progress to be, from the place being changed.

7. *Anything Proceeding slowly, seems to go backward; but proceeding swiftly it seems to run forward.*

So the Moon indeed being moved more swiftly, she apparently moves more slowly than the Clouds under her; seems at another time to move most swiftly.

8. *A thing near at hand wholly standing still; seems to meet with you or to go backward, by coming to it or departing from it most swiftly,*

swiftly, and beholding it obliquely; but being most remote, it seems to proceed, all under one.

If any one Rides nigh Trees, or Sails over a River approaching to Trees, or to the Port or Harbor, they seem to meet with him; if he depart from them, they seem to depart; so the Moon by reason of the unproportionate distance from the Earth, seems in the Night to make a Journey with the Traveller: Because, always after some and other Mountains, it is beheld Sited in the same manner: Thus supposing one could be transferred into the Moon, whose Motion is a cause of the greatest Velocity, then the Moon would seem to stand still, but the Earth to move, with the same Motion that the Moon does, but the contrary way.

9. *A Thing at rest seems to move, either from the Motion of the Head, or from an Heterogene mean or middle, being moved: And all things are thought to turn round.*

For from a swift Motion, the Eyes are so affected, that if either they behold for some time, a steep Torrent, or a Potters Wheel, or any such like thing in swift Motion, they think other Immoveable things also to move with the same Motion, the Eyes being turned to them; for the optic Spirits being vehemently agitated by that turning Motion, as yet run hither and thither, although the Body rest; or a Top that Boys Whip, which by reason of the communicated force, Runs or Spins for some time although the Hand of the Agitant rest: Or even as from the swift course of a Ship conveyed, although for divers Months she be at rest or lye in an Harbour, yet still it seems to move it self: Likewise if we behold somewhat, through a moveable loop hole, you will think a thing that is at rest, to move in the same manner.

10. *Equal circular Motion, seen from another than the proper center, appears unequal.*

For

For from any other Center, the moveable will pass through either equal Arches, in unequal time, or in equal time, unequal Arches; and therefore will seem to move unequally.

11. *The Motion of a Man, which is not perceived in the Body, by the Argument of Succession; or which he knows not to proceed from his inward moveable faculty, is accounted for rest.*

So when a Man is snatcht away from the slow and equal motion of a Ship, himself resting, he will think the neighbouring Shoar to move towards him.

12. *Those things which appear greater, and either exceed the Eye, or Possess a great compass, are thought to rest; but those things which in respect of them appear little, those on the contrary seem to move.*

So the most small Stars, according to Sight move swiftly between great Sheets of Clouds, but the Clouds are esteemed to rest.

13. *As the inequality which shews a moving thing sometimes in a swifter, sometimes in a slower Motion while it performs its Revolution, is begat from the distance of the thing seen, from the Principle of equality, that is from the Center; so that inequality which shews a thing sometimes going forward, sometimes backward, argues not only the distance of the thing seen from the principle of Equality, but also the Motion of the thing discerned about that Center.*

For the Progress and Regress are begat from the reciprocation or going back of apparent Motions, and from the various Position of the Motion of Seeing to the thing seen, being moved unequally in respect of him that sees: From these and such like optic Axioms the Followers of Copernicus, defend their Astronomical Hypotheses: By which they dispatch a wonderful Compendium, those things which others scarcely represent by many Wheels and Circles.

Part. II. Of CATOPTRICS.

C H A P. I.

Of Definitions.

1. **C**atoptric, is that which treats concerning Reflected Vision.
2. *Reflected Vision*, is that which comes forth from some Superficie, the Rays being Reflected, or beaten back; to wit, from a Looking-glass or Speculum in the same Medium.
3. *A Looking-glass* or Speculum, is a thin body, polished by Nature or art, Reflecting the Images of things, and casting them before the Eyes, from which, right Lines may be drawn into it.
4. *Polishing*, is the continuity of the parts of the Superficie of the Polisht body, without sensibility of Pores or of Division.
5. And a *Looking-glass* or Speculum, is either Natural or Artificial.
6. *Natural*, is that which Nature it self produces and Polishes.
Such are Air, thinner and thicker, Clouds, Water, some Gems: The Eye also, in which any one may contemplate himself, as in an Artificial Looking-glass.
7. *Artificial*, is that which the industry of art hath prepared.
These things have their Original from natural things, when art Imitates nature and is as it were the Ape of it.
8. Moreover Artificial Mirrors or Speculums, are either Regular or Irregular.
9. *Regular*, are those which have Uniform Figure: *Irregular*, which have deformed.
10. *Regular*, are either Plain, or Convex, or Concave, viz. of which the Superficie is Plain, Convex, or Concave.
11. *Catoptric Radiation* or shining, is in this manner, viz. from a Visible Ray of Incidence on a Mirror or Speculum, and from the Ray of Reflection from the Mirror it Radiates on the opposite part.

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12. *A Ray of Incidence*, is that, according to which, the Species of the Visible thing falls on the Superficie of the Mirror or Speculum : For Example ; see Fig. 212. c falling from the Ray c e, on the Glas or Speculum f e d.

13. *A Ray of Reflection* is that, according to which, the Image of a thing is Reflected to the sight or the Eye, as e a.

14. *A Point of Incidence* or of *Reflection*, is the common Term of these Rays, as e.

15. The perpendiculars also in this place to be considered are three.

Of Obliquation, of Incidence, and of Reflection.

16. *A Perpendicular of Obliquation* is, that which being drawn out from the point of the Visible thing, on the Superficie of the Looking-glass, or Speculum, and being produced, makes equal Angles on both sides with the Glas or Speculum, as c f g.

18. *A Perpendicular of Reflection*, is a right Line falling from the Eye on the beheld Superficies at right Angles, as a d.

19. From these Lines, Angles are constituted ; one of *Incidence*, the other of *Reflection*.

20. *An Angle of Incidence*, is that which constitutes the Ray of Incidence with the Neighbouring Superficie of the Looking-glass or Speculum, as c e f.

21. *An Angle of Reflection*, is that which constitutes the Ray of Reflection with the Neighbouring Superficie of the Looking-glass or Speculum, as a e. d.

22. From these Angles, is made the *Superficie of Repercussion* or of *Reflection*, which is determined by three points (According to which, if they are not posited in the same right Line, necessarily it places the peculiar Planes, viz. from the Center of sight, to the point of the thing seen, and to the point of Repercussion or of Reflection a c e.

Of some affections of Rays, and of Catoptric Angles.

1. **T**He Reflection of Rays of Incidence is made on the opposite Part.

For even as a Ball, strongly cast against a Solid Wall, rebounds unto the opposite Coast or part, so much the more as the force of casting is greater : because indeed the passing further is denied to it, nor yet is all the moving force abolished : So Rays gone forth from Lucid or Coloured bodies, and being always apt to move and spread abroad, when by an objected Superficies they cannot proceed further ; they are Reflected into Another part, and in this manner they Labour and endeavour to continue their spreading abroad.

2. *A Ray of Incidence at the Superficies, to which it passes by Perpendicularly, or at right Angles, is Reflected into it self ; but falling Obliquely, tis Reflected, at an Oblique Angle.*

For since Reflection is made into the opposite part, and equal force Argues equal motion, the discovery or sign of which is the equality of Angles ; Therefore which way the force is moved, or by what Angle the Ray is moved from the Point of Reflection, in which the force is the same, it will moreover be moved, unless it be hindred by passing from a Reflective Superficie, or from a Mirror or Speculum, 'tis carried by the same force and the same Angle with the Superficie of the Mirror or Speculum, to wit, being right, tis carried into the opposite part ; which cannot be done, unless the Perpendicular Ray falling in at right Angles, be Reflected in it Self.

3. *An Oblique Ray falling on a Reflective Superficies ; is also Reflected Obliquely, and makes the Angle of Incidence and of Reflection equal.*

The demonstration is the same with the former : for let the point, *a*, Radiate obliquely on the Reflective Superficie *d e f*. the Ray of Incidence will be *a e* : Unless therefore the Reflective Superficies *d e f* resist it, it will be moved further even to *g*, by constituting an Angle *f e g*, equal to the Angle of Incidence *a e d*, but since it cannot proceed beyond

proceed beyond the Reflexive Superficie, it is Reflected in the point of Reflection e , to the opposite part c , in the Angle of Reflection cef , equal to the Angle feg , which is equal to the Angle of incidence aed .

4. *Reflections from thicker Superficies being polished and made regular, are perceived rightly alone.*

Thicknes is required, least the Rays penetrate ; And polishing, or Smoothness and Regularity, that they may Reflect more evidently : For such as the Reflexive body is, the same also is the Reflection ; from irregular and rough bodies, the Reflection also is irregular and rough, deformed and confused : But from polished and regular, the more genuine forms of things are wont to recoil or reflect.

5. *Light and Colour Reflected, are weaker, by Radiating directly.*

For being more removed from their Fountain, to wit, from a Lucid or Coloured body ; from whence, the farther Rays go forth, they are made the weaker, which also helps weakness, if they fall in obliquely ; for right motion also is stronger than oblique.

6. *Light Reflected, although it be weaker than the first light, which flows from the Luminous body, yet it is stronger than the Second Light, which proceeds from that Light by communication.*

Experience shews this, for if in some Chamber being illustrated by a second Light, a Looking-Glass be placed opposite to the Sun, that it cast the Reflected Rays on the second Light, it will appear most clearly whether of the two is stronger.

7. *Yet Reflected Light by being collected, may be made stronger than incident Light.*

For because force united is always stronger than dispersed, therefore also direct Rays but dispersed, are of less Efficacy, than Reflected gathered together : Hence tis that from some Glasses placed to the

Sun, Fire kindles, if all the Reflected Rays be gathered into one point ; which disperfed Rays would not do.

C H A P. III.

Of some affections of reflected Vision it self in general.

1. **N**ot the thing, but the Image of the thing, is seen by the Reflection in the Glass or Speculum.

For the Eye cannot see it self, because to the Eye, there is wanting, that whereby it should see it self: And the Eye which you behold in a Looking-glass, is not thy Eye, but the Image of thy Eye shining from the Glass.

2. Yet not always the genuine Image, but oftentimes light only confused, or fading Colour appears.

To wit, when in Reflection, there is some roughness and inequality of parts, as in a Wall Illustrated by the Sun, which doth not reverberate the Image of the Sun, but only Reflects Light into all parts of some obscure Chamber.

3. The Images of things are discern'd in Glasses or Speculums, not by some Impression, even as the Seal of Wax, and a blot impress'd on Paper: but in the same Plain manner whereby other things incur to the Eyes; but that here they appear without the place, and the Flux of Species, restrain'd by the Glass is reverberated to the Eye.

For 1. Speculums made of Iron, Steel, or Adamantine, also shew the Image from themselves by Reflection, which by Reason of hardness admit of no Impression.

2. If such Image should be impressed on the Glass or Speculum, it would be Reflected and discern'd, not at one certain place, but every where, neither would the place of the Image be varied according to the variated Site of the Eye, which nevertheless is done, as shall be Demonstrated by and by.

3. Neither lastly, can the Images of great things,

as

as of Mountains, and of Buildings, be comprehended in a small Glass or Speculum.

4. *Either from the moving of him that beholds, or from the moving of the Object it self that is beheld, the Image also is moved, or rather the place of the Image in the Glass, although it self rest.*

For the Superficies of Reverberating, or of Reflection is one, in which also the Center of Sight, and the Point of the thing seen, and the Point of Reflection are together; of which, from one being changed, all Vision must needs be taken away or changed.

5. *Many Men beholding the same thing together in the same Glass or Speculum; they behold it sited, not in the same, but in divers places; and there will appear in the Glass, so many Images as there are Eyes looking on, but to each Eye, not above one.*

For from one visible Point to one Point of Reflection in one Eye, there can belong but only one Ray: Therefore to another Eye there belongs another Ray, and another reflected Point, and consequently also it sees another Place of the Image, and another Image.

6. *The Image in the Glass or Speculum, resembles the thing, as to the Matter and Form.*

For if Matter, for Example, Glass, be Coloured or but a little Tinged, the Face also of him that beholds, will appear Tinged with the same: So if by reason of the Form, the Looking-glass be not exactly Plane, but makes a blunt Angle, or leaves off in an obtuse Boss or Knob, the Face will be beheld, divided through the middle, either as an Asses or a Dogs Face, or deformed with a Swines Snout.

Hence also it comes to pass, that right Lines in convex Mirrors or Glasses, appear crooked; so if you look on a strait and polished Sword, the Face will seem an Oblong, according to the manner of the Polishing of it.

Which reason of Polishing, also helpeth or abateth much to the Image : For to what difference of Position, the Pores are smoothed or Polished, 'tis necessary also that the Images should be formed according to that difference : From whence, Mirrors or Speculums are wont to be Polish'd circularly, accordingly to which manner, fewer Deformities appear : But in the general, the more like in Site and Figure, the Visible or Thing seen, and the Mirror or Speculum is, the truer the Image will be.

7. *Often times we behold the differences of place, turned upside down.*

The right Hand appears the Left ; and that which is the right Hand part of the Members to us, seems in Mirrors or Speculums the Left : If you place a Mirror, or Looking-glass, or Speculum, opposit to a Book, you will see in it, the Letters not only Inverted, but after the manner of the *Hebrews* to be read from the right Hand towards the Left ; so also, upwards appears downwards, as we see Buildings in running Waters : But first of all, concave Mirrors or Speculums deceive very much here, in which the Visible is sometimes turned, sometimes inverted ; except when the Image of one Mirror, be reflected into another Mirror or Speculum, and so again inverted ; for then we behold truly the Image of the Image.

8. *An whole Mirror or Speculum shews one Image, except concaves and uneven ones : But a broken Mirror Multiplies the Species, if the parts of it lye not direct or strait.*

For although in any Mirror or Speculum, many Rays may light upon it from the objected visible Thing, yet not all of them will be Reflected to the Eye, but another way : And therefore from one reflected Ray, but one only is seen : Yet concave Mirrors are hence excepted, in which, the same Visible, may have many Images ; and Uneven or rough ones are also excepted, which not always shew the Image, but sometimes the Light only, or a confus'd Colour.

But if the Mirror or Speculum be broke in Pieces, and they

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lie not direct or straight, but so, that all are Perpendicularly opposed to a middle Point, radiating between the Eye, and the thing in the Glass or Mirror; Then the Eye will see all the Images of one Visible, in several broken Pieces; and by one Person looking on these Pieces, several People will seem; for the Angles of Reflection and of Incidence, in this manner will be made equal and meet in the Eye: But if these broken Pieces of the Looking-Glass be sited directly or straight, then the Visible or thing seen, will appear in one Image only, the other Images will reflect towards another place.

9. *Images are seen to stick or adhere, not in the Superficies of the opposit Glass or Speculum, but to some Interval behind them.*

This is manifest to all Men that behold a Mirror or Looking-glass accurately: For the farther any thing is from the Glass on the out-side, the farther also it will be discerned to be sited from the Superficie of the Glass on the in-side: And even as we depart from the Glass, or draw nearer to it, the Image also it self, will seem to depart into the Gulf or depth of the Glass or Mirror, or to draw near from the inner depth to the outer Superficie: The Cause of this, *Th. Campanella* assigns thus; That not only our Visage, but the space of Air also, placed between us and the Mirror, and other things near, standing round about, Shine again into the Mirror, which also in the Glass shew some Imagination of distance: But the true and optic Cause of the Place of the Image, may be gather'd from the following Theorem.

10. *For the Images of things reflected from the Glass to the Eye, are seen in the concurrence or meeting of the Perpendicular of Incidence, and of continued Reflection, Fig. 212 and 213.*

In this Scheam let the Mirror or Looking-glass be *fcd*; The thing appearing in the Mirror *c*, reflected from the Point *e*, to the Eye *a*: And from the Point *c*, drawing the Line *cfg*, which let be Perpendicular to the Superficie of the Mirror or Glass: Then continue the reflected Radius *ae* unto this Perpendicular, in the concurrence or meeting of which, to wit, in the Point *g*, the Image of the Visible thing *c* will be discerned. 223 The

The reason is the same in hollow Mirrors, as in Fig. 214, for where the Perpendicular of Incidence bd , and the continuation of the Reflection ea , meet in c , without the Superficie of the Mirror or Speculum, there the Image will appear as it were hanging in the free Air: And this is the reason of it: 1, Because the Eye, although it perceive the Reflected Rays, yet the Reflection from it self, which is made in the Point e , it comprehends not at all: 2, And because at another time, concerning the coast whence the Visible arrives, he Judges from the direction of the Eye to the visible or thing seen, according to the Ray which falls into the Eyes: Therefore also in the Reflection of the Coast or part of the Image, he judges from the reflected Ray, not subsisting in the Point of Reflection e , but in Planes going forth further than the Mirrors, even to the same Perpendicular bcd , which from the Visible (which there is e , but here b ,) falls in on the reflecting Superficies, and Terminates the reflected Superficies.

CHAP. IV.

Of the Properties of Vision, Reflected from Mirrors or Speculums in Particular.

I. **O**F Natural Mirrors or Speculums.

1. *In close and thick Air, one may see the Image of himself.*

That is wont to be done sometimes in Evening and Morning Twilights, especially by those whose Sight is dull or thick: Even as *Aristotle* writes of *Antipheron*, Book 3 of *Meteors*, that he saw his Image always reflected from the Air.

2. *Those that Inhabit in Higher Places may see the Image of the Sun beneath them.*

To wit, if Vallies are filled with thick Clouds, they who inhabit in an higher Mountain, may see the Image of the Sun reflected from a Cloud.

3. *In the Eye of another, one may see the Image of himself.*

For such little Images are not those Images from which Vision is made in the Eye, much less any thing sticking in the Eye, as the Vulgar think: And old Women, in the troubled Eyes of whom these kind of little Images appear not, from unadvised

vis'd Judgment, they are accounted for Presaging ; But they are the Images of the Man standing before the Eyes of the other, which are depicted in his Eyes, as it were in a Mirror or Looking-glass.

II. Of Artificial Mirrors or Looking-glasses.

4. Natural Mirrors or Speculums, are the Original of Artificial ones.

For unless Nature had given Men some hints before, and taught them that Plainness or Slickness, and Umbrage or Shadowy, are necessary to Mirrors or Speculums, perchance Man had not arrived at the art of making of Mirrors or Looking-glasses.

III. Of Plane Mirrors or Looking-glasses, otherwise called Speculums.

5. In whole Plane Mirrors, one Image only appears.

Indeed, one Point of a Visible falls in by many Rays on the whole Glass, all which also, according to the manner of Incidence, are reflected to the opposite Part: But because not but one Ray alone is reflected to the same Point of the Eye, because from one Point to one Point, but one right Line only can be drawn ; And that which is the reason of one Point, is the same also of all the rest : Hence it comes to pass also, that the whole Visible reflects only one Image to the Eye.

6. The continuation of Reflection at the Image, is equal to the Ray of Incidence.

For the Point c falls on the Mirror fed , by the Ray of Incidence ce , which is reflected into the Eye a : If this Ray of Reflection be continued, this continuation eg will be equal to the Ray of Incidence ce : For because in the Rectangled Triangles cef and gef , the two sides cf and fg are equal, and the side fe common to both ; Therefore by *Element 1 Prop. 4. of Euclide*, the base eg , the continuation of reflection, will be equal to the Base ce the Ray of Incidence.

7. Hence in plane Mirrors, the distance and the true Image, is also equal to the Visible.

ZZ 4 For

For because Triangles from Rays of Incidence and of Reflection continued, likewise being made Perpendicular, are equal: The Image also and the Visible will be equal, as well in Reason of Quantity as of Distance; Therefore the Image in plane Mirrors is truer than in Gibbous, in which this equality hath no place.

8. *A Plane Mirror or Looking-glass, being Illuminated from some Lucid thing, reflects the same Rays, which the same Lucid casts forth at the same Site (in respect of the Glass) through the Glass it self, as if it were an hole.*

As if in the aforesaid plane Scheam; the Lucid *a*, should fall on the Mirror *def*, it would reflect on the Wall *efg*, the same Rays at *e*, which it would cast forth on the opposit part at *g*, if the Mirror were perforated, or a hole made through in *e*.

Hence, by a Mirror or Speculum placed Level to the Horizon, the Altitude of the Sun may be alike taken by reflected Rays, as by Rays of Incidence; And so the Angle of Reflection would be found by Experience to be always equal to the Angle of Incidence.

9. *The same thing may be seen in many Mirrors or Speculums, by one only and at once; and also if something be placed behind one, or in remote Places, whereby a direct admittance appears not to the Sight.*

That is done, if plane Mirrors or Speculums be so placed, that the reflected Ray reach not to the Sight, but be received from another Mirror, which delivers it again to the other, until by many intricate Passages, the last at length is offer'd to the Eye.

10. *Also one and the same thing, may be seen manifold and often times in many Mirrors.*

For even as a broken Mirror multiplies species as aforesaid; so the same also, augmented or multiplied Mirrors perform, yet not in a direct Site: For then, by reason of the distinct Superficies of Mirrors, also distinct Perpendiculars of Incidence are made, as well of divers Rays of Incidence, as of Reflection

to

to the Sight, to the number of which, 'tis also necessary to multiply or increase the Images.

11. *Two Mirrors or Speculums may be so placed, that in one you may see the Image coming to, and in the other, going away.*

That may be done if you joyn them at a right Angle, and the Eye fix'd over against the Angle: For the thing being moved from one extreame of the side to the other, it will seem in one Mirror to come to the Angle, and in the other to go from the same.

12. *In many Mirrors or Speculums placed together; from one Leaping or Dancing, you may behold a Ball of many Leaping or Dancing together.*

For if, of an Arch of a Circle, the Subtense Chords be divided into aliquot Parts, and upon each one of them, a Mirror or Looking-glass be Perpendicularly erected; whatsoever a Spectator acteth, being constituted in the Center, that will seem there, so many times as there are Mirrors: And if he Dance, In those Mirrors he will see many Dancing together, and each Mirror will have an uniform aspect to the Sight existing in the Center: And so also one Candle placed in the Center, will shew so many Candles as there are Mirrors.

13. *The Site of the Eye of him that beholds, is proportional to the Site of the visible thing.*

In the former plane Scheam.

As the Radius ea , to the Eye a , reflected from the Point e to its Perpendicular ad , or the Stature of the Spectator from the Eye to the feet: So is the Ray of Incidence ce , to its Perpendicular cf , and in like manner.

Again, as the distance ed , to the Altitude ad : So is the distance ef , to the Altitude cf , and the contrary: For the Angles of Incidence and of reflection cef and acd are equal: And on both parts is one right Angle cfe and

and *ade*, and therefore both the remaining Angles also, *dae* and *fce* will be equal to each other, and therefore they have proportional sides reciprocally.

From the Foundation of this Proposition we may measure the Altitudes and Distances of things by Mirrors or Speculums as is said before in *Geometry*.

IV. *Of Convex Speculums or Looking-glasses.*

14. *In these, the distance, and the Image of the Visible, falling obliquely, are always less than Just.*

That is, As the Image it self appears less than the visible thing it self is; so also the distance of the Image from the Superficie of the Mirror, is less than the true distance of the visible thing from the same Superficie: The reason is, because the Lines of Incidence terminating the same Image, meet and are made sharp in the Center of the Speculum.

But if the thing to be beheld, be Sited directly against the Convex, and the Eye stand in the same Line, then the place of the Image is so far distant from the Superficie, as the thing to be beheld, is it self.

15. *By how much less, the Globe of a Convex Mirror is, by so much the less also, are as well the distance, as the Image; And they will be made so much the greater, as you come nearer to the Visible.*

For by how much less, the Circle of Convexity of the Mirror is, by so much also the place of the Image, is nigher to the Center in which the Perpendiculars meet, and by consequence also the less: And the nearer the Visible comes to the Mirror, the greater it constitutes the Angle at the Center, and therefore it will be seen greater.

V. *Of Concave Mirrors or Speculums.*

16. *The Eye existing in the Center of the Concave Mirror, sees it self only: Within and without the Periphery it seems no where.*

For because all the Rays fall Perpendicularly from the Center on the Concave Superficie, and therefore

fore are reflected into themselves, and again meet in the Center: The Eye therefore constituted in the Center discerns nothing besides it self: But in the Circumference or without it, the Eye being constituted, which way soever it Radiates, yet the Rays will reflect another way, and you cannot behold the Image of it: From whence also it follows, that the Image of no one thing existing in the Center, can be seen by the Eye existing elsewhere.

17. *If the Eyes be so constituted in the Diameter of a concave Mirror, that both be equally distant from the Center, neither will discern it self, but one will behold the other.*

Fig. 215. In the concave Mirror *b a d*, distant in the Diameter *d e b*, of the Eyes *i* and *f* equally from Center *e*: The Eye *i* will Radiate in *a*, which will reflect into the Eye *f*: And again, if the Eye *f* radiate in *a*, the Ray of it will be reflected into *i*, and therefore neither Eye will see it self, but the Eye *i*, will behold the Eye *f*, and the Eye *f* the Eye *i*.

18. *A concave Mirror, shoots or darts out the Images without it self, especially Lucid ones, so that they stick as it were hanging in the Air.*

The reason is, because the place of the Image, is in the meeting of the Perpendicular of Incidence, and of the continuity of Reflection, which meeting, in Concaves, is made in the Air without the Mirror, as is also shewn before in Fig. 214. at page 710.

Let *b* be the Light shining on the concave Mirror or Speculum *e d*, and let *e* be the Point of Reflection of the Ray *b e* into the Eye *a*, and the Perpendicular of Incidence will be *b c d*, cutting the Ray of Reflection *e a* in the Point *c*: There then will be the Image of Light, and the Medium Air will be thought to shine: And observe, that by how much you are nigher to the Center of the Mirror, so much the Image appears greater, being separated quite from the Glass or Mirror, that it seems to be touched by the

the Hands: And if in the place of the Image *c*, you apply a Paper or other Solid, all things will appear more evident: But if in the Place *b*, you place Snow or Fire, the Eye being placed in the Point *c*, it will also sensibly perceive the cold of the Snow or Heat of the Fire, which are reflected together with the Ray of the Visible.

16. *A concave Mirror or Speculum exposed to the Sun, burneth; in the whole Axis indeed, but most strongly in the Center, See Fig. 216.*

Suppose some Rays to proceed from the Center of the Sun, through the whole circular Superficie of the concave Mirror, and to make with the Axis, every where like Angles on the Superficie of the Mirror, which being Reflected, will meet in a certain Point, and by reason the force of Heat is united, it will burn there; yet more weakly, because in one Point, none but the Ray of one Periphery meets: And the greater and larger the Circumference is, or the more incident Rays it receives, the stronger likewise is the burning, and this is made according to the Axis.

But the central burning is thus proved: let the concave Mirrour be *DEFGH*, and the Sun *ABC*, Radiating on it by the Center of the Mirror, by the Rays *AIG*, *BIF*, *CIE*; I say that all these Rays, intercepted between the points *E* *G*, meet in the Center *I*: For because they are all Perpendiculars, therefore they are reflected into themselves, and intersect one another anew, in the middle Point of the Glass or Mirror *I*: If there be placed or applyed combustible Matter, as Gunpowder, or Sulphur, it will be kindled Immediately from the Heat of the Sun: But because the Portion of the Mirror *EG* is very small, nor therefore capable to receive many Rays, and all the rest of the fallen solar Rays, meet indeed elsewhere in the Axis, but without the Center, beyond and on this side

side it, therefore the operation of hollow Mirrors is more weak and Languid.

VI. Of Irregular Mirrors.

Irregular Mirrors are various, some more simple as the *Cylindrical*, *Conical*, *Elliptical*, *Hyperbolic*, *Parabolic*, &c. Or compounded from Regulars, or altogether out of order and rough, and all these polish'd either within or without.

And what a Cylinder and a Cone is, is said before in *Geometry*: The *Hyperbola*, *Parabola*, and *Ellipsis*, are Lines or Figures, mixt from crooked and right Lines, participating of right and crooked: The *Parabola* from equal or alike, the *Hyperbola* from more of Rectitude, The *Ellipsis* from more of Crookedness.

20. *A Parabolic Mirror or Speculum burns most Vehemently.*

For all the Rays falling from the Sun, of which there are infinite, being Reflected, are here united; when in a Concave, only some are gather'd into one Point: And by such a Mirror we read that *Archimedes* and *Proclus* in old time, burnt the Gallies of their Enemies.

21. *Compounded Mirrors or Speculums, burn indeed also, but faintly.*

'Tis impossible to burn somewhat, by a simple plane, or by a Convex one; because the Rays are stretched out divers ways, neither do they ever meet: Yet if many, either Plane, or Convex, or Concave, be so aptly conjoined, that to each compounded, a circle may be circumscribed, it must needs be, that the Rays of the Sun, reflected from all the Surfaces, being gather'd into one Point, viz. the Center, it will there kindle.

22. *Irregular Mirrors or Speculums, follow the Affections of Regular, as much as may be.*

So the Cylindric, outwardly according to the Length imitates a Plane, according to its breadth, it Imitates a convex Mirror, because 'tis compounded of both.

23. *And*

23. *And they always yield monstrous Images.*

For Images are made like to the Mirrors, and therefore Irregular Mirrors, cannot reflect Regular and genuine Images.

24. *In general, we have very little certainty of reflex Vision, but are almost constantly mistaken about it.*

For not the Visible it self, but only the Image of it is seen, yea oft times, not the Image indeed, but only some confusion of Colour: And if the Image it self appears, yet it may be changed by the reason of Colour, as well as by the reason of Figure, and most of all by the Matter and Form of the Mirror: Lastly, neither is any thing seen in its Place, but all in another Place.

From whence the optic Image, in as much also as 'tis most intire, they call reflected from the Mirror, when things themselves are beheld indeed by their Colours, and Figures by their Parts, but by another Site, arrayed in any place, and by alien Quantities, and by an unapt proportion of Parts of Figure: In short, a reflected Image is the Vision of any thing joined with the Error of the Faculties agreeing to Sight.

Part III. Of OPTICS.

Concerning MESOPTICS or DIOPTRICS.

CHAP. I.

Containing certain Definitions.

1. **M***esoptics or Dioptrics* is, that which treats concerning refracted Vision.

2. *Refracted Vision* is, when we discern somewhat by a refracted Medium.

3. *A Refracted Medium* is, either two divers Surfaces, having divers thinness and thickness; or one, affected in divers manners. *Such*

Such Mediums are Sky and Air; Air and Water; Air and perspicuous Glass, or Spectacles, or Tubes through which we see; also pure Air, Cloudy, &c.

4. *A Dioptric Ray, is either of Incidence or of Refraction.*

5. *A Ray of Incidence, is that which falls from a visible thing through one Medium, on the Superficies of divers Diaphanous things, Fig. 217.*

As bd, from the visible b, falling on divers Diaphanous Bodies fcd.

6. *A Ray of Refraction, is that according to which, it being already entred into a divers Medium, 'tis Refracted.*

As dh, or dl, bending or swerving from the rectitude of the line bdi.

7. *For Refraction, is the bowing of a Ray of Incidence, to the constituting of an Angle.*

8. *A Point of Refraction, is the common Term of these two Rays; as the Point d.*

A Superficies refracted, is the first extream of a following Diaphanous, as cdm.

9. *A Perpendicular of Incidence, is that which is drawn from the Point of the thing seen, to and through the Superficies of divers Mediums at right Angles.*

Such is bcf in a plane Superficie; bcg in a crooked.

10. *A Perpendicular of Refraction, or of Obligation, is that which passes through the Point of Refraction at right Angles, as the Line adg; And 'tis the rule or square of Refraction.*

11. *An Angle of Incidence, is that which constitutes in the Superficie of a diverse Medium, a Ray of Incidence with the Perpendicular of Refraction; such is the Angle bda.*

12. *A Refracted Angle, is that which constitutes a Ray of Refraction, with the Perpendicular of Obligation or bending aside, in the Superficie of a divers Medium, as the Angle gdh.*

13. *An*

13. *An Angle of Refraction*, is that, which in the same Superficie of a divers Medium, constitutes a Ray of Refraction with the Ray of Incidence, by continuing a right Line; as the Angle hdi .

14. *Refraction at the Perpendicular*, is then said to be, when the Ray of Refraction comes near to the Perpendicular of Refraction, as bdb .

15. *Refraction from the Perpendicular*, when it goes away farther from the same than the continued Ray of Incidence, as $bdkl$.

16. Moreover Refraction is twofold, *Simple* and *Mixt*.

17. *Simple Refraction*, is when nothing else happens to the Rays besides Refraction.

18. *Mixt Refraction*, when the Rays are not only Refracted in a divers Mean, but are also Reflected.

CHAP. II.

Of the Affections of Simple Refraction.

1. **A** Perpendicular Ray passes also through a divers Medium unhurt from Refraction: But an oblique Ray is refracted in a divers Medium, and so much the more, the obliquier it is.

The reason is, because a Ray falling Perpendicularly is stronger and less; but falling obliquely, is weaker and suffers greater violence from the Medium: And although a Perpendicular Ray is not refracted, yet 'tis more pressed together in a thicker Medium; but in a thinner, it wanders about freely, and spreads it self abroad.

2. A Ray falling obliquely on a thicker Medium, is Refracted at the Perpendicular, the more for being thicker.

As if the Medium $fcim$ in Fig. 217. be appointed thicker than the Medium $cbda$, the Refraction of the Ray bd , which is dh , is made at the Perpendicular adg .

3. *A Ray in a Thin Medium falling obliquely, is Refracted the more by the Perpendicular, the thinner it is.*

As if you suppose the Medium *fcim* to be thinner than the Medium *c b d a*; The refraction of the Ray *b d*, which is *dl*, is made in the Perpendicular *adg*: Beyond the Ray of Incidence continued right forward, viz. *b d i*.

The reason of this diversity is, because the thicker Medium, more resists and presses the Rays: But in a thinner, they have freer Passage, and may spread themselves abroad.

4. *Hence also an oblique Ray falling in a thicker Medium, the refracted Angle *gdh* is the less, but the Angle of Refraction *hd i* is made the greater, the thicker the Medium is, and interchangeably, the less thick the Medium is, the greater the refracted Angle is made, but the Angle of Refraction less.*

For there, the refracted Angle comes near to the Perpendicular, but departs further from the Ray of Incidence continued: But here, it departs further from the Perpendicular, and comes nearer to the Ray of Incidence continued.

5. *But an oblique Ray falling in a thin Medium, makes both, as well the refracted Angle, as the Angle of Refraction the greater, by how much thinner the Medium is, yet the refracted Angle, always exceeding the Angle of Refraction.*

For in this thinner Medium the Angle of Refraction is always part of the refracted Angle, and therefore less than the whole.

6. *In Refraction, the Visible is not seen, but only the Image of it: Nor always Genuine, but sometimes very confused.*

For what's said before concerning Reflections, that also happens in Refractions.

7. *A Refracted Image is seen in the meeting of the Perpendicular of Incidence, and of the Ray of Refraction continued, Fig. 218.*

A a a

Let

218

Let the Visible be *a*, falling from a thicker Medium *et la*, on a Thinner Superficie *bc*, which in the Point *e* passes not directly to *i*, but is Refracted from the Perpendicular of Refraction *fiL*, and is made the refracted Ray *ek*, which being continued even to *h*, to wit, to the Perpendicular of Incidence *acd*, it shews there the Image of it self *a*, in *h*, without its place, in the meeting of the Perpendicular of Incidence and of Refraction continued.

8. *Somewhat may be seen by fracted or broken Rays, which cannot be discern'd by direct.*

So the Visible *a* if it should exist in the same mean with the Eye, and no refraction Intervene, it cannot be discerned directly on the Left-side, unless from the Point *i*; But because the Visible *a*, is in a thicker Medium, and the Eye being supposed to consist in a thinner, it may also be seen from *k*, not indeed the visible it self *a*, but its Image *b*, Refracted in a thicker Medium.

219 *Fig. 219.* An Experiment of this we have in Money cast into clear Water: For if you depart from it, even until you cannot see the Money in the bottom of the empty Vessel being placed of an indifferent height, by reason of the sides of the Vessel lying between it and your Sight; Then that Vessel being filled up with clear Water, suddenly the Image of the Money will appear again, and insinuate it self to your Sight, without its true Place, and greater than it is in it self.

167 *Fig. 167.* So by reason of Vapours ascending from the Earth (which the Air next to us renders a little thicker and less Diaphanous than is the upper Portion of it) the Sun and all the Stars nigh the Horizon appear to us not directly but refracted: And therefore are always beheld more Elevated than indeed they are, and are beheld often times above the Horizon when as yet they lie hid beneath it:

From

From which cause also the *Hollanders Wintering in Nova Zembla*, when they have wanted the presence of the Sun some Months, yet 14 Days before the expected and just time of Apparition, they have beheld the Image of the Sun intirely, viz. when as yet he lay hid beneath the Horizon according to Astronomical Calculation five Degrees and more.

Also besides from this the manner of the Suns sooner rising, and of the greater form of the Sun, as if great plenty of Vapors, presupposing that Day to be resolved into Rain, the Vulgar sort are wont to predict showers of Rain, this Prognostication seldom failing, being confirmed by use.

9. 1. When the thing seen and the Eye consist in the same Medium, the apparent place falls in with the true.

2. But when the thing seen consists in a more thick Medium, but the Sight in a thinner, the apparent Place is always nearer to the Sight than the true: Then because those things which are beheld from a nearer place, seem greater because they are comprehended under a greater Angle: But those things seen from a more remote place, because the Angle of Vision is less, appear less; it comes to pass also, that those things which are seen from a thinner in a thicker Medium, seem greater: But those things that are beheld from a thicker Medium in a thinner, appear less.

For this reason, Stars seem from a troubled and more thick Air, in a more thin and subtile Sky, appear higher and less than just: Yet nevertheless by that former reason, if as aforesaid, there be about the *Eastern* or *Western* Horizon great plenty of thick Vapours, as is wont be; And we behold the Sun or Stars Oriental or Occidental from a more pure and thinner Air, they will appear also higher and greater than just.

10. Right Lines or things seen through a double Medium, seem broken.

The cause is manifest from what's said : And may be infer'd by Experience ; for a Staff immersed in Water, that part of it which is under the Water seems more Elevated, than that which is without the Water ; From whence it receives the Form of broken or bowed.

11. *The meeting of broken lucid Rays, begets also Fire, and is kindled from Combustibles.*

For the Virtue of every one is gather'd together and united, and made stronger and increas'd, that it may excite Fire in combustible Matter : If the convex Superficie of a Chrystalline optic Glass receive the Rays of the Sun, they are broke in this thicker Medium at the Perpendicular ; afterwards going forth from the optic Glass into the Air, by a thinner Medium they are broken again, but more from the Perpendicular ; and so being twice broken, they must needs by little and little come together in one Point, where being united, they kindle combustible Matter.

C H A P. III.

Of the Accidents and Effects of Mixt Refraction.

1. **P**OLISHED Refractives are also Reflexive or turned back, by reason that Refraction and Reflection may be mixt.

Thus Pellucid Bodies, as Water, Glass, Chrystal, have also often times polished Superficies, and therefore, together in part they break in Peices the falling in Rays, and in part they Reflect them : So the Tunicles of the Eyes are refractive Bodies, because they break the received Rays of Visibles : And all under one being Reflected, in which beholding the Image of any thing as it were in a Glass, it is Depainted and Reflected.

2. *Hence the same visible may be beheld with a double Image, the one stronger by Reflection, the other weaker by Reflection and Refraction together.* So

So if the Sun Shining, there be placed upon any Vessel, a Mirror or Looking-glass, and Water poured upon it, there will be seen a double Image of the Sun, the first also the greater, which from the Reflexion of the Rays of the Sun will come down to the Eye: The other which arises with the Rays also, pass through the Water to the Perpendicular of the Refracted, and there Shining again in the Mirror, are Reflected again from the same; from whence being entered into the Air, by a new Refraction from the Perpendicular; they meet in the Eye, shewing another Image of the Sun, but from that multiplied Obliquation, much weaker: Which other Image seems to some, (but amiss,) to be a Star nigh the Sun.

3. *From the Doctrine of Refraction, the cause is to be sought, why Myopes, viz. those which see things near at Hand distinctly, but remote things confusedly, see more clearly by hollow Spectacles; and Presbytas, viz. those which see remote things distinctly, and nigh at hand confusedly, see more clearly by Convex Glasses or Spectacles; and some indeed by those that are more hollow or more convex, and others by them that are less, according to the diversity of their Eyes; see Fig. 221.*

1. *For Presbytas, accustom their Eyes to see remote things: Which if therefore they use convex Glasses, their Sight is changed by the Glass put between; and 'tis thought as if it came from afar off: For let the Eye of Presbytas be QRS, and the visible thing near at hand L, to which one looks that cannot see Nigh at Hand, and using a convex Spectacle or Glass MNOP: Then the Rays of the Visible thing L (besides that in the Glass it self being thick, they are Refracted by degrees) going forth from the Glass, depart more from the Perpendicular, and make Q, Parallel to OQ and PS: Therefore the Eye being accustomed to a Parallel Vision, be-*

holds that thing more accurately, because he thinks the Rays as it were to come from afar, from IK, and knows not what Rays in the mean while happen in the Heterogene Pellucid Medium between.

2. On the contrary, the Eye of *Myops* is accustomed to see things nigh at Hand, which if therefore it be used to a concave Glass, remote things will seem nigh unto it, for the Points A, B, Radiate afar off, as it were by Parallel Lines AD and BC, which (besides that in an hollow Glass they are a little Refracted at the Perpendicular) without the Glass between it and the Eye they go away more from the Perpendicular, and they will be DF and EH; the Eye therefore because it thinks the Image to come forth from the nearer Point C, therefore it sees remote things more distinctly.

And hence also the cause appears, wherefore all things appear greater through convex Glasses, and less through hollow ones; to wit, because the visive Rays through these bow together, but through those they part asunder.

4. *Multangled Glasses (invented not of necessity but for the sake of pleasure) shew so many Images of one thing, as it consists of Superficies.*

For from each oblique Superficies, in which every visible Radiates equally, every one makes Refractions, and so the visible Species or Form several times multiplied penetrates to the Eye.

5. *From this mixture of Reflexion and Refraction, many causes may be given of Meteors, to wit, of Solar and Lunar Rainbows, of Gulfs, of two or three Suns appearing at once, of Circles about the Sun, Moon or Stars; of Colours, of Clouds and of Twilights.*

I. Concerning the Rainbow.

We have a noble Example, as well of Reflexion as of Refraction of Rays of Solar Light, in the Rainbow that most fair heavenly Arch, which God hath

hath placed as a Symbol of his grace against our Eyes, a most pleasant Spectacle, to know the cause of which, and the manner of its Generation is worth our while.

1. The material Cause, in which 'tis Generated, is Clouds, not thick nor continued, but like as of melting dewy Vapor, and loosed into innumerable most small drops; beyond which is some Umbrage or Shadyness, having the reason of Lead or Quick-silver on the back-side of Looking-glasses, to wit, that the Images may appear the more expressly.

2. The Efficient cause are the Solar Rays, spread by pure Air to that opposit dewy Cloud.

3. The formal Cause is as well the manifold Refraction of the Solar Rays in that dewy Cloud, as the Reflexion to the Eyes of Spectators.

For a Rainbow is not Generated, but of some transparent and diaphanous Matter, (such is that multitude of small Drops) placed over against the Sun, which indeed receives the Rays but yet hinders them to Penetrate straight on: Whence 'tis necessary first that the Refraction of Rays be made, because the Light is carried through a thinner mean: And the Reflexion also is made together, because carried into a thicker Medium or Mean.

And this Reflexion is made from infinite contiguous little drops of dewy Clouds, which although they are infinite, yet because they go here contiguously, neither have they sensible Separation from one another, therefore they are esteemed as it were for one continued Glass.

Nor also not only once, but several times in each, also the Ray falling first into the small Drops is Refracted and Reflected, from which it goes out and extends to the Sight; And so not only from the Collection and Aggregation of fracted Rays, plenty of Light is increased in the deep Superficie of the Vapor, that it may be extended more strongly to the Sight; But also the Reflexion it self increases

and strains it, therefore, because from Reflexion, the Ray is made of more efficacy and force than it was before: See *Fig. 222.*

For in this Figure, let the little round Drops of Water be ABCDEFGH, because Water or a homogeneous Body, is Imbodied into the Figure of its Element; let some Ray fall on LH, it being Reflected from two Places, it will return to the Eye M, to wit, from the convex Superficie H unrefracted, and from the hollow Superficie of the little Drops, yet first something changed, and in the ambit of the small Drops several times reflected, to wit, from H in E, from thence to the like Angles in B, afterwards in G, hence into D; from D into A, thence into F, again into C; from C at last through B, 'tis brought to the Sight M, by a repeated Reflection eight times Circlewise.

The Case is the same of the Ray KA, which going forth from G into the Eye M, agrees with the former, and is accounted altogether parallel to it, by reason of the littleness of the small Drops, and the unproportionate distance of the Eye.

4. *The Figure of a Rainbow is Circular.*

Indeed the whole Superficies of the dewy Vapor being expos'd to the Sun, is Irradiated or Lightned; because from every Point in which the direct Rays of the Sun fall, Reflexion is made; yet from irradiated Vapor than no more is beheld, than the Form of an Arch: For the Eye perceives not the Form of any thing, unless it consists in the Point of the Axis of Illumination, to which the Rays falling into the Vapours at equal Angles, are Reflected: By which Reflection, according to equal Angles they Form a Pyramid of Reflexion, the Base of which is circumscribed by an Arch; and only exhibits the Form of an Arch to the Eye, which is called a *Rainbow*: In which always two Pyramids are constituted, one of

Chap. III. Of *DIOPTRICS*, &c. 729

of Illumination, the other of Reflexion and Vision: the *Pyramid of Illumination* hath its Vertex in the Center of the Luminous Body of the Sun: From which the Rays go forth, to the Superficie of the Cloud, some of which fall in obliquely, but one Perpendicularly at right Angles, which is the Axis of the *Pyramid of Illumination*, to a certain Point of which, the Rays falling in obliquely, at equal Angles, are Reflected, and constitute the *Pyramid of Reflexion*, whose Base also, is in the Superficie of the Cloud, but the Vertex is in the Eye, and the Axis common to both.

Therefore, for as much as 'tis demonstrated by Mathematicians, and approved also by Sense, if from one Point, into divers Parts, right Lines are drawn forth, the Line which joins the extrems of these, is the Periphery of a Circle, also the Points of Reflection, joining the Line in the Superficie of the Cloud, must needs be a Circle.

From whence also follows, that which *Vitellius*, Demonstrates at Proposition 64 of his 10th Book, many ways, that necessarily the Eye is in one right Line, between the Center of the Sun, and the Center of the Rainbow; so that the Center of three Bodies, to wit, of Light, of Vapor, and of Sight, consist in one right Line.

And that we see not a whole Circle formed, that happens not from a Fault of the Rainbow, but of the Eye, and the Impediments of objects, which hide the desired part of the Circumference of the Rainbow from us.

Indeed from the Sun occupying or beginning to approach the Horizon, we see Precisely a Semicircle Rainbow, for then the greatest height of it (which from the Hypothesis is the Semidiameter) is observed to be 45° , which constitutes half a right Angle, by which all the Rays are equally Reflected

to

to the Eye: And by how much the Sun is Elevated above the Horizon, so much the Center of the Rainbow is depressed beneath the Horizon, so that by the Sun, no natural Rainbow can be discerned higher than 45° : But if the Eye could be Elevated above the Sensible Horizon in some Mountain, so far off, that he may look 45° downwards into a valley where are Clouds full of Dew, then he would behold not a mangled, but the whole and intire Rainbow of a perfect Circle, that which also appears to be true to the Eye in Artificial Rainbows; to wit, if in a dark Chamber, in which nevertheless through the Window by some opening, the Rays of the Sun may enter, then plenty of fair Water being drawn into ones Month and thrown out Drop by Drop into the contrary coast of the Sun, for so you will see some Image of a Rainbow, not arched or mangled only, such as Nature is wont to shew to us, but intirely round, or only that part of it mangled or cut off, which passes through the Shadow of thy Body.

5. *And a Rainbow is multiplied according to the multitude of Eyes.*

For if from a Rainbow appearing, several thousands of Men should be Spectators, yet they would every one behold one, and every one peculiar; even as also happens in all Glasses, that beholding them from a divers Place, they discern the Image sited in a divers manner; or even as one Word according to the multitude of the Heavens is multiplied without diminishing it: That which any one may gather from, what we have said, that the Center of the Sun, of the Rainbow and of the Eye consist in one right Line; therefore how many Eyes, so many Lines, so many divers Rainbows; yet with this difference.

1. Of the Spectators, according to the length only

only of differences, right on from the Vapor towards the Sun, those which are farther distant from the Vapor, acquire a greater Base, a greater Angle at the Sight, longer Rays, the meeting of the reflected Rays further removed from the Rainbow: Wherefore also they see a greater Rainbow; But those which come near to the Vapor, see a less, by reason of the contraries of these.

2. Those which differ according to Breadth only, and bend to the side; indeed they keep the equality of Rays, of Angles, of Bases, and of Rainbows; but they acquire divers Axes, divers Centers, and in like manner they are beheld divers in divers Places.

3. Those which differ as well in Length as in Breadth, have all things divers: Therefore necessarily, they behold a divers Rainbow, and of unequal quantity.

6. *The chief Colours of a Rainbow are four; The extream or out-most is Red, next to this in the middle Yellow, Green succeeds, afterwards follows Purple Coloured, which at last is finished in Blew or Azure.*

The Cause of this diversity is the disorder of the Vapors; for the Light now distilling into the Vapors, being refracted into Drops, is Painted with unlike Colours: For the Shadows from these unlike Drops of scatter'd and intermixt Light being aggregated and refracted, express unlike Colours, which the reflected Ray brings together with it self to our Sight.

7. *Next after the Principal Rainbow, sometimes also another of the second sort is seen.*

This being the Eighth part of a rect Angle (that is $11^{\circ} : 15'$) is higher; encompassing the Principal or Prime one, the Semidiameter of which therefore is made $56^{\circ} : 15'$ (which you measure from the Sun Rising or Setting) and is to the Semidiameter of the first or principal Rainbow which is 45° as 5 to

4: Therefore when the Sun is not higher than so many Degrees, then two Rainbows may be seen in a dewy Cloud.

And at all times, the Semidiameter of any Rainbow may be found in this manner: Take the Altitude, as well of the Rainbow as of the Sun, by some fit Instrument, join the Arches, of both the Altitudes, the Sine of which joined Arches is the Semidiameter of the Rainbow; whence, from the Altitude of the Rainbow, the Altitude of the Sun also may be known; viz. if you take away the Altitude of the second Bow, from $56^{\circ} : 15'$, it leaves the Altitude of the Sun.

There are some who think, that the second Rainbow is begat from the Reflexion of the first, from this Argument, that the Colours of it are beheld in an inverted Order: But they are deceived; for if it should be the Image of the first, not only the order of the Colours would be beheld inverted, but they would be also turned back from one another horned, as is seen in the 222 Figure or Scheme: Moreover it may happen, that the first being absent (to wit, when the Sun is higher than 45° and yet lower than 65°) a second may be seen; the Image, without that which makes it, the Type without the Antitype, or a Copy without the Original; which nevertheless frequently happens in Summer-time: From whence 'tis concluded that the second sort of Rainbow, is also Immediately generated from the Sun, no otherwise than the Principal one.

8. *Concerning the time in which Rainbows are generated of the Sun, the following things are to be noted.*

1. In Winter more frequently, Rainbows may be generated at any Hour: The cause is, because the Altitude of the Sun is less, and therefore the Solar Rays are also less in rectitude, and less in strength, than that they can consume and scatter the

the Vapors: Therefore in that time, is more of Vapors, which are consumed by the Heat of Summer.

2. In the Summer about Sun Rising or Setting, Rainbows are made, but not at Noon time.

For from the Suns Rising and Setting, Vapors are not so easily dissipated, as at Noon-time or when the Sun is in the Meridian; by reason there then happens a greater rectitude and strength of Rays: Moreover, too much Meridian Altitude of the Sun hinders the generation of a Rainbow, for as much as the place of its Generation is forced down beneath the Horizon.

And concerning this, from the Altitude of the Sun, we must note a Rule of *Vitellonius*, which he explains at Prop. 79 of the 10. Book, when the Arch of the Suns Altitude is greater than the Semidiameter of the Rainbow, (which how it is found is said a little before) a Rainbow is not generated: But by how much that Sine is less than the Semidiameter of the Rainbow, by so much the more will appear of the Rainbow.

9. Concerning the place of the Rainbow, the Rules are these.

1. From the Sun being posited about the East, the Rainbow is seen about the West, and Interchangably.

2. From the Sun being near the Meridian, the Rainbow will be seen on the North Part: And since the Sun from the Site of our Sphere never declines to the North, we never see a Rainbow towards the South, unless the Solar Rays fall on Solid watery Clouds, and posited overagainst the Sun, and reflected from these into a dewy Cloud, and placed at the side of the Sun, and lastly, from hence reverberated or beaten back to the Sight.

10. Many Rainbows may be Generated at once, in a twofold manner.

1. When

1. When one encompasses and includes another, so that they have as it were one Axis, and one Center, and of Colours either the same, or sited contrary in the reason of Order.

2. When one includes not another, but they have divers Axes and divers Centers, which may be supposed to be done, when the Rays of the Sun fall strongly, on one or more watery and solid Clouds; and from hence are reverberated to one or more Superficies of dewy Vapors, and from these dewy Vapors reflected to the Sight: But when, as happens not altogether many Arches of the Rainbow are beheld wonderfully full of turnings, the cause of which cannot be given, 'tis rather referr'd to strange things and wonders, than among natural things.

11. *In the Rainbow also we may take notice, that if you fly away, it follows you, if you approach towards it, it meets you, and if any one proceed to one side, it also seems to move to the side:*

The reason of this appearance is this: 'Tis manifest, that a Rainbow always appears the greater to him that goes away from it: Therefore by a continued augmentation of this same sort of departing from the Rainbow, it seems to follow, but coming to a Rainbow 'tis lessen'd and decreas'd: From whence, to go away or depart, is also esteemed as it were to fly away: Which access and recess from hence appears, that greater things are better seen, and therefore nearer than less: And contrarily, that less things affect the Sight less and more weakly, and are thought to be further distant: And if any one decline to a side, with him the Axis of the Pyramid will be changed to the same side; the Point of the Superficie of the Luminous Body, or the vertex of the Pyramid of Illumination, the Circle of Altitude, and the Aggregate of Rays, or the
Vertex

Vertex of the Pyramid of Reflexion are all changed with him; Wherefore also the Rainbow is discern'd to move to the side.

12. *Lastly, Lunar Rainbows are seen, but seldom.*

For to the Generation of these, three things are required; 1. A fit Vapor; 2. The Moon at full or near full Moon; 3. The same Rising or Setting, being overwhelm'd by the Rays of the Sun beneath the Horizon.

And since these things seldom meet together, 'tis manifest that Lunar Rainbows are very seldom seen; yet oftner in Northern parts than in Southern, in which the Matter of the Rainbow is consumed from the Heat of the Sun.

II. *Concerning other Meteors of the Air having optic Causes.*

1. *Gulfs*, appear when a thick Cloud bursts asunder into more thin Parts; nigh to the bursting it exhibits a Lucid Colour, that the Heaven seems as from a Fire to open wide.

2. *Two or three Suns appearing at once, or two or three Moons*, when thicker Clouds are posited to the side of the Sun or of the Moon; like the Images of them in a Glass they are reflected to our Sight: Then more Suns or Moons appear.

3. *But Circles about the Sun, Moon, or Stars*, appear, when the cloud under the Sun or the Moon is very thin, which the Star Illustrates by its Rays round about: For then they shew the Circles of these kind of refracted Rays, every where at equal Angles; and the Circular Refraction is made every where at half a right Angle, so that the Diameter of the Circle is 45° .

4. *The Colours of Clouds*, are begat from the mingling together of Lucid Rays of the Sun with the obscure thickness of the Cloud.

For because Clouds are unequal in themselves, as well by reason of thickness and thinness, as also by reason of their

their Site to the Celestial Lucid Bodies; therefore from hence depends all the variety of Colours: And the more Light falls into it, the brighter the Cloud is; by how much the less of Light, 'tis so much the blacker: Yet for the most part these Colours are ill-favoured.

5. Of Twylights, by John Kepler in his *Epitome of Astronomy*.

1. The Causes are many: First the Efficient cause, which incites all the rest, is the Sun it self; notwithstanding that it is conversant under the Earth, neither carries our Eyes, either by a direct or a refracted Ray.

2. Another Cause, is the celestial golden Sky compassed about near to the Sun, at the Amplitude of some Degrees, Shining by reason of the Suns nearness, sometimes more, other times less, which time is somewhat imployed in arising, until the Sun it self at last is risen.

3. A third Cause, is of Smoak, and of dry Exhalations, excited from the Heat of the Sun by Day, or from Harmonical Radiations of the Stars; and stretched out by Heat and by its thinness, upon the Superficie of the Air; which when they are higher from us, are sooner Illuminated from the Sun, and from the cover'd brightness of the Sun, than when lower, and so being Elevated above our Horizon, like lighted Lamps, they impart the Light receiv'd from the Sun to us.

4. A fourth Cause, is the Air it self, caus'd two ways: For Air, makes bright and clear, the direct Rays of the Sun, and revibrates them, as being a thicker Sky; or is instead of a Glass to the solar Rays, and Reflects them.

But lastly, to conclude, we were willing to delineate these few things, in which are the most pleasant, and most useful things chiefly necessary to all Philosophy; the optic Science is Collected from Writers

ters of the best Note, which by reason of its great Amplitude, we Judge no mortal Man is able to deliver it fully and perfectly : Yet he that understands these Rudiments and Principles of it, will not be altogether Ignorant of the Optics ; but will have enough of help to understand the more accurate Writings of the Optics, in which he will find chiefly, the most natural Causes of many admirable things, which with the Vulgar are accounted for monstrous strange things, or juggling Tricks, which they believe can never be perform'd without the help of Dæmons : But let them the more admire and celebrate the Wisdom, the Goodness, and the Power of God himself, which hath not reserv'd such abstruse and hidden things, only in his own Power, but hath made out also the Nature of them, and left the searching out of the most secret things of Nature, to Man : To the most wise, most Excellent, most great Author of Nature ; To God himself, be Praise, Honour and Glory, World without end, Amen.

The End of OPTICS.

OF
STATICS.

The PREFACE.

I. **S**Tatics, is a Science which artificially explains the way of finding out the weight of Bodies by Instruments : II. Which Science that it may be Methodical, we ought to deliver : 1. The Definitions and Theorems : 2. The Structure of the Instruments : 3. The Practice itself.

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PART

Part I. Of STATICS.

CHAP. I.

DEFINITION I.

B*odies of the same, or of equal Weight or Gravity, are those things, which in the same mean tend downwards by an equal Endeavor, Impediments being taken away.*

All those are called heavy Bodies which are borne downwards of themselves, and by their own Nature, this is known by natural Philosophy and Sense almost common to all Men; yet they are in great difference; neither as in bulk, so also in Weight are they always alike: For a small Mass or bulk of Lead, weighs down a great Sack of Wool or of Feathers: Yea those things which are of the same kind or Species, they do not always make equal the Mole to Weight, for how much diversity of Gravity or Weight is there, either of the Watery Element alone? Observe, as the Figure is changed, the Weight together is changed: A Bullet of Lead readily descends in Water; but the same being hammer'd into a thin Plate like a Leaf, sinks not so much: That therefore, if the exquisite equality of Weight be required, The Species also and the Figure, and all other things ought to be alike.

II. *A Body of known or of given Weight, is either of Gravity, the heaviness of which is expressed by a certain Number: Or to the Knowledge of which, some proportion of Weight is to be known to us.*

For neither do we know surely a Body to be (for Example) of one Pound, unless either it be mark'd that we can believe it; or of which we have the Proportion to another known Weight, certainly known to us.

III. *The Center of Gravity, is a Point in any Body, sited within or without, about which, the parts on every side consist of equal Moments.* There-

Therefore if by Division through such Center, by passing through right forward, howsoever a Body be cut, it will always be cut into two Parts of equal Weight.

IV. *Those Bodies are Equilibrall, or Weigh equal, which being hung at each end of a Ballance, constitute the same Parallel to the Horizon, whether they are of equal Weight or not.*

V. *A Weight is said to be rais'd, it being hung on the weighing Instrument or Ballance, whatsoever is the cause of elevating or raising the Weight on high, whether it be a ponderous Body, or some impetus; 'tis also call'd, Power, in Mechanics.*

VI. *A Leaver from the consideration of its reason, is any length, which leaning on some Prop, on one Point of it is applied an heavy thing to be moved, and on the other, it hath a moving Power applied to it.*

Wherefore in a Leaver, three Points are most notable in the use of it; to wit, the Point of the Prop, the Point of the Weight, and the Point of the Power; from the various Combination of which, arise three kinds of Leavers: No Man will think we go beyond the Scope of *Statics*, when we speak concerning the Leaver, for they are akin; and *Mechanicks* and *Statics* lean upon almost the same Foundations; and a Ballance, when the Scales are of unequal Arms, is the same as a Leaver, in which, the Axis passing through the Center, is the Prop; and from the reason of this flows the demonstration of that; whence if sometimes we touch upon *Mechanics*, it makes the *Static* Artifice more clear.

C H A P. II.

T H E O R E M I.

OF Circles, of Spheres and of Rectangles; the Center of Gravity is in the common Section of the two Diameters.

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In

II. *In all Equilateral Figures which may be inscrib'd in a Circle, the Center of Gravity is the same with the Center of the Circle, in which they are inscrib'd.*

Yet, by supposing these heavy things, to have every where equal thickness, and to consist of Matter of equal Weight in all their Parts.

III. *In all Trigons, or Triangles, the Center of Gravity is in the common Section of the three Lines, each of which, coming to each side from the opposit Angles, divide them equally.*

For 'tis necessary, that these should meet in one and the same Point, by a mutual Section.

Fig. 223. For the Sides AB and AC being divided in the middle in D and E; and CD, BE, being drawn, and through the common Section AGH; and DE, viz. Parallel to the third Side; the Triangles BEC and CDB will be equal, and the Basis BC the same; therefore BCG the common, being taken away, CEG will be equal to DBG, to which, seeing AGE and AGD are equal, inasmuch as they are on equal Bases and between Parallels; AGE, and AGD, will also be equal between themselves: And since they are upon the same Line AG, they will be of equal Altitude; which is also the Altitude of the Triangles FGD, and FGE, which consist on the same Base FG; and therefore they are equal between themselves: But because BC is Parallel to DE; DGE and BGC will be Equiangled, and the Proportion of BG to GE, as of CG to GD; from the same also, BGH and GEF are alike, and also CGH and DGF: Wherefore the Proportion of the Triangle BGH to EFG, is as BG to GE, that is double; and CGH to DFG, as CG to GD, which is double: And CG is to GD as BG to GE, wherefore BGH is to EFG, as HCG to DFG: Therefore when DFG and EFG are equal, also BGH and CGH will be equal: And since they are
between

between Parallels, the Bases also BH, HC will be equal : Therefore all the Triangles ABH, ACH, CDB, CDA, BEC, BEA, will be the middle of the Triangle ABC ; which therefore being hung from the Point G, or from the Center of Gravity, may be changed into many Parts.

IV. *The Center of Gravity in Conoid Proportions, divides the Axis of the right Angle so, that the part which terminates at the Vertex, is double of the other part which ends in the Base.*

V. *Equal Weights, being hung equally on both Arms of the weighing Instrument, are in Equilibrio ; but unequal Weights so hung, do not Equiponderate, but the heavier outweighs : On the contrary, equal Weights hung at an unequal distance, do not Weigh equally, but that outweighs, which is hung at a greater distance.*

VI. *Truly the Reason of the Leaver is wonderful, and the Cause hid, why, the Power, the farther 'tis distant from the Prop, the Leaver is the more efficacious and moves the easier.*

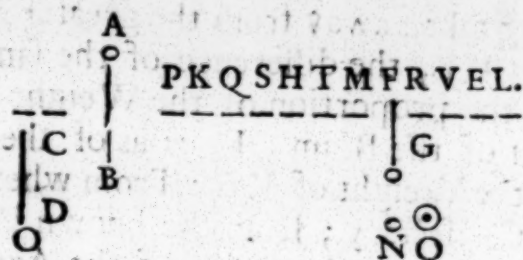
Aristotle thinks the cause of this to be, that the greater Radius, or more distant from the Center, describes also a greater Circle or a greater Arch of a Circle : And because in the same time, as well the Weight nigh the Prop, as the Power remote from the same, pass through, indeed like Arches of Circles, but unequal, the Motion of the Power is swifter than the Motion of the Weight, and therefore overcomes it.

This Opinion of *Aristotle* is variously explain'd and refuted by Artificers ; but all these things do not bring the reason of this compendium exactly to be discerned : Yet I think it is certain that we may require also the Physic causes to his Demonstration of this Mathematick Physick : The Principal force is of the Power of the first movent, which Machines and Instruments, nevertheless according to

this artificial Application of it to them, may very much help, yet so far, as also the Power of the Mover, and the matter of the Instruments permits, and is able to do good: So in the Leaver, the further the moving Power is from the Prop or Hypomochlion, as in the Point of rest, and of hindering or stopping; this is of more Force, as less hindred, and so moves by a greater proportion of Operation; and indeed, the more the back of the Prop is sharpned into a Line, and like a Prisme leaves the breadth, this Motors action succeeds more happily, for the back of the Prop being broad, resists the Power from many Points; (when the distance from the Prop is referr'd as to a Point) a Weight by the like reason, the further 'tis distant from the said Point of Hinderance, this more increases the natural bending of it downwards, contrary to the Power of the Mover: But instantly lifting up the Prop, it raises up the neighbouring Weight more aptly, when the Power reaches out the helping Hand; yet the breadth of the Prop doth not so much hinder the raising the Weight, as it doth hinder the Power, the Action of which draws almost every Line.

VII. *By Pullies, Skrews, Wheels and Wedges, the proportion of the Power does partly increase, and in part is more intimately and conveniently applied to moving; is partly multiplied, and by that means it is divided into several Principles of moving, so as more Hands, Leavers and Props may be applied: And the Weight be more equally divided among many,*

VIII. *A Beam is some kind of Leaver, the proper Instrument of Statics, and therefore to be consider'd accurately here, the whole Reason of which depends on the three following Theorems; the first of which is Archimedes, and 'tis thus.*



The Proportion of Weights that are Equibrall, is as that of their distāces from the Cock of the Ballance.

This Theorem succeeds, if the Rule of the Ballance or Beam be supposed altogether of no Weight, as if it should be of most light Wood, or of some matter of No, or scarce sensible Gravity :

For Example, a Weight D being hung in C, makes an Equilibrium with a Weight G, being hung in F: And the proportion of the Arms FB to BC is fourfold or Quadruple : I say on the contrary the Weight D is Quadruple of the Weight G ; this is truly the Golden Rule which *Aristotle* renders in his *Mechanicks*.

IX. But because the Beam or Rod of the Ballance for the most part is Weighty, being made of solid Wood, yea of Iron or Brass ; the Ratio also that is to be had of this, is as this following Theorem explains : *When the Burden or Weight shall be hung only to the extreame or end of the shorter Arm, and the Beam or Rod is ponderous, being equal in Magnitude and Weight, and the Ballance being so dispos'd will consist in Equilibrio ; the Proportion of the Weight hung, will be to the Weight of the whole Beam, as the difference of Parts, or of the Arms of the Rule or Beam, to the double of the Weight of the lesser Arm :*

Example, The Weight D, hung in C, makes an Equilibrium with the Rod or Beam BL, without any other Weight, and so both Arms of the Ballance BL and BC, consist or stand Parallel to the Horizon : And make BK equal to the less Arm BC,

Bbb 4

which

which being taken away from the greater Arm BL, there is left KL, the difference of the same : I say then, that the proportion of the Weight at D, to the Weight of the Beam CL, is as of the Weight of LK, to the Weight of KC : From whence flows an elegant Confectary ; from the known Weight of the whole Beam, and of one of the Arms of it, To find the Gravity of a Weight to be hung that shall make the Ballance stand in Equilibrio : For if we multiply KL, which for Example let be 40, into it self, it makes 1600, which Number we divide by the Weight of CK, which let be 16, and it produces 100, to which add the former Number KL 40, and it produces 140, the Weight at D sought.

X. *If the Beam be supposed without Weight, and from the part, which is the difference of the Arms from the hollow or hole, wherein the Beam of the Ballance turns, from equal Weight extended through that whole Rod, that will have equal Gravity with it self, being hung in a Point distant from the Center of the Ballance, by the half of the whole Beam.*

This Theorem flows from the two Antecedent ones, the Sense of which is this ; let the Beam CL be of no Weight, and CB equal to BK, and some Weight equally co-extended on the Rod, that under the form of a Square it makes an Equilibrium with D hung in C, and to the co-extended Weight equal to G, hung from M, the Term of the Rod BM, which is the middle part of the whole CL ; I say the Weight G, will here also make the Ballance in Equilibrio, as the co-extended of the whole KL : Let it be therefore, that in M, it will make Equilibrium with D, then it will be by Theorem the 8, that MB will be to BC, as D to G : Also because it makes D in Equilibrium with G, with the co-extended KL ; if therefore the whole also CK, shall be equally heavy, it will still make Equilibrium or
be

be Equilibrall, because B C and B K are equal, and so the proportion of the Weight D to the Weight K L will be, as of the Weight L C to the Weight C K, by exchanging the proportion at Theorem 9: Therefore as of the length L C to C K, because the Weight is equally distributed: But as the Weight D, to the Weight L K, so D to G, because by the Hypothesis G and L K are equal; as therefore L C to C K, so M B to B C; wherefore by changing, as C K to C B; so L C to M B; but C B is the half of C K, therefore B M is the half of L C, which was to be Demonstrated. But because C B is the half of C K, and B M the half of C L, it follows, that M K is the half of K L; and so the whole Rod K L, only Burdens or Weights, as if the Weight of the same were hung from the middle of it self: the Consequence of this Theorem is that Proposition of Archimedes.

Every Weight, although unequal, as for example a Triangle, the little Rod of the Ballance, or of any thing co-extended, causes so much Gravity, as the thing suspended from the Point of the Rod, which hangs over Perpendicularly in the Center of Gravity of the co-extended Weight.

Which Proposition of Archimedes is general, viz. although the Weight extends not as far as the Tongue of the Ballance, but for Example, if it should be co-extended only through L F, nevertheless being hung or suspended in E, straight from its Center of Gravity, it will Equiponderate.

P A R T II.

Of the Furniture of Instruments.

C H A P. I.

Of Weights.

TO Weigh without Weights, is as to Write without Ink ; therefore the Ratio of all corporeal Weight and Gravity, ought to be well known first of all ; but because this matter is already copiously explained in the second Part of Practical Arithmetick from Page 90 and the following, it may therefore be fetcht from thence.

C H A P. II.

Of the Ballance or Scales to Weigh with.

A Ballance is an Instrument to examine the Gravity of Bodies ; so prepared, that if the Weight be known that is put in one Scale, and the Body to be examined in the other, the Equality of the Weight of both, constitutes the Ballance in Equilibrio : But if one Scale or Bason of the Ballance be more depressed than the other, it argues the inequality of the same according to Gravity.

There are chiefly two Instruments with which Statics use to Weigh the Gravity of Bodies :

1. The Ballance already describ'd : 2. The Beam, of which the first is most simple and more uniform, and most fit to examine the smallest things of Weight ; from whence, precious things are Weighed in Scales, rather than to produce their Weight by hanging them on a Beam ; for a Beam is an Instrument more various and scarcely descends beneath a Drachm, in examining of Gravity : Yet this it hath before that, that by one Weight only, although diversly hung, we may Weigh greater or less

less Weights or Gravities; when in Scales, the Weights are always to be equally added to the things to be Weighed, as we shall hear by and by : Therefore the form of common Scales ought duly to consist of these parts ; thus, Fig. 224.

AC is called the Beam of the Scales : *BC*, *BA* the length of the equal Arms : *B* the Center ; *BE* the little Tongue or Cock, the Examiner, *BD* the hole or hollow wherein the Cock turns ; *F* and *G*, the Scales or Basons of equal Weight, hanging by small Chords, from *A* and *C* : The little Tongue *BE*, is joined unmoveably to the Beam or Rod at right Angles : The hollow place *BD*, sustain'd by the little Axis conveyed through the Center *B*, and through the holes of the hollow place answering to the Center on both sides, upon which little Axis the agitation of the Beam of the Scales is made.

Two doubts arise about Scales, which being explain'd, perhaps the Nature of them may appear more clearly : 1. Why the Arms although existing equally, and equiponderating Burdens put in each Scale, the Beam with the Scales being moved into Equilibrio, always Radiates at a Parallel Site to the Horizon ?

Fig. 225. The Moderns report the cause thus, that unless the Line of Direction (which is the right Line generally passing through the Center of Gravity of any Body) should connect two Centers of Gravity and of Suspension, the Beam of the Balance cannot rest : *Cardan* endeavours to Demonstrate it thus : Let the Scales be, whose hollow place hangs from *A*, the Axis passes through the Center *B*, about which moves the Beam *CD*, with the Scales hung to it, and the Angles *ABC* and *ABD* are right Angles : Therefore the Weight that is hung by a thread in *C*, is heavier than in any other place of the Semicircle *ACQ*, (for the Beam being moved Circularly about the Center *B*) the Burden *C*, is constituted certainly from two Reasons :

sons : The first is taken from what's Demonstrated concerning the Leaver and Ballance, for the more the Weight is distant from the Cock or hollow place, this he appoints to be the heavier ; and the nearer it is to the same, the Lighter ; and the Weight is no where further distant from the Cock than in C, for that is the greatest of all the Subtenses of right Lines, in the Circle which passes through the Center, *by the 15 Prop. of the 3 book of Euclid*, therefore in C is the heaviest ; and this is manifest from Reason and Experience it self, the farther a Weight descends from A towards C, the heavier it becomes and moves the swifter : By the contrary reason from C towards Q, the Weight becomes the lighter, and its Motion slower : By the second Argument he evinces, that a Weight comes nigher to the Center of the Universe, and so is heavier while 'tis moved from C into E, than from F to G, although it passes through the equal Arches CE and FG in equal time ; for it will make the Arches also HC, and FG equal, and by drawing the Lines, they make the Triangles FGL and HCK ; and because the Angles K and L are right Angles, and the Angles BFG, and BCH equal ; the Angle FGL, will be less than the Angle KCH, and therefore the side HK, greater than the side FL : And in the same time the Weight comes nearer to the Center of the World, through each Line, and therefore more by the longer HK, therefore 'tis heavier in C than in F : Therefore the Weight moves as well in F as in R, by its natural Gravity downwards ; but it always grows more and more heavy through the interval AC, and on the contrary 'tis made Lighter from D in Q, and by a twofold reason, the Weight F Weighs heavier and raises R, until they rest in CD in Equilibrio.

2. *Why the Scales being inverted, so that the Beam*
FR,

FR, lies upon the Iron *QB*, which they hold in the hand; *D* which already hath descended in *R*, whether the Scales are empty, or equally loaded, not only not returns to the Equilibrium *CD*, but rather descends more towards *Q*? Aristotle Answers, that the Angle *QBF*, which is made when the Cock is above the Scales, is greater than the Angle *QSR*: And when the Cock is beneath the Beam, the Angle *RBA*, is greater than the Angle *FBA*, but the greater Angle always renders the Weight heavier, and so in the former case the Weight *F*, in the second, the Weight *R* is the heavier, and always draws the Beam downwards, although the Moderns debate this Question many ways in both Arguments.

C H A P. III.

Of the Beam. Fig. 226.

THe Beam is a Static Instrument, fit to examin the Gravity of Bodies, of which a Weight being hung to the extream of the less, or shorter Arm, and another Weight of known Gravity, and mov'd through the greater Arm, so long until the Beam stand in Equilibrio, it will shew the Weight.

The Form of it is such as Fig. 226 shews: The parts of which are: 1. *BC* the Beam, being a Triangular Pillar, Quadrangular, or any other Figure at the will of the Artificer, only the length must be straight; and it it may be made of Wood, of Iron, Steel, Brass, or any other Matter, so that the Gravity of it be known: 2. *ED* the Cock or Tongue, dividing the Beam unequally, and affixt to it in the same manner as in Scales: 3. *BA* the Hook or Scale, also the Weight of this ought to be known: 4. *G* the Weight hung to the Beam, moveable to and fro, and this also ought to be known: The Center of the Weighing Instrument, is where the Cock is bored through, the little Axis (in which *R* Weighs or

or Poises) receives the Arm EC, Being always longer than the Arm EB: 'Tis always divided into divers Parts in a certain Ratio, which is shewn in the 9 and 10 Theorems: Notwithstanding it may briefly be done thus: (see the Scheme of Theorem the 8) Let the Beam be CL, equal in Magnitude and Gravity thro'out the whole; of the Weight of 7 Ounces only, or $1\frac{1}{2}$ of a Pound: And the Center of Weighing or Poiling in B, so that KL be 10, and KC in proportion, and the Weight G one Pound: Now then by the 9 Theorem, seek the Gravity of the Weight D, viz. which being hung from the Beam, may make an Equilibrium, in this manner: multiply the Weight LK, which is of 5 Ounces, into it self, and it will be the Number 25, which divide by the Weight KC, which is $1\frac{1}{2}$, and you will have in the Quotient $12\frac{1}{2}$, to which LK $1\frac{1}{2}$ being added, produces $17\frac{1}{2}$, viz. the Weight D desir'd; which because C is wont to be hung to the Iron Hook CD, let the Hook be of Ounces or twelfths $5\frac{1}{2}$ which taken from the whole Weight, leaves the Weight LK of one Pound: G therefore in K will be equal to one Pound: but because LK is also equal to one Pound, and therefore D, unless it be of two Pound, cannot direct the Beam Parallel to the Horizon; K therefore shews two Pounds, and from the Gravity of G according to the distance from B, in the Center of the Ballance equally increasing, we inscribe S 3, T 4, F 5, V 6, L 7, of known Pounds: And from thence it appears, that all these Spaces may also be subdivided into Ounces, besides the first BK; which yet also may be distinguished by less Marks or Notes, if G in the Center it self B, be understood to avail or signifie a Pound, and so the interval BK will increase by equal Spaces of Ounce by Ounce, and the first Mark or Sign will shew 13, the second 14, &c. until you come to K the Mark or Sign of two Pounds.

PART III.

The Practice.

PROBLEM I.

To Examine Weights by the Ballance or Scales.

1. **I**Nto any Scale or Balance, put any Merchandise or Commodity, whose Weight you desire to know : Into the other Scale or Balance put so much Weight of known Gravity to you, until the little Tongue or Cock BE of *Fig. 224.* exactly agree with the *Ansa*, or that you hold it by BD, and make an Equilibrium, and it will be the whole weight or Gravity of the Commodity or thing to be weighed.

2. Or if the Weights or Poise, by chance being something greater than the Commodity to be Weighed, or outweigh it ; you may put in the Scale it self, which bears the Commodity, a less Weight by so much, as may make it in Equilibrio, and this being Subtracted from the greater Weight of the other Scale, will leave the Gravity of the thing.

PROBLEM II.

To examine Weights by the Beam.

Hang the Thing or Commodity to be Weighed, to the Hook BA of *Fig. 226.* and the Weight G being known, which is moveable ; move to and fro on the Beam, until it consist in Equilibrio, and then the Weight will detain the moveable ; the mark on the beam will shew you the Weight of the thing.

PROBLEM III.

To find the reason and manner of Perfection of a Beam.

The Balance or Scales as being more Simple, so also 'tis more accurate : The Beam as being more troublesome, so 'tis less exact : Hence we Weigh precious or costly things, not on a Beam, but in
Scales

Scales or a Ballance : Nevertheless the manner or way whereby to make a Beam most perfect will be this : Let the Beam by help of the moveable Weight G , be divided in the aforesaid manner, and make G in F to Equiponderate to D (see the Scheme at *Theorem* the 8th) and so D to Equiponderate to G in F , and being hung together, of the difference of the Arms KL : And divide the Weight D , into two portions N and O , so that they make an Equilibrium, the portion N with LK , and the other part O with G ; Then according to the 8 *Theorem*, the Proportion of O to G , is as FB to BC : But the proportion of N to LK by *Theorem* the 10, as to the same Weight hung from M : And by *Theorem* the 9, as LC to CK : If then the Weight N be hung directly in D , and the Beam stand in Equilibrio, and moreover BL being loaded by a Weight hung in G , the Equilibrium of the Beam cannot otherwise be maintained, than if the Weight N , being augmented by the Portion O , is so to G , as the part LB to BC , and so according to the equal increase of BL , increases O , N always remaining the same, wherefore according to the equal increase of the parts BL , the Weight D increases, and will make the Beam alike, if in the extream of it C , a Weight be hung, making Equilibrium with LK ; then divide the greater Arm of the Beam from the Cock to L , by equal Spaces.

PROBLEM IV.

To find the correction of the Beam.

The precedent way of finishing the Beam, is even as the Idea of some perfection of the same : But because for the most part the Weight D depends not from the Term it self, or from the point C of the Beam, but the Hook being fixt within CB : the former division of the Beam is something troubled, and the first mark shewing also the Weight LK , necessarily retires towards B , and will be in P : For if the Weight as well LK as D should be two pounds

according to the former *Ratio*, the first mark in *K* would shew or signify four Pounds; but when *G*, depending from *K*, (to wit, being conveyed in *P*, but in the point of four Pounds, from which it can make an Equilibrium) shall make four Pounds, these being added to the weight *L K* of two Pounds, makes six Pounds, and therefore the mark *K* deceives; nor from thence doth it make an Equilibrium: The mark therefore of four Pounds must be made in *P*, for there, both also *G* and *L K*, the weight of two Pounds, being conjoined, weigh four Pounds, and the Beam is made parallel to the Horizon; and so the first mark is less distant from the Cock, or that you hold by, than the other between themselves, which will be equidistant; as if the second be in *Q*, the third will be in *H*, the fourth in *M*, the fifth in *R*, the sixth in *E* and *C*, and so *G* in *E* will shew twenty four Pounds.

P R O B L E M V.

To weigh at the Beam, the weight of most vast Bodies, yea and of all the whole Globe of the World.

The Great Archimedes boasted, if he had place to set his Feet he would move the whole Earth; if we consider Mathematical Principles, and those already shewn, it is not only possible to raise the whole Orb of the Universe, and remove it from its place, but most easily and almost with no power; this only being presupposed to be granted, from whence either to hang the Cock of the Beam, or (if to be moved by a Leaver) a place to set the Hypomochlion or Prop upon: For since, according to Theorem the VIII. the *Ratio* of Weights standing in Equilibrio, is as the mutual distances of them from the Prop, or from the Cock of the Ballance. The distance of the Earth from the Prop or Hypomochlion may be so lessened, till at last the length of the lesser Arm to the length of the greater, is scarce perceivable, and so will the proportion

* * *

portion be of the weight hung to the greater Arm, to the whole Earth; and so by a Beam, or by an indifferent Leaver, by a small weight, or by a little strength opposed, the whole Burden of the ponderous Globe of the Earth may be raised; yea, by the IX. Theorem, either from the proportion of the greater Arm alone, or the other part of the Leaver, prolonged in the said manner, beyond the hanging of the weight, or by the help of force, the whole Earth may be sustained: But all this Undertaking seems to be in vain, if we judge Physically, and from Natural Principles; for although by Divine Power we should have an Hook, to which we might hang or join, the Handle or Cock of the Beam; or a Base, on which we may place the Prop or Hypomochlion, yet the Work would not succeed: And presupposing neither the weight hung on the Beam, by its natural gravity, to endeavour downwards, according to the Line of the Cock, or parallel to the handle, which necessary Hypothesis falls in the beginning of this Labour; for the Earth indeed by invincible gravity, endeavours perpendicularly towards the Center of the World, but this of raising from a weight hung to the greater Arm and opposite, will incline towards the Earth and the Cock it self, and indeed by how much the more this Arm is prolonged, it will make this Angle of inclination the sharper, and at last this weight on the Beam will hang parallel and perpendicular on the Cock it self, and so both the weight, and part of the Leaver, opposite to the moving power; for unless the power and the weight to be moved, should be accounted to be equally at the Center of the Universe, motion would not follow: But in a Leaver of the second kind, of which the power could only move upward, that by any given proportion it may overcome the natural gravity of the Earth, a word or two; in it, straightway by the application of the Statick Mechanick Instrument, there is the same center of the Earth, or of the Body to be moved, and of the motion of the opposite weight, or of the power; the cause is of stability, not of motion; as all heavy things, by the community of motion of the Center, by a Spherical Congruity, contain themselves in rest to Eternity; so the Earth, being firmly founded on its Base, shall never be removed.

Problem

P R O B L E M VI.

To weigh the two most light Elements Fire and Air.

Water and Earth, and such Bodies where these Elements predominate, are easie to be weighed, but those Elements which in comparison of these are said to be light, have here no sensible gravity, no more than Water has in its own place; nevertheless the way of examining the weight of them, if there is any, this seems the best: Take a Viol or a Glass Retort of Ashes, or of Sand, from the Fire, only not red hot, and forthwith put it into the Scale of a Ballance; also in the other Scale put Weights to gravitate to an Equilibrium, and so let them remain while the Glass grows cold, and you will see the Scale which bears it, something to descend, and the other to rise; then you must load it with a known weight to bring it to an Equilibrium again, and so from this last weight, the weight of the Air may be gathered, and withal, the difference of gravity of both the Elements; for red hot Glass is filled with nothing but Fire, or Air extended to the subtilty of Fire, till it is cooled, and then it is full of Air only.

P R O B L E M VII.

To find the Deceit of Fallacious Ballances.

The making of a Ballance or a pair of Scales may be corrupted divers ways, that they may deceive; perhaps it may suffice to remember from Aristotle one of the ways. A certain Seller of Rich Purple, in those times most precious, used deceitful Scales, so composed, that by the Ware and by the Weights just over against it, both Scales were loaded to an Equilibrium, yet the pound of Purple that he presented to the Buyers, was less than just, by an whole Ounce, viz. the Beam AC in Fig. 224, was divided into 23 equal parts, within the little Axis or Cock 11 parts, and the bearing five to the twelfth part; both the Arms stretched forth unequally,

yet the longer being made smaller, or by the Scale weighing lighter, the Ballance seemed equal; but neither did the whole Weight of twelve Ounces which was in the Scale of the shorter Arm, weighing against the other Scale of the larger Beam, wherein was the Wares of eleven Ounces, destroy the Equilibrium; but this and all other Deceits made by Scales, may easily be detected, by changing of the Weights and Wares into the contrary Scales.

P R O B L E M VIII.

To find out the naughtiness of deceitful Beams.

Generally Beams may be made to commit a fault three ways. 1. If the Beam be not exactly equal in magnitude, thickness and weight throughout the whole length. 2. If the division of the Beam be not exactly equal. 3. If the weight hung about the extrem of the lesser Arm disorder the Ratio of the Beam.

The End of Staticks.

Now follows the *Astronomical* and *Geographical* TABLES.

A CANON of Sines, Tangents, Secants and Verfed Sines.

| D | Sine. | Sine
Com. | Tan. | Tange.
Compl. | Secan. | Secant
Compl. | Verfine | Verline
Comple. | D |
|----|--------|--------------|--------|------------------|---------|------------------|----------|--------------------|----|
| 1 | 1745 | 99985 | 1745 | 5728 | 100015 | 5729 | 13 | 98254 | 89 |
| 2 | 3490 | 99939 | 3492 | 2863 | 100061 | 2865 | 92 | 96510 | 88 |
| 3 | 5233 | 99883 | 5241 | 1908 | 100137 | 1910 | 05 | 94766 | 87 |
| 4 | 6976 | 99756 | 6993 | 1430 | 100244 | 1433 | 60 | 93024 | 86 |
| 5 | 8715 | 99619 | 8749 | 1143 | 100382 | 1147 | 53 | 91284 | 85 |
| 6 | 10453 | 99452 | 10510 | 9514 | 100551 | 9566 | 81 | 89547 | 84 |
| 7 | 12187 | 99255 | 12278 | 8144 | 100751 | 8205 | 39 | 87813 | 83 |
| 8 | 13917 | 99027 | 14054 | 7115 | 100983 | 7185 | 19 | 86082 | 82 |
| 9 | 15643 | 98769 | 15838 | 6313 | 101246 | 6392 | 17 | 84356 | 81 |
| 10 | 17365 | 98481 | 17633 | 5671 | 101543 | 5758 | 22 | 82635 | 80 |
| 11 | 19081 | 98163 | 19438 | 5144 | 101872 | 5240 | 28 | 80919 | 79 |
| 12 | 20791 | 97815 | 21256 | 4701 | 102234 | 4809 | 24 | 79208 | 78 |
| 13 | 22495 | 97437 | 23087 | 4331 | 102630 | 4445 | 00 | 77504 | 77 |
| 14 | 24192 | 97029 | 24933 | 4010 | 103061 | 4133 | 43 | 75807 | 76 |
| 15 | 25882 | 96592 | 26795 | 3732 | 103528 | 3863 | 42 | 74118 | 75 |
| 16 | 27564 | 96126 | 28674 | 3487 | 104030 | 3627 | 83 | 72436 | 74 |
| 17 | 29237 | 95630 | 30573 | 3270 | 104569 | 3420 | 52 | 70762 | 73 |
| 18 | 30902 | 95106 | 32492 | 3077 | 105146 | 3236 | 35 | 69098 | 72 |
| 19 | 32557 | 94552 | 34433 | 2904 | 105762 | 3071 | 10 | 67443 | 71 |
| 20 | 34202 | 93969 | 36397 | 2747 | 106418 | 2923 | 74 | 65797 | 70 |
| 21 | 35837 | 93358 | 38386 | 2605 | 107114 | 2790 | 56 | 64163 | 69 |
| 22 | 37461 | 92718 | 40403 | 2475 | 107853 | 2669 | 61 | 62539 | 68 |
| 23 | 39075 | 92050 | 42447 | 2355 | 108636 | 2559 | 51 | 60926 | 67 |
| 24 | 40674 | 91354 | 44523 | 2246 | 109464 | 2458 | 45 | 59326 | 66 |
| 25 | 42262 | 90631 | 46631 | 2144 | 110338 | 2366 | 22 | 57738 | 65 |
| 26 | 43837 | 89879 | 48773 | 2050 | 111260 | 2281 | 60 | 56162 | 64 |
| 27 | 45399 | 89101 | 50952 | 1962 | 112233 | 2202 | 35 | 54600 | 63 |
| | Si. C. | Sine | Ta. C. | Tange. | Se. Co. | Sec.int. | Ver. Cn. | Verfine | |

A CANON of Sines, Tangents, Secants and Versed Sines.

| D. | Sines | Sine
Com. | Tang. | Tang
Comp. | Seca. | Secan.
Comp. | Versine | Verline
Comple. | D. |
|----|--------|--------------|---------|---------------|--------|-----------------|----------|--------------------|----|
| 28 | 46947 | 88295 | 53171 | 188073 | 113257 | 213005 | 11705 24 | 53052 84 | 62 |
| 29 | 48481 | 87462 | 55431 | 180405 | 114335 | 206266 | 12538 03 | 51519 04 | 61 |
| 30 | 50000 | 86602 | 57735 | 173205 | 115470 | 200000 | 13397 46 | 50000 00 | 60 |
| 31 | 51504 | 85717 | 60086 | 166428 | 116663 | 194160 | 14283 27 | 48496 19 | 59 |
| 32 | 52992 | 84805 | 62487 | 160033 | 117918 | 188708 | 15195 19 | 47008 08 | 58 |
| 33 | 54464 | 83867 | 64941 | 153986 | 119236 | 183608 | 16132 94 | 45536 10 | 57 |
| 34 | 55919 | 82904 | 67451 | 148256 | 120622 | 178829 | 17056 24 | 44080 71 | 56 |
| 35 | 57358 | 81915 | 70021 | 142815 | 122077 | 174345 | 18084 80 | 42642 36 | 55 |
| 36 | 58778 | 80902 | 72654 | 137638 | 123607 | 170130 | 19098 30 | 41221 48 | 54 |
| 37 | 60181 | 79863 | 75355 | 132704 | 125213 | 166164 | 20136 45 | 39818 50 | 53 |
| 38 | 61566 | 78801 | 78128 | 127994 | 126902 | 162427 | 21198 92 | 38433 85 | 52 |
| 39 | 62932 | 77715 | 80978 | 123490 | 128676 | 158902 | 22285 40 | 37067 96 | 51 |
| 40 | 64279 | 76604 | 83910 | 119175 | 130541 | 155572 | 23395 56 | 35721 24 | 50 |
| 41 | 65606 | 75471 | 86929 | 115037 | 132501 | 152425 | 24529 04 | 34304 10 | 49 |
| 42 | 66913 | 74314 | 90040 | 111061 | 134563 | 149448 | 25685 52 | 33086 94 | 48 |
| 43 | 68200 | 73135 | 93251 | 107337 | 136733 | 146628 | 26864 63 | 31800 16 | 47 |
| 44 | 69466 | 71934 | 96569 | 103553 | 139016 | 143956 | 28066 02 | 30534 16 | 46 |
| 45 | 70711 | 70711 | 100000 | 100000 | 141421 | 141421 | 29289 32 | 29289 32 | 45 |
| | Si. C. | Sine | Ta. Co. | Tang. | Se. Co | Secan. | Ver. Co. | Versines | |

This TABLE needs no other Explanation than this: If you want a Sine, or Tangent, or Secant, or Versed Sine, above 45 Degrees, you must seek your Number of Degrees in the first Column on the Right Hand, and your Sine, Tangent, Secant or Versed Sine, at the bottom of the Table, proceeding upwards; as on the contrary, the Degrees of the first Column to the Left Hand, proceed downwards: The Reason is, because here you have the Complements joining to their respective Sines, Tangents, &c.

A TABLE of the Declination of the Ecliptick; and of the Angles of the Ecliptick and Meridian Circle.

| Deg. of Ec. | ♈ | | Angle. of Ecl. & Mer. | ♉ | | Angle. of Ecl. & Mer. | ♊ | | Angle. of Ecl. & Mer. | Deg. of Ec. |
|-------------|----|----|-----------------------|----|----|-----------------------|----|----|-----------------------|-------------|
| | D. | " | | D. | " | | D. | " | | |
| 0 | 0 | 00 | 00 | 66 | 29 | 11 | 30 | 42 | 69 | 21 |
| 1 | 0 | 23 | 56 | 66 | 30 | 11 | 51 | 48 | 69 | 32 |
| 2 | 0 | 47 | 53 | 66 | 30 | 12 | 12 | 40 | 69 | 45 |
| 3 | 1 | 11 | 49 | 66 | 31 | 12 | 33 | 21 | 69 | 57 |
| 4 | 1 | 35 | 49 | 66 | 33 | 12 | 53 | 50 | 70 | 10 |
| 5 | 1 | 59 | 37 | 66 | 34 | 13 | 14 | 05 | 70 | 23 |
| 6 | 2 | 23 | 28 | 66 | 36 | 13 | 34 | 07 | 70 | 36 |
| 7 | 2 | 47 | 16 | 66 | 39 | 13 | 53 | 57 | 70 | 51 |
| 8 | 3 | 11 | 04 | 66 | 42 | 14 | 13 | 32 | 71 | 05 |
| 9 | 3 | 34 | 47 | 66 | 45 | 14 | 32 | 53 | 71 | 20 |
| 10 | 3 | 58 | 28 | 66 | 49 | 14 | 51 | 59 | 71 | 34 |
| 11 | 4 | 22 | 34 | 66 | 52 | 15 | 10 | 50 | 71 | 50 |
| 12 | 4 | 45 | 37 | 66 | 56 | 15 | 29 | 26 | 72 | 06 |
| 13 | 5 | 09 | 05 | 67 | 01 | 15 | 47 | 47 | 72 | 22 |
| 14 | 5 | 32 | 29 | 67 | 07 | 16 | 05 | 51 | 72 | 38 |
| 15 | 5 | 55 | 47 | 67 | 12 | 16 | 23 | 39 | 72 | 54 |
| 16 | 6 | 18 | 55 | 67 | 18 | 16 | 41 | 09 | 73 | 10 |
| 17 | 6 | 42 | 06 | 67 | 24 | 16 | 58 | 22 | 73 | 27 |
| 18 | 7 | 05 | 06 | 67 | 30 | 17 | 15 | 18 | 73 | 45 |
| 19 | 7 | 28 | 00 | 67 | 38 | 17 | 31 | 54 | 74 | 05 |
| 20 | 7 | 50 | 46 | 67 | 46 | 17 | 48 | 14 | 74 | 23 |
| 21 | 8 | 13 | 26 | 67 | 53 | 18 | 04 | 14 | 74 | 41 |
| 22 | 8 | 35 | 58 | 68 | 01 | 18 | 19 | 57 | 75 | 00 |
| 23 | 8 | 58 | 20 | 68 | 10 | 18 | 35 | 18 | 75 | 19 |
| 24 | 9 | 20 | 34 | 68 | 20 | 18 | 50 | 21 | 75 | 39 |
| 25 | 9 | 42 | 41 | 68 | 28 | 19 | 05 | 04 | 75 | 59 |
| 26 | 10 | 04 | 38 | 68 | 38 | 19 | 19 | 26 | 76 | 20 |
| 27 | 10 | 26 | 24 | 68 | 48 | 19 | 33 | 26 | 76 | 40 |
| 28 | 10 | 48 | 02 | 68 | 59 | 19 | 47 | 07 | 77 | 01 |
| 29 | 11 | 09 | 27 | 69 | 10 | 20 | 00 | 26 | 77 | 23 |
| 30 | 11 | 30 | 42 | 69 | 21 | 20 | 13 | 22 | 77 | 44 |
| | ♊ | ♋ | | ♌ | ♍ | | ♎ | ♏ | | |

The Use of the Table of Declinations.

IF the Sign of which you want the Degrees of Declination be found in the superior Line of the Table, you must take the Degree in the Left Hand part of the Table; but if the Sign shall be found in the inferior line of the Table, the Degree must be taken on the Right Hand part, and forthwith, in the common meeting of the Sign and of the Degree, you will light upon the Degree and Scruples of Declination. *Example*, Would you know how much 17° of ♈ declines from the Equator? in the Left Hand part of the Table I take 17° of ♈ , (for this Sign is placed in the upper part of the Table) and in the common Angle under ♈ , I find $16^{\circ} : 58' : 22''$. so much therefore I pronounce the Declination to be of 17° of ♈ : Likewise if you would find how much Declination 23° of ♋ hath, then because this Sign is in the lower part of the Table, I find in the Right Hand part, the said 23° , above the Sign ♋ , to answer to $21^{\circ} : 33' : 22''$. and so much is the Declination sought; but if you have Minutes proposed, as well as Degrees, you must take the Declination of the next lesser and the next greater Degree, and by the difference of them, find out the part proportional, which must be added to the Declination of the next lesser Degree, if the Sign proposed be in the upper part; but subtracted from the same next lesser Declination, if the Sign shall be beneath at the bottom of the Table.

An Example of both: I would know the Declination of $4^{\circ} : 27'$ of the Sign ♈ , then because I have no Minutes in the Table, I take the difference of Declination of 4° and 5° of the Sign ♈ , which difference contains $10' : 44''$. which I reduce into Second Minutes, and they make $644''$. then I say by the Rule of Proportion, if $60'$ give $644''$. what will $27'$ give? *Answer*, $4' : 57''$. which must be added to $21^{\circ} : 1' : 25''$ the Declination of 4° of ♈ , because the Sign ♈ is Superior in the Table, and it makes $21^{\circ} : 6' : 22''$. and so much is the Declination of $4^{\circ} : 27'$ of ♈ .

The Use of the Table of Declinations:

IN like manner I would know the Declination of $5^{\circ} : 32'$ of the Sign ϖ ; then, because the Sign proposed is at the bottom of the Table, we seek the Degrees on the right side, and the part proportional must be subtracted: The difference between the Declination of 5° and 6° of ϖ , is $2' : 30''$. which reduced, is $150''$. Say then, if 60° give $150''$, what will $32'$ give? *Answer*, it gives $1' : 20''$, to be subtracted from $23^{\circ} : 25' : 48''$, and there remains $23^{\circ} : 24' : 28''$. for the Declination of $5^{\circ} 32'$ of ϖ from the Ecliptick.

A TABLE of Right Ascensions, to the Tychoonick Obliquity $23^{\circ}31'$.

| D. | Υ | Δ | Co | δ | m | Co | Π | τ | Co |
|----|------------|----------|----|----------|-----|-----|-------|--------|-----|
| D. | D. | De. | S. | D. | De. | Sc. | D | De. | Sc. |
| 0 | 0 | 180 | 0 | 27 | 207 | 54 | 57 | 237 | 48 |
| 1 | 0 | 180 | 55 | 28 | 208 | 51 | 58 | 238 | 51 |
| 2 | 1 | 181 | 50 | 29 | 209 | 49 | 59 | 239 | 53 |
| 3 | 2 | 182 | 45 | 30 | 210 | 46 | 60 | 240 | 56 |
| 4 | 3 | 183 | 40 | 31 | 211 | 44 | 61 | 241 | 59 |
| 5 | 4 | 184 | 35 | 32 | 212 | 42 | 63 | 243 | 3 |
| 6 | 5 | 185 | 30 | 33 | 213 | 40 | 64 | 244 | 6 |
| 7 | 6 | 186 | 25 | 34 | 214 | 38 | 65 | 245 | 9 |
| 8 | 7 | 187 | 21 | 35 | 215 | 37 | 66 | 246 | 13 |
| 9 | 8 | 188 | 16 | 36 | 216 | 36 | 67 | 247 | 17 |
| 10 | 9 | 189 | 11 | 37 | 217 | 34 | 68 | 248 | 29 |
| 11 | 10 | 190 | 6 | 38 | 218 | 33 | 69 | 249 | 25 |
| 12 | 11 | 191 | 2 | 39 | 219 | 33 | 70 | 250 | 29 |
| 13 | 11 | 191 | 57 | 40 | 220 | 32 | 71 | 251 | 34 |
| 14 | 12 | 192 | 53 | 41 | 221 | 31 | 72 | 252 | 38 |
| 15 | 13 | 193 | 48 | 42 | 222 | 31 | 73 | 253 | 43 |
| 16 | 14 | 194 | 44 | 43 | 223 | 31 | 74 | 254 | 47 |
| 17 | 15 | 195 | 40 | 44 | 224 | 31 | 75 | 255 | 52 |
| 18 | 16 | 196 | 35 | 45 | 225 | 31 | 76 | 256 | 57 |
| 19 | 17 | 197 | 31 | 46 | 226 | 32 | 78 | 258 | 2 |
| 20 | 18 | 198 | 27 | 47 | 227 | 32 | 79 | 259 | 7 |
| 21 | 19 | 199 | 23 | 48 | 228 | 33 | 80 | 260 | 12 |
| 22 | 20 | 200 | 20 | 49 | 229 | 34 | 81 | 261 | 17 |
| 23 | 21 | 201 | 16 | 50 | 230 | 35 | 82 | 262 | 22 |
| 24 | 22 | 202 | 12 | 51 | 231 | 36 | 83 | 263 | 28 |
| 25 | 23 | 203 | 9 | 52 | 232 | 38 | 84 | 264 | 33 |
| 26 | 24 | 204 | 6 | 53 | 233 | 40 | 85 | 265 | 38 |
| 27 | 25 | 205 | 2 | 54 | 234 | 41 | 86 | 266 | 44 |
| 28 | 25 | 205 | 59 | 55 | 235 | 41 | 87 | 267 | 49 |
| 29 | 26 | 206 | 57 | 56 | 236 | 46 | 88 | 268 | 55 |
| 30 | 27 | 207 | 54 | 57 | 237 | 48 | 90 | 270 | 0 |

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A Table of right Ascensions to the
Tychonick Obliquity $23^{\circ} : 31'$.

| D. | $\odot$
Deg | $\vee$
Deg | Co
Sc. | Ω
De. | π
Deg | Co
Sc. | η
Deg | χ
Deg | Co
Sc. |
|----|----------------|---------------|-----------|-----------------|--------------|-----------|---------------|---------------|-----------|
| 0 | 90 | 270 | 0 | 122 | 302 | 12 | 152 | 332 | 6 |
| 1 | 91 | 271 | 5 | 123 | 303 | 14 | 153 | 333 | 3 |
| 2 | 92 | 272 | 11 | 124 | 304 | 16 | 154 | 334 | 1 |
| 3 | 93 | 273 | 16 | 125 | 305 | 19 | 154 | 334 | 58 |
| 4 | 94 | 274 | 22 | 126 | 306 | 20 | 155 | 335 | 54 |
| 5 | 95 | 275 | 27 | 127 | 307 | 22 | 156 | 336 | 51 |
| 6 | 96 | 276 | 32 | 128 | 308 | 24 | 157 | 337 | 48 |
| 7 | 97 | 277 | 38 | 129 | 309 | 25 | 158 | 338 | 44 |
| 8 | 98 | 278 | 43 | 130 | 310 | 26 | 159 | 339 | 40 |
| 9 | 99 | 279 | 48 | 131 | 311 | 27 | 160 | 340 | 37 |
| 10 | 100 | 280 | 53 | 132 | 312 | 28 | 161 | 341 | 33 |
| 11 | 101 | 281 | 58 | 133 | 313 | 28 | 162 | 342 | 29 |
| 12 | 103 | 283 | 3 | 134 | 314 | 29 | 163 | 343 | 25 |
| 13 | 104 | 284 | 8 | 135 | 315 | 29 | 164 | 344 | 20 |
| 14 | 105 | 285 | 13 | 136 | 316 | 29 | 165 | 345 | 16 |
| 15 | 106 | 286 | 17 | 137 | 317 | 29 | 166 | 346 | 12 |
| 16 | 107 | 287 | 22 | 138 | 318 | 29 | 167 | 347 | 7 |
| 17 | 108 | 288 | 26 | 139 | 319 | 28 | 168 | 348 | 3 |
| 18 | 109 | 289 | 31 | 140 | 320 | 27 | 168 | 348 | 58 |
| 19 | 110 | 290 | 35 | 141 | 321 | 27 | 169 | 349 | 54 |
| 20 | 111 | 291 | 39 | 142 | 322 | 26 | 170 | 350 | 49 |
| 21 | 112 | 292 | 43 | 143 | 323 | 24 | 171 | 351 | 44 |
| 22 | 113 | 293 | 47 | 144 | 324 | 23 | 172 | 352 | 39 |
| 23 | 114 | 294 | 51 | 145 | 325 | 22 | 173 | 353 | 35 |
| 24 | 115 | 295 | 54 | 146 | 326 | 20 | 174 | 354 | 30 |
| 25 | 116 | 296 | 57 | 147 | 327 | 18 | 175 | 355 | 25 |
| 26 | 118 | 298 | 1 | 148 | 328 | 16 | 176 | 356 | 20 |
| 27 | 119 | 299 | 4 | 149 | 329 | 14 | 177 | 357 | 15 |
| 28 | 120 | 300 | 7 | 150 | 330 | 11 | 178 | 358 | 10 |
| 29 | 121 | 301 | 9 | 151 | 331 | 9 | 179 | 359 | 5 |
| 30 | 122 | 302 | 12 | 152 | 332 | 6 | 180 | 360 | 0 |

The use of the Table of right Ascensions.

YOU must take the Sign in the Head of the Table, and on the left side, the Degree of the Sign, for in the common meeting of the Sign, and of the Degree proposed, you will find the Degree and Minute of the Equator, which arises together with the given Degree of the Ecliptick; so you will see, with 19° of Ω , that is, with the Arch of the Ecliptick, begun from the beginning of Υ , and ending in the 19° of Ω , which comprehends 139° , in a right Horizon, to arise $141^{\circ} : 27'$ of the Equinoctial Circle: But if Minutes adhere to the given Arch, you must seek out the part proportional, answering to the given Minutes, as is said in the use of the Table of Declinations, and that must be added to the Ascension of the Arch of whole Degrees of the next lesser. *Example.* Lets seek the right Ascension of the Arch of the Ecliptick, containing $125^{\circ} : 40'$, that is, the Ascension of $5^{\circ} : 40'$ of Ω : Subtract the Ascension of 5° of Ω , viz. of $127^{\circ} : 22'$ from the Ascension of 6° of Ω , to wit, from $128^{\circ} : 24'$, and there will remain $1^{\circ} : 2'$, the difference of both Ascensions, which suits with $60'$. Wherefore, according to the Rule of Proportion, in Minutes, will be given Minutes $41\frac{1}{2}$, which, if you add to the Ascension of 5° of Ω , you will have the Ascension of the proposed Arch of $128^{\circ} . 3'$ almost.

But if you inquire the Ascension of an Arch of the Ecliptick, beginning not from the beginning of Υ ; for Example, of an Arch of the Zodiack, beginning from 10° of δ , and ending in 18° of Π , which contains 38° , you must subtract the Ascension of 10° of δ , to wit, of $37^{\circ} 34'$, from the Ascension of 18° of Π , that is, from $76^{\circ} 57'$, and there will be left the Ascension of the proposed Arch of $39^{\circ} : 23'$. Thus by the preceding Table you may find out the right Ascensions.

In this manner we may easily gather the right Ascension of any Sign of the Zodiack, taken by it self, as appears in the form following.

The

The Ascensions of the Signs in a right Sphere

| | | | | |
|-----------|----------|-------------|--------------|-----------|
| ♈ Aries. | ♍ Virgo. | ♎ Libra. | ♐ Pisces. | 27° : 54' |
| ♉ Taurum. | ♌ Leo. | ♏ Scorpio. | ♑ Aquarius. | 29 : 54 |
| ♊ Gemini. | ♋ Cancer | ♏ Sagitary. | ♒ Capricorn. | 32 : 12 |

Hence it plainly follows, in a right Sphere, that the four Signs which stand about the two Solstitial Points, arise rightly, but the other eight obliquely; also those Arches arise more oblique which are nearer to the Equinoctial Points, but those more rightly which exist nearer to the Solstitial Points: The same may be said concerning the Descensions and Mediations of the Heaven; therefore in a right Sphere always four Signs have equal Ascensions and equal Descensions, and Mediations of Heaven, all which indeed may be demonstrated from Spherical Triangles: By this you see which Signs arise rightly in a right Sphere, and which obliquely.

Note, that the Column in the Table of right Ascensions, whose Title is Common Scruples, serves to both the adjoining Columns on the Left Hand.

From this Table of Right Ascensions, and the following Table of oblique Ascensions, you may find ☉ Rising and Setting, with the length of the Day and Night: for the Ascensional difference of the ☉, (which you have, by subtracting the right and oblique Ascensions one from the other) being added to the Semidiurnal Arch in a right Sphere, that is, to 90° in the Northern Signs, or subtracted from it in the Southern, their Sum or Difference will be the Semidiurnal Arch, which doubled is the right Arch, which bisected, is the time of ☉ Rising, and the day Arch bisected, is ☉ Setting.

As when ☉ is in 0 Degrees of ♊, his Ascensional Difference is 27° : 36', which being added to 90°, because the declination is North, the Sum will be 117° : 36', the Semidiurnal Arch.

The double whereof is 235° : 12', the diurnal Arch, which being converted into time, makes 15 Hours, 41 Minutes, for the length of the day, whose Complement to 24, is 8 Hours, 19 Minutes, the length of the Night; the half whereof is 4 Hours, 9 Minutes, 30 Seconds, the time of ☉ Rising.

A TABLE of the oblique Ascensions
of all the Points of the Ecliptick for
the Latitude of *London*, being
51°. 32'.

| De.
° | ♈
De. ' | ♉
De. ' | ♊
De. ' | ♋
De. ' | ♌
De. ' | ♍
De. ' |
|----------|------------|------------|------------|------------|------------|------------|
| 0 | 0 00 | 13 04 | 30 12 | 56 48 | 94 35 | 137 15 |
| 1 | 0 24 | 13 32 | 30 54 | 57 54 | 95 59 | 138 42 |
| 2 | 0 49 | 14 01 | 31 38 | 59 01 | 97 24 | 140 08 |
| 3 | 1 14 | 14 30 | 32 21 | 60 08 | 98 46 | 141 34 |
| 4 | 1 39 | 15 01 | 33 06 | 61 16 | 100 10 | 143 00 |
| 5 | 2 04 | 15 30 | 33 50 | 62 25 | 101 35 | 144 26 |
| 6 | 2 29 | 16 00 | 34 35 | 63 35 | 102 59 | 145 52 |
| 7 | 2 54 | 16 31 | 35 22 | 64 46 | 104 23 | 147 17 |
| 8 | 3 19 | 17 02 | 36 08 | 65 57 | 105 48 | 148 43 |
| 9 | 3 45 | 17 33 | 36 57 | 67 10 | 107 13 | 150 09 |
| 10 | 4 10 | 18 05 | 37 46 | 68 22 | 108 38 | 151 34 |
| 11 | 4 36 | 18 37 | 38 35 | 69 35 | 110 03 | 153 00 |
| 12 | 5 01 | 19 10 | 39 26 | 70 49 | 111 29 | 154 25 |
| 13 | 5 26 | 19 42 | 40 16 | 72 03 | 112 54 | 155 51 |
| 14 | 5 52 | 20 14 | 41 08 | 73 19 | 114 21 | 157 16 |
| 15 | 6 17 | 20 48 | 42 01 | 74 35 | 115 46 | 158 44 |
| 16 | 6 43 | 21 22 | 42 54 | 75 52 | 117 11 | 160 07 |
| 17 | 7 09 | 21 58 | 43 49 | 77 09 | 118 37 | 161 32 |
| 18 | 7 35 | 22 33 | 44 44 | 78 28 | 120 03 | 162 58 |
| 19 | 8 02 | 23 08 | 45 39 | 79 46 | 121 28 | 164 23 |
| 20 | 8 28 | 23 43 | 46 36 | 81 04 | 122 55 | 165 48 |
| 21 | 8 55 | 24 20 | 47 33 | 82 24 | 124 21 | 167 14 |
| 22 | 9 22 | 24 58 | 48 31 | 83 43 | 125 47 | 168 39 |
| 23 | 9 49 | 25 35 | 49 30 | 85 03 | 127 14 | 170 04 |
| 24 | 10 16 | 26 13 | 50 30 | 86 23 | 128 40 | 171 30 |
| 25 | 10 44 | 26 51 | 51 31 | 87 44 | 130 06 | 172 54 |
| 26 | 11 11 | 27 30 | 52 33 | 89 06 | 131 32 | 174 20 |
| 27 | 11 39 | 28 09 | 53 36 | 90 28 | 132 58 | 175 45 |
| 28 | 12 07 | 28 49 | 54 39 | 91 51 | 134 24 | 177 10 |
| 29 | 12 35 | 29 31 | 55 43 | 93 13 | 135 50 | 178 35 |
| 30 | 13 04 | 30 12 | 56 48 | 94 36 | 137 15 | 180 00 |

A Continuation of the TABLE of
oblique Ascensions of all the Points
of the Ecliptick for the Latitude of
London, being $51^{\circ} 32'$.

| De. | ♈ | ♉ | ♊ | ♋ | ♌ | ♍ |
|-----|--------|--------|--------|--------|--------|--------|
| De. | De. | De. | De. | De. | De. | De. |
| 0 | 180 00 | 222 45 | 265 24 | 303 12 | 329 48 | 346 56 |
| 1 | 181 25 | 224 10 | 266 47 | 304 17 | 330 29 | 347 25 |
| 2 | 182 50 | 225 36 | 268 09 | 305 21 | 331 11 | 347 53 |
| 3 | 184 15 | 227 02 | 269 32 | 306 24 | 331 51 | 348 21 |
| 4 | 185 40 | 228 28 | 370 54 | 307 27 | 332 30 | 348 49 |
| 5 | 187 06 | 229 54 | 272 16 | 308 29 | 333 09 | 349 16 |
| 6 | 188 30 | 231 20 | 273 37 | 309 30 | 333 47 | 349 44 |
| 7 | 189 56 | 232 46 | 274 57 | 310 30 | 334 25 | 350 11 |
| 8 | 191 21 | 234 13 | 276 17 | 311 29 | 335 02 | 350 38 |
| 9 | 192 46 | 235 39 | 277 36 | 312 27 | 335 40 | 351 05 |
| 10 | 194 12 | 237 05 | 278 56 | 313 24 | 336 17 | 351 32 |
| 11 | 195 37 | 238 32 | 280 14 | 314 21 | 336 52 | 351 58 |
| 12 | 197 02 | 239 57 | 281 32 | 315 16 | 337 27 | 352 25 |
| 13 | 198 28 | 241 23 | 282 51 | 316 11 | 338 02 | 352 51 |
| 14 | 199 53 | 242 49 | 284 08 | 317 06 | 338 38 | 353 17 |
| 15 | 201 16 | 244 14 | 285 25 | 317 59 | 339 12 | 353 43 |
| 16 | 202 44 | 245 39 | 286 41 | 318 52 | 339 46 | 354 08 |
| 17 | 204 09 | 247 06 | 287 57 | 319 44 | 340 18 | 354 34 |
| 18 | 205 35 | 248 31 | 289 11 | 320 34 | 340 50 | 354 59 |
| 19 | 207 00 | 249 57 | 290 25 | 321 25 | 341 23 | 355 24 |
| 20 | 208 26 | 251 22 | 291 38 | 322 14 | 341 55 | 355 50 |
| 21 | 209 51 | 252 47 | 292 50 | 323 03 | 342 27 | 356 15 |
| 22 | 211 17 | 254 12 | 294 03 | 323 52 | 342 58 | 356 41 |
| 23 | 212 43 | 255 37 | 295 14 | 324 38 | 343 29 | 357 06 |
| 24 | 214 08 | 257 01 | 296 25 | 325 25 | 344 00 | 357 31 |
| 25 | 215 34 | 258 25 | 297 35 | 326 10 | 344 30 | 357 56 |
| 26 | 217 00 | 259 50 | 298 44 | 326 54 | 344 59 | 358 21 |
| 27 | 218 26 | 261 14 | 299 52 | 327 39 | 345 30 | 358 46 |
| 28 | 219 52 | 262 36 | 300 59 | 328 22 | 345 59 | 359 11 |
| 29 | 221 18 | 264 01 | 302 06 | 329 06 | 346 28 | 359 36 |
| 30 | 222 45 | 265 24 | 303 12 | 329 48 | 346 56 | 360 00 |

The

The Use of the Table of oblique Ascensions.

From this Table you may find the oblique Ascension of any Arch, in the same manner as in the use of the Table of right Ascensions is shewn, nevertheless they must be taken in that Table of oblique Ascensions of the elevation of the Pole, in which you seek the Ascensions; but however, you may find the descension of any Arch in any oblique Sphere, for in a right Sphere the Ascension and Descension of the same Arch are equal; it is shewn that the Ascension of any Arch is equal to the Descension of the opposite Arch, and the Descension of any Arch is equal to the Ascension of the opposite Arch; therefore if the Descension of any Arch be sought, you must find the Ascension of the opposite Arch, for this will be the Descension of the proposed Arch. *Example.* If we want the Descension of the Arch from γ , even to 8° of ϖ , of *Rome* where the Pole is elevated 42° , the opposite Arch is from ϖ , even to 8° of κ ; and because 8° of κ ascend with $347^\circ : 29'$ of the Equator, beginning from γ , if you subtract 180° , viz. a Semicircle from γ even to ϖ , there will remain the Ascension of the Arch from ϖ unto 8° κ , that is, the Descension of the Arch from γ even to 8° ϖ , $167^\circ : 29'$: In like manner let's seek the Descension of the Arch from the beginning of $\mathfrak{m}$ even to 20° of $\mathfrak{z}$, the opposite Arch, is from $\mathfrak{z}$ even to 20° of Ω ; and because 20° of Ω , beginning from the beginning of $\mathfrak{z}$, ascends with $111^\circ : 15'$ of the Equator, so much we say is the Descension of the Arch contained between the beginning of $\mathfrak{m}$ and 20° of $\mathfrak{z}$: In like manner we find the Descension of the last Degree of $\mathfrak{z}$, that is, the Arch contained between the beginning of γ and

The Use of the Table of Oblique Ascensions.

and the last Degree of $\uparrow$, the Arch opposite to this Arch, is contained between the beginning of α and the end of Π ; for the first points of the said Arches, and not the Extrems, are opposite by the diameter in a Sphere; and the Arch from α unto the end of κ , ascends with 180° of the Equator, and the Arch from γ to the end of Π , with $66^\circ : 57'$ of the Equator, to which, if we add 180° , we shall have the Ascension of the Arch, from the beginning of α unto the end of Π , that is, the Ascension of the Arch, from the beginning of γ unto the end of $\uparrow$ $246^\circ : 57'$, and so of the rest.

Also the Descension of any Arch, beginning from the beginning of γ , may be found thus; take away from the Ascension of the point, which is opposite to the extream point, by the diameter of the proposed Arch, an half Circle, that is 180° ; but if the Subtraction cannot be made, first add 360° , that is, an whole Circle, to the Ascension of the opposite point, for that which remains will be the Descension sought. *Example.* Let's seek the Descension of 8° of Ω at Rome; from the Ascension of 8° of α , that is, from $327^\circ : 45'$, subtract 180° , and there remains the Descension of the Arch from γ even to 8° of Leo or Ω of $147^\circ : 45'$. Again, would we find the Descension of 20° of M , we add to the Ascension of 20° of γ , viz. to $30^\circ : 46'$, an whole Circle, or 360° , and from the compounded number, that is, from $390^\circ : 46'$, we take away 180° , and there will be left the Descension of the Arch from γ even to 20° of M , of Degrees 210, Minutes 46, &c.

*

A

A TABLE of the longest Day and longest Night, between the *Polar Circle* and the *North Pole*.

| Altit.
of No.
Pole. | Contin.
Day in
Sumer. | Contin.
Night
in Win. | Altit.
of No.
Pole. | Contin.
Day in
Sumer. | Contin.
Night
in Win. |
|---------------------------|-----------------------------|-----------------------------|---------------------------|-----------------------------|-----------------------------|
| De. ' Da. H. Ni. H. | De. ' Da. H. Ni. H. | De. ' Da. H. Ni. H. | De. ' Da. H. Ni. H. | De. ' Da. H. Ni. H. | De. ' Da. H. Ni. H. |
| 67 0 | 23 18 | 22 1 | 79 0 | 127 19 | 121 7 |
| 67 30 | 33 17 | 31 13 | 79 30 | 130 17 | 124 2 |
| 68 0 | 41 14 | 39 2 | 80 0 | 133 13 | 126 20 |
| 68 30 | 48 6 | 45 8 | 80 30 | 136 8 | 128 4 |
| 69 0 | 54 3 | 50 22 | 81 0 | 139 3 | 132 7 |
| 69 20 | 59 12 | 56 0 | 81 30 | 141 21 | 135 0 |
| 70 0 | 64 11 | 60 16 | 82 0 | 144 14 | 137 17 |
| 70 30 | 69 4 | 65 2 | 82 30 | 147 8 | 140 9 |
| 71 0 | 73 13 | 69 6 | 83 0 | 150 0 | 142 23 |
| 71 30 | 77 17 | 73 5 | 83 30 | 152 16 | 145 13 |
| 72 0 | 81 17 | 77 1 | 84 0 | 155 8 | 148 4 |
| 72 30 | 85 14 | 80 17 | 84 30 | 158 0 | 150 18 |
| 73 0 | 89 8 | 84 6 | 85 0 | 160 15 | 153 8 |
| 73 30 | 92 22 | 87 18 | 85 30 | 163 5 | 155 22 |
| 74 0 | 96 10 | 91 2 | 86 0 | 165 19 | 158 12 |
| 74 30 | 99 21 | 94 9 | 86 30 | 168 9 | 161 2 |
| 75 0 | 103 5 | 97 14 | 87 0 | 170 23 | 163 15 |
| 75 30 | 106 11 | 100 17 | 87 30 | 173 13 | 166 4 |
| 76 0 | 109 16 | 103 19 | 88 0 | 176 2 | 168 16 |
| 76 30 | 112 20 | 106 20 | 88 30 | 178 16 | 171 0 |
| 77 0 | 115 22 | 109 20 | 89 0 | 181 5 | 173 19 |
| 77 30 | 118 22 | 112 17 | 89 30 | 183 19 | 176 9 |
| 78 0 | 121 22 | 115 14 | 90 0 | 186 17 | 178 22 |
| 78 30 | 124 21 | 118 11 | | | |

A TABLE of the Length of the Day and Night under the Meridian of *Tubing*, Latitude 48°. 34.

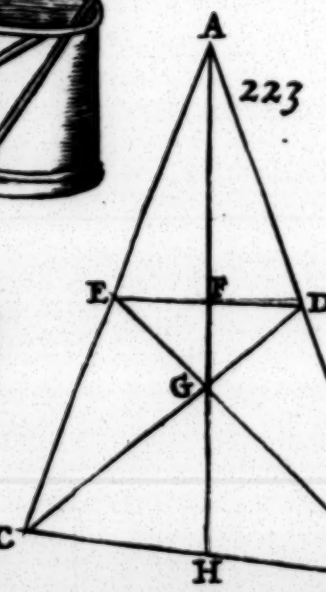
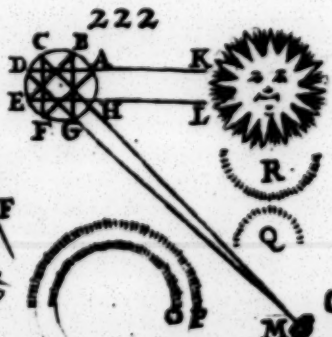
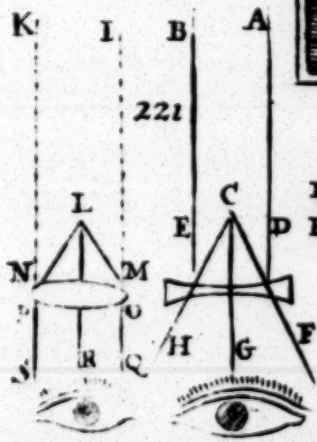
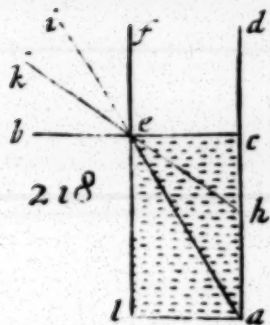
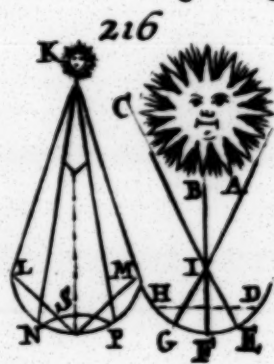
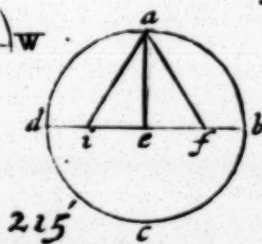
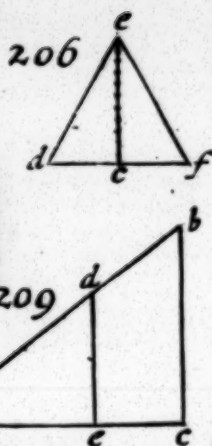
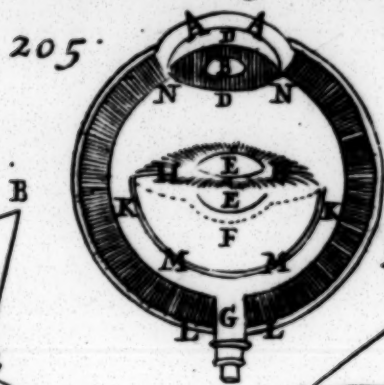
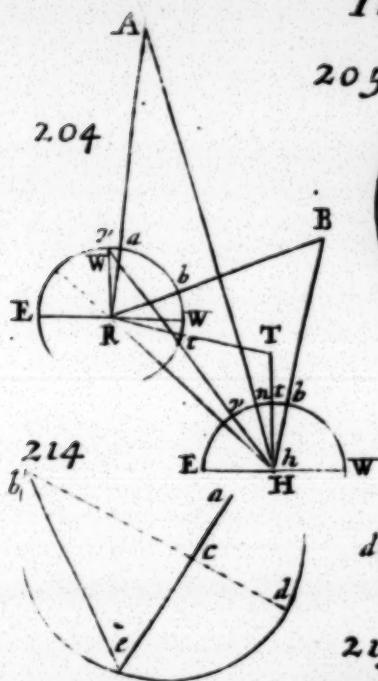
| ☾ | ☿ | ♈ | ♉ | ♊ | ♋ | ♌ | ☿ |
|-----|-------|-------|-------|-------|-------|------|-----|
| Ho. | Ho. | Ho. | Ho. | Ho. | Ho. | Ho. | Ho. |
| 0 | 12 0 | 12 0 | 13 40 | 10 14 | 15 16 | 8 41 | 50 |
| 1 | 12 4 | 11 56 | 13 50 | 10 10 | 15 18 | 8 42 | 29 |
| 2 | 12 8 | 11 52 | 13 54 | 1 6 | 15 22 | 8 38 | 28 |
| 3 | 12 10 | 11 50 | 13 55 | 10 4 | 15 24 | 8 36 | 27 |
| 4 | 12 14 | 11 46 | 14 00 | 10 0 | 15 26 | 8 34 | 2 |
| 5 | 12 18 | 11 42 | 14 04 | 9 56 | 15 28 | 8 32 | 25 |
| 6 | 12 22 | 11 38 | 14 06 | 9 54 | 15 30 | 8 28 | 24 |
| 7 | 12 26 | 11 34 | 14 10 | 9 50 | 15 52 | 8 28 | 23 |
| 8 | 12 28 | 11 32 | 14 14 | 9 46 | 15 34 | 8 26 | 22 |
| 9 | 12 32 | 11 28 | 14 15 | 9 44 | 15 35 | 8 24 | 21 |
| 10 | 12 36 | 11 24 | 14 20 | 9 40 | 15 38 | 8 22 | 20 |
| 11 | 12 40 | 11 20 | 14 22 | 9 38 | 15 40 | 8 20 | 19 |
| 12 | 12 44 | 11 16 | 14 26 | 9 34 | 15 42 | 8 18 | 18 |
| 13 | 12 46 | 11 14 | 14 30 | 9 30 | 15 42 | 8 18 | 17 |
| 14 | 12 50 | 11 10 | 14 32 | 9 28 | 15 44 | 8 16 | 16 |
| 15 | 12 54 | 11 6 | 14 36 | 9 24 | 15 46 | 8 14 | 15 |
| 16 | 12 58 | 11 2 | 14 38 | 9 22 | 15 46 | 8 14 | 14 |
| 17 | 13 02 | 10 58 | 14 42 | 9 18 | 15 48 | 8 12 | 13 |
| 18 | 13 04 | 10 56 | 14 44 | 9 16 | 15 50 | 8 10 | 12 |
| 19 | 13 08 | 10 52 | 14 48 | 9 12 | 15 50 | 8 10 | 11 |
| 20 | 13 12 | 10 48 | 14 50 | 9 10 | 15 52 | 8 8 | 10 |
| 21 | 13 16 | 10 44 | 14 54 | 9 6 | 15 52 | 8 8 | 9 |
| 22 | 13 18 | 10 42 | 14 56 | 9 4 | 15 52 | 8 8 | 8 |
| 23 | 13 22 | 10 38 | 14 58 | 9 2 | 15 54 | 8 6 | 7 |
| 24 | 13 26 | 10 34 | 15 02 | 8 58 | 15 54 | 8 6 | 6 |
| 25 | 13 30 | 10 30 | 15 04 | 8 56 | 15 54 | 8 6 | 5 |
| 26 | 13 32 | 10 28 | 15 06 | 8 54 | 15 56 | 8 4 | 4 |
| 27 | 13 36 | 10 24 | 15 10 | 8 50 | 15 56 | 8 4 | 3 |
| 28 | 13 38 | 10 22 | 15 12 | 8 48 | 15 56 | 8 4 | 2 |
| 29 | 13 42 | 10 18 | 15 14 | 8 46 | 15 56 | 8 4 | 1 |
| 30 | 13 46 | 10 14 | 15 16 | 8 44 | 15 56 | 8 4 | 0 |
| ☿ | ☿ | ☿ | ☿ | ☿ | ☿ | ☿ | ☿ |

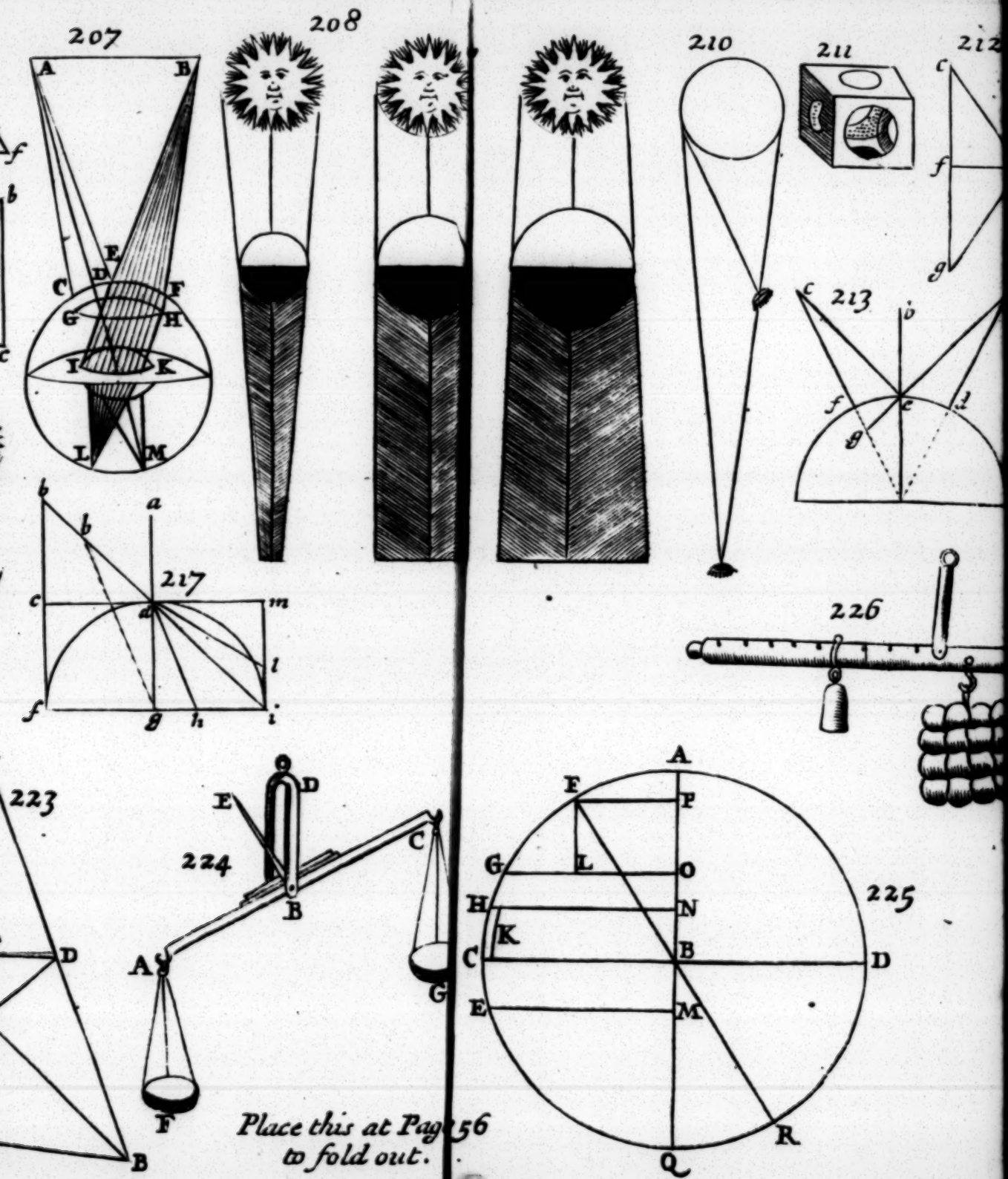
772
 Catalogue of the Longitude, of the Latitude and Magnitude
 of notable fixed Stars, from the Observations of Tycho, reduced
 to the beginning of the Julian Period.

| Names of STARS. | Their Longitude. | | Their Latit. | | Magni-
tude. |
|---|------------------|-------|--------------|-----|-----------------|
| | Sig. | Deg. | Deg. | " | |
| I. Stars of the Zodi. | | | | | |
| 1. The southern in
the former horn of $\vee$ | 9 | 28 10 | 57 7 8 | 30N | 4 |
| 2. The bright
Star in the top of
the Head. | 10 | 02 39 | 57 9 57 | 00N | 3 |
| 3. The Eye of δ
<i>Palitium, Aldebaran</i> | 11 | 04 45 | 57 05 31 | 00S | 1 |
| 4. <i>Castor</i> , in the
higher Head of π . | 0 | 25 14 | 57 10 02 | 00N | 2 |
| 5. In the lower
Head of π <i>Pollux</i>
<i>Hercules</i> | 0 | 18 16 | 57 06 38 | 00N | 2 |
| 6. The Northern
Als. | 1 | 02 31 | 57 03 08 | 00N | 4 |
| 7. The Southern
Als. | 1 | 03 51 | 57 00 04 | 00S | 4 |
| 8. The Heart of
Ω , <i>Regulus</i> , or <i>Ba-
siliscus</i> . | 1 | 24 50 | 57 00 26 | 30N | 1 |
| 9. The bright
Star of the Tail of Ω . | 2 | 16 35 | 57 42 18 | 00N | 1 |
| 10. <i>Spica</i> μ . | 3 | 18 49 | 57 01 59 | 00S | 1 |
| 11. The South-
ern Balance. | 4 | 10 04 | 57 00 26 | 00N | 2 |
| 12. The North-
ern Balance. | 4 | 14 21 | 57 08 35 | 00N | 2 |
| 13. The Heart μ .
<i>Antares</i> . | 5 | 04 46 | 57 04 27 | 00S | 1 |
| 14. In the left
Shoulder γ . | 6 | 07 24 | 57 03 31 | 00S | 4 |
| 15. The North-
ern in the former
Horn $\vee$. | 6 | 28 51 | 57 07 02 | 00N | 3 |
| 16. The Southern
in the same Horn. | 6 | 29 04 | 57 04 41 | 00N | 3 |
| 17. The middle
between | 6 | 29 24 | 57 06 53 | 00N | |
| 18. In the Left
Shoulder of α . | 7 | 18 24 | 57 08 42 | 00N | 3 |

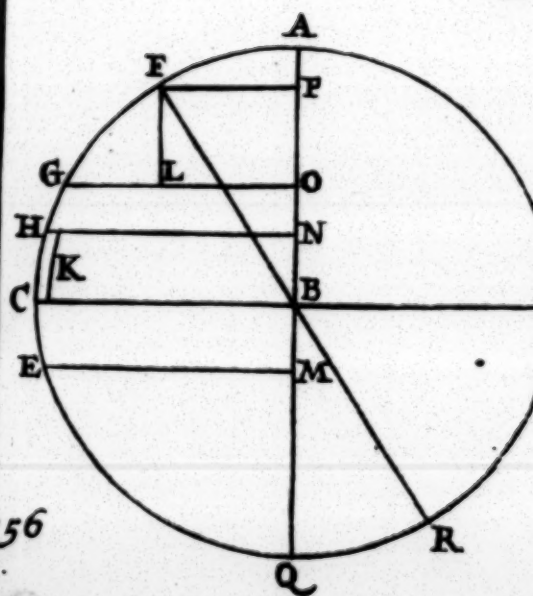
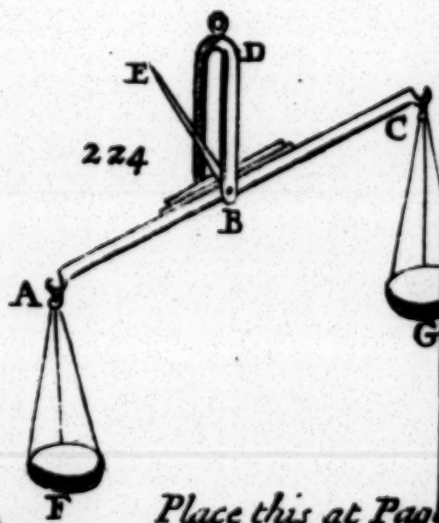
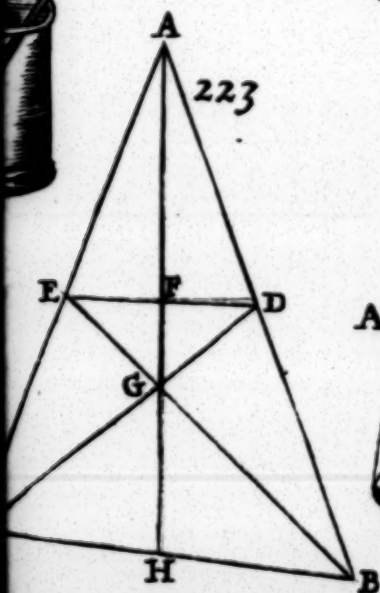
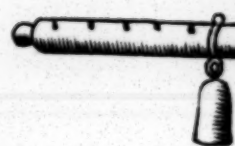
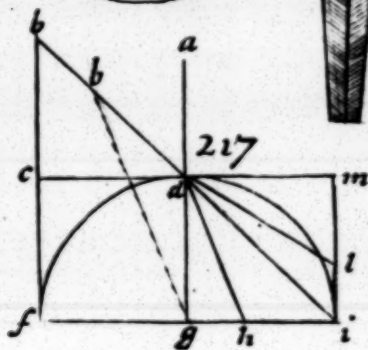
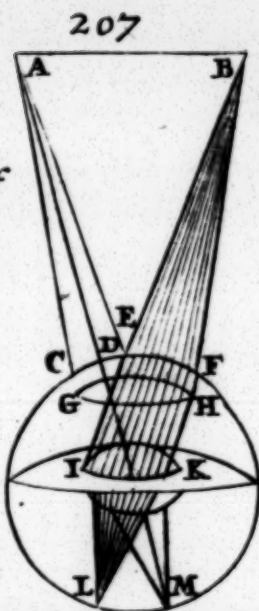
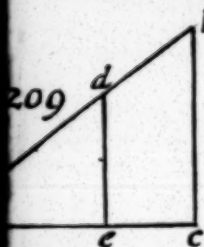
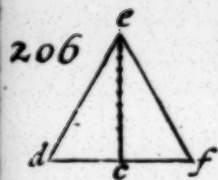
Names

The Scheme of an Eie

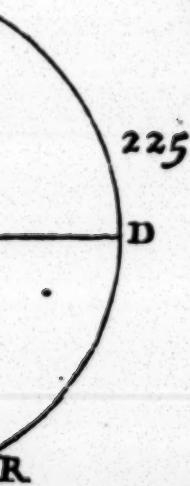
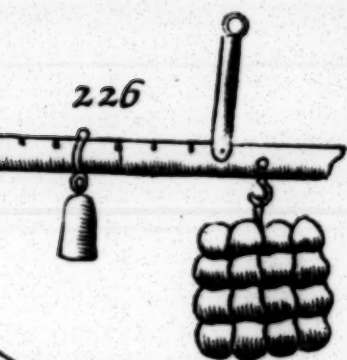
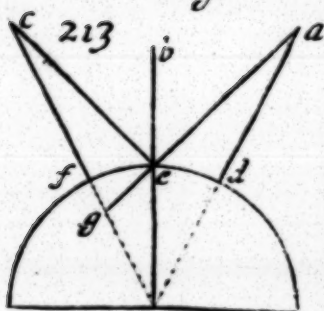
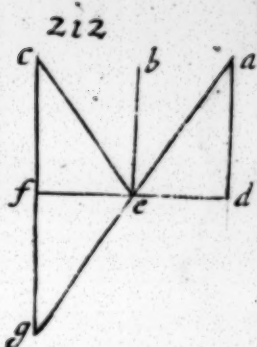




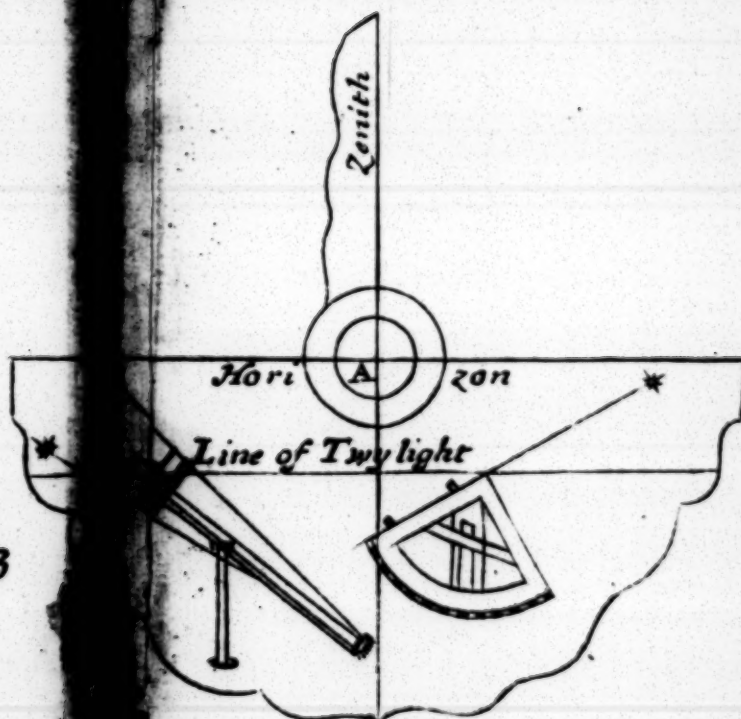
Place this at Page 56
to fold out.



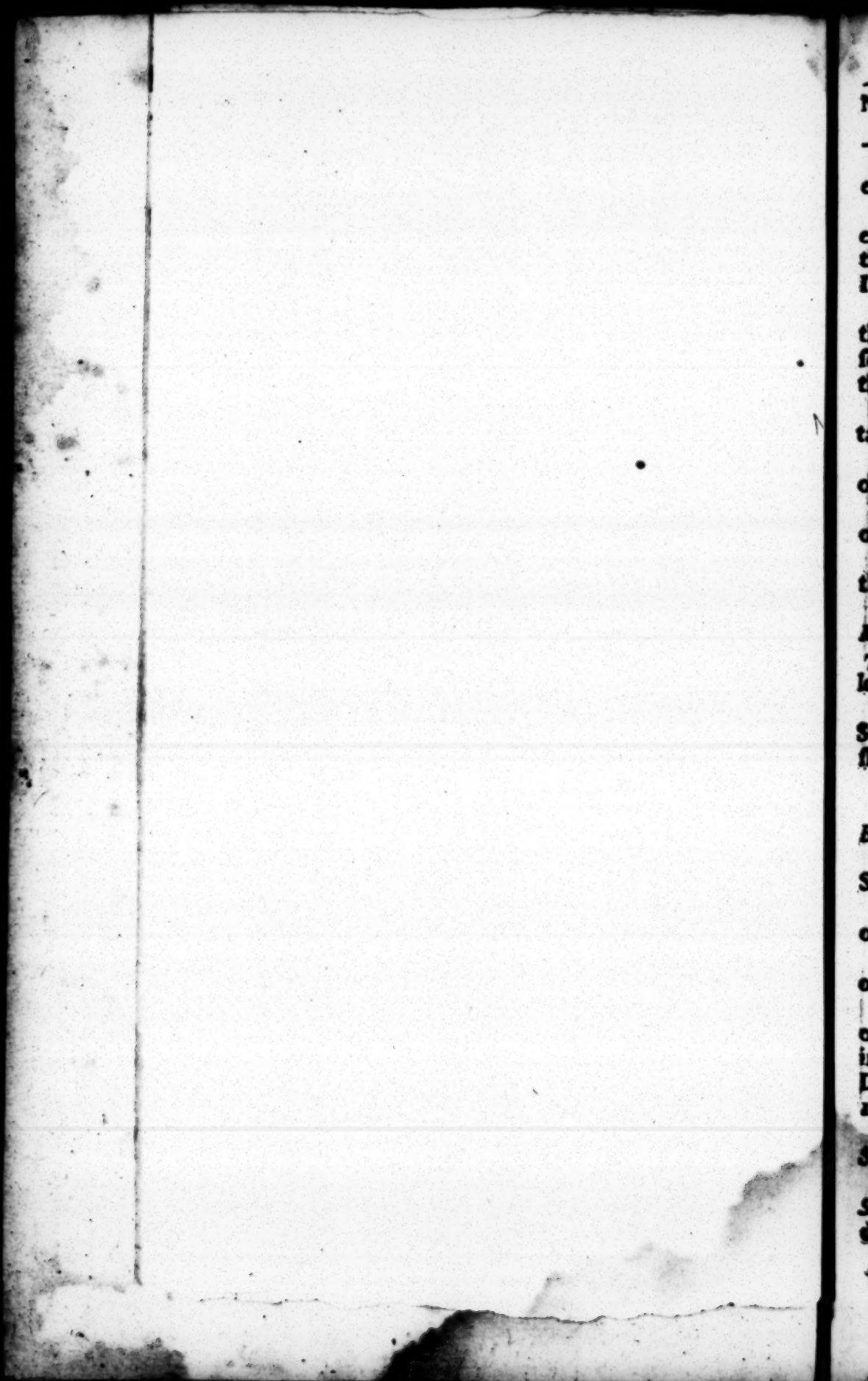
Place this at Page 56
to fold out.



143



is to turn round on Fig. 142



Names of STARS:

| | Their
Sig. | Longitude
Deg. | Their
Latit.
Deg. | Magni-
tude. |
|--|---------------|-------------------|-------------------------|-----------------|
| 19. The last in the
effusion, <i>Fumabant.</i> | 7 | 28 45 27 | 21 0 0S | 1.8 |
| 20. The bright-
er, in joining toge-
ther of both Nets. | 9 | 24 21 27 | 09 4 30S | 3 |
| II. Northern Stars. | | | | |
| 1. In the end of
the Tail of the les-
ser Bear; <i>Cynosura</i>
the Polar Star. | 11 | 23 36 27 | 65 2 0N | 2 |
| 2. The last of the
tail of the great bear. | 2 | 21 45 57 | 54 25 0N | 3 |
| 3. In the Breast
of <i>Cassiopea</i> , <i>schedir</i> , | 10 | 02 50 27 | 46 35 30N | 3 |
| 4. In the Girdle
of <i>Andromeda</i> . | 9 | 25 22 57 | 25 59 0N | 2 |
| 5. In the top of
the Triangle. | 10 | 01 52 57 | 15 49 30N | 4 |
| 6. The head of
<i>Medusa</i> or <i>Algol</i> . | 10 | 21 10 57 | 22 22 0N | 3 |
| 7. The Goat in the
left shoulder of <i>enoch</i> | 11 | 16 49 57 | 22 50 30N | 1 |
| 8. The bright
Star in the Right
Shoulder. | 11 | 26 25 57 | 21 27 30N | 2 |
| 9. <i>Coma Berenices</i> | 2 | 18 50 57 | 28 25 0N | 3 |
| 10. <i>Arcturus</i> , of
<i>Boots</i> . | 3 | 19 12 57 | 01 02 30N | 1 |
| 11. The bright
Star of the Crown. | 4 | 07 11 27 | 44 23 0N | 2 |
| 12. The Head
of <i>Hercules</i> . | 5 | 11 04 57 | 37 23 0N | 3 |
| 13. The Head
of <i>Opheus</i> . | 5 | 17 23 57 | 35 57 0N | 3 |
| 14. The former
of the light Stars
in the Head of the
Dragon. | 5 | 06 53 27 | 75 21 0N | 3 |
| 15. The following. | 5 | 22 57 57 | 75 3 30N | 3 |
| 16. The bright
Star of the Harp. | 6 | 10 16 57 | 61 47 50N | 1 |
| 17. The bright
Star in the shoul-
der of the Eagle. | 6 | 26 42 57 | 29 21 30N | 2 |

Names

| Names of STARS. | Their
Sig. | Longitude.
Deg. | Their
Deg. | Latitude.
Deg. | Magni-
tude. |
|--|---------------|--------------------|---------------|-------------------|-----------------|
| 18. In the Breast
of the Swan. | 7 | 19 58 57 | 57 | 09 30 N | 3 |
| 19. In the Tale
of the Swan. | 8 | 00 27 27 | 59 | 56 30 N | 2 |
| 20. In the mid-
dle of the Flying
Horse, the Head
of <i>Andromeda</i> ,
III. Of Southern
Stars. | 9 | 09 20 57 | 25 | 42 0 N | 3 |
| 1. The bright
Star of the Jaw of
the Whale. | 10 | 09 20 57 | 12 | 37 0 S | 2 |
| 2. The bright
Star in the Tail
of the Whale. | 8 | 27 29 57 | 20 | 47 0 S | 2 |
| 3. The right
Shoulder of <i>Orion</i> . | 11 | 23 45 57 | 16 | 06 0 S | 2 |
| 4. The left shoul-
der of the same. | 11 | 15 36 57 | 16 | 53 0 S | 2 |
| 5. The first of
the Belt. | 11 | 17 24 27 | 23 | 38 0 S | 2 |
| 6. The middle
Star of the same. | 11 | 18 27 57 | 24 | 33 30 S | 2 |
| 7. The last of
the Belt. | 11 | 19 40 27 | 25 | 21 30 S | 2 |
| 8. The left foot
of <i>Orion</i> , <i>Regel</i> . | 11 | 11 50 57 | 31 | 11 30 S | 1 |
| 9. The greater
Dog <i>Syrinx</i> . | 0 | 09 09 27 | 39 | 30 0 S | 2 |
| 10. Of the les-
ser Dog <i>Procyon</i> . | 0 | 20 52 57 | 15 | 57 0 S | 2 |
| 11. <i>Canopus</i> . | 0 | 08 33 57 | 73 | 0 0 S | 1 |
| 12. The bright
Star, or the Heart
of the Water Ser-
pent. | 1 | 23 19 27 | 22 | 24 0 S | 1 |

A Table of the motion of the Fixed Stars.

| years | sig. | Deg. | | Years. | sig. | De. | |
|-------|------|------|----|---------|------|-----|----|
| 1 | o | o | 51 | 60 | o | 0 | 51 |
| 2 | o | o | 42 | 80 | o | 1 | 08 |
| 3 | o | o | 33 | 100 | o | 1 | 25 |
| 4 | o | o | 24 | 200 | o | 2 | 50 |
| 5 | o | o | 15 | 300 | o | 3 | 15 |
| 6 | o | o | 06 | 400 | o | 5 | 40 |
| 7 | o | o | 57 | 500 | o | 7 | 05 |
| 8 | o | o | 48 | 600 | o | 8 | 30 |
| 9 | o | o | 39 | 700 | o | 9 | 55 |
| 10 | o | o | 30 | 800 | o | 11 | 20 |
| 11 | o | o | 21 | 900 | o | 12 | 45 |
| 12 | o | o | 12 | 1000 | o | 14 | 10 |
| 13 | o | o | 03 | 2000 | o | 28 | 20 |
| 14 | o | o | 54 | 3000 | o | 12 | 30 |
| 15 | o | o | 45 | 4000 | o | 1 | 26 |
| 16 | o | o | 36 | 5000 | o | 2 | 10 |
| 17 | o | o | 27 | 6000 | o | 2 | 25 |
| 18 | o | o | 18 | 7000 | o | 3 | 09 |
| 19 | o | o | 09 | 8000 | o | 3 | 23 |
| 20 | o | o | 00 | 9000 | o | 4 | 07 |
| 40 | o | o | 34 | 10000 | o | 4 | 21 |
| Janu. | o | o | 5 | July. | o | o | 30 |
| Febr. | o | o | 9 | August. | o | o | 24 |
| Mar. | o | o | 13 | Septeml | o | o | 18 |
| Apri. | o | o | 17 | October | o | o | 43 |
| May. | o | o | 21 | Novem | o | o | 47 |
| June. | o | o | 25 | Deceml | o | o | 51 |

A Table of the Suns entrance into $\odot$ r.
The Radixes at the beginning of the
Julian Period, and the days from
Midnight compleat.

| Ra. 1 Bissext. | Days | Ho. | Mi. | Se. | Rad. | Days | Ho. | M. | Se. | | |
|----------------|------|-----|-----|-----|----------|-------|------|----|-----|----|---|
| 2 Common. | 118 | 13 | 59 | 15 | 3 Comon. | 118 | 01 | 59 | 45 | | |
| | 117 | 19 | 59 | 30 | 4 Comon. | 118 | 08 | 00 | 00 | | |
| Years current | 1 | 0 | 0 | 11 | 15 | Years | 40 | 0 | 07 | 30 | 0 |
| | 2 | 0 | 0 | 22 | 30 | | 60 | 0 | 11 | 15 | 0 |
| | 3 | 0 | 0 | 33 | 45 | | 8 | 0 | 15 | 0 | 0 |
| | 4 | 0 | 0 | 45 | 0 | | 100 | 0 | 18 | 45 | 6 |
| | 5 | 0 | 0 | 56 | 15 | | 200 | 1 | 13 | 30 | 0 |
| | 6 | 0 | 1 | 07 | 30 | | 300 | 2 | 08 | 15 | 0 |
| | 7 | 0 | 1 | 18 | 45 | | 400 | 3 | 03 | 0 | 0 |
| | 8 | 0 | 1 | 30 | 0 | | 500 | 3 | 21 | 45 | 0 |
| | 9 | 0 | 1 | 41 | 15 | | 600 | 4 | 16 | 30 | 0 |
| | 10 | 0 | 1 | 52 | 30 | | 700 | 5 | 11 | 15 | 0 |
| | 11 | 0 | 2 | 03 | 45 | | 800 | 6 | 6 | 0 | 0 |
| | 12 | 0 | 2 | 15 | 0 | | 900 | 7 | 0 | 45 | 0 |
| | 13 | 0 | 2 | 26 | 15 | | 1000 | 7 | 19 | 30 | 0 |
| | 14 | 0 | 2 | 37 | 30 | | 2000 | 15 | 15 | 15 | |
| | 15 | 0 | 2 | 48 | 45 | | 3000 | 23 | 10 | 0 | 0 |
| | 16 | 0 | 3 | 0 | 0 | | 4000 | 31 | 6 | 45 | 0 |
| | 17 | 0 | 3 | 11 | 15 | | 5000 | 39 | 1 | 30 | 0 |
| | 18 | 0 | 3 | 22 | 30 | | 6000 | 46 | 21 | 0 | 0 |
| | 19 | 0 | 3 | 33 | 45 | | 7000 | 54 | 16 | 30 | 0 |
| | 20 | 0 | 3 | 45 | 0 | | 8000 | 62 | 12 | 0 | 0 |

The Suns entrance into the Twelve Signs of the Zodiack.

| | | | | | | | | | |
|--------------|-----|----|----|----|-------------|-----|----|----|----|
| Υ | 0 | 0 | 0 | 0 | $\cap$ | 186 | 18 | 26 | 8 |
| $\odot$ | 30 | 14 | 30 | 22 | m | 217 | 01 | 0 | 26 |
| II | 61 | 17 | 11 | 2 | $\dagger$ | 246 | 19 | 18 | 25 |
| III | 93 | 03 | 47 | 7 | VS | 276 | 05 | 40 | 37 |
| IV | 124 | 15 | 23 | 12 | m | 305 | 14 | 34 | 48 |
| V | 155 | 22 | 32 | 28 | X | 335 | 04 | 13 | 17 |
| | | | | | Y | 365 | 05 | 49 | 0 |

A Table of the equal motion of ☉ Longitude in Years and Months.

The Epocha or Radix at the beginning of the Julian Period, viz. the Midnight of the Calends of January.

Sig. Deg.
8 : 1 : 59 : 38

| Biflex. | Sig. | Deg. | Years | Sig. | Deg. |
|----------|------|------|-------|-----------|-------------|
| 1 | 0 | 0 | 44 49 | 40 | 0 0 18 2 |
| 2 | 0 | 0 | 30 29 | 60 | 0 0 27 12 |
| 3 | 0 | 0 | 16 09 | 80 | 0 0 36 16 |
| 4 | 0 | 0 | 01 49 | 100 | 0 0 45 20 |
| Bifl. | 5 | 0 | 46 38 | 300 | 0 1 30 41 |
| 6 | 0 | 0 | 32 18 | 300 | 0 2 16 01 |
| 7 | 0 | 0 | 17 58 | 400 | 0 3 01 23 |
| 8 | 0 | 0 | 03 38 | 500 | 0 3 46 42 |
| Bifl. | 9 | 0 | 48 27 | 600 | 0 4 32 2 |
| 10 | 0 | 0 | 34 7 | 700 | 0 5 17 23 |
| 11 | 0 | 0 | 19 47 | 800 | 0 6 2 43 |
| 12 | 0 | 0 | 5 27 | 900 | 0 6 48 4 |
| Bifl. | 13 | 0 | 50 16 | 1000 | 0 7 33 24 |
| 14 | 0 | 0 | 35 56 | 2000 | 0 15 6 48 |
| 15 | 0 | 0 | 21 36 | 3000 | 0 22 40 12 |
| 16 | 0 | 0 | 07 16 | 4000 | 1 0 13 36 |
| Bifl. | 17 | 0 | 52 4 | 5000 | 1 7 47 0 |
| 18 | 0 | 0 | 31 44 | 6000 | 1 15 20 24 |
| 19 | 0 | 0 | 23 24 | 7000 | 1 22 53 48 |
| 20 | 0 | 0 | 9 4 | 8000 | 2 0 27 12 |
| Months. | | | | Months. | |
| January. | 1 | 0 | 33 18 | July. | 6 28 57 26 |
| February | 1 | 28 | 09 11 | August. | 7 29 30 44 |
| March. | 2 | 28 | 42 30 | September | 8 29 04 54 |
| April. | 3 | 28 | 16 39 | October | 9 29 38 12 |
| May. | 4 | 28 | 40 58 | November. | 10 29 12 22 |
| June. | 5 | 28 | 24 08 | December. | 11 29 45 40 |

In the Biflexile Year after February add to the Months 49' : 8"

A TABLE of the equal Motion of Longitude of ☉ in Days, Hours and Minutes.

| Da | De | Da | De | Da | De |
|----|---------|----|----------|----|----------|
| 1 | 2 58 28 | 11 | 15 47 32 | 21 | 20 41 49 |
| 2 | 1 58 17 | 12 | 11 47 40 | 22 | 21 41 03 |
| 3 | 2 57 25 | 13 | 13 48 48 | 23 | 22 40 12 |
| 4 | 3 55 33 | 14 | 13 47 59 | 24 | 23 39 20 |
| 5 | 4 55 42 | 15 | 14 47 05 | 25 | 24 38 28 |
| 6 | 5 54 50 | 16 | 15 46 13 | 26 | 25 37 37 |
| 7 | 6 53 58 | 17 | 16 45 22 | 27 | 26 36 44 |
| 8 | 7 53 07 | 18 | 17 44 30 | 28 | 27 35 52 |
| 9 | 8 52 15 | 19 | 18 43 38 | 29 | 28 35 02 |
| 10 | 9 51 23 | 20 | 19 42 47 | 30 | 29 34 10 |

In Hours and Minutes.

| Ho | De | Ho | De | Ho | De |
|----|---------|----|---------|----|----------|
| 1 | 0 02 28 | 11 | 0 51 45 | 41 | 01 41 2 |
| 2 | 0 04 56 | 22 | 0 54 13 | 42 | 01 41 30 |
| 3 | 0 07 42 | 23 | 0 56 40 | 43 | 01 41 58 |
| 4 | 0 09 51 | 24 | 0 59 08 | 44 | 01 42 25 |
| 5 | 0 12 19 | 25 | 1 01 36 | 45 | 01 50 53 |
| 6 | 0 14 47 | 26 | 1 04 04 | 46 | 01 51 20 |
| 7 | 0 17 15 | 27 | 1 06 32 | 47 | 01 51 48 |
| 8 | 0 19 43 | 28 | 1 09 00 | 48 | 01 52 16 |
| 9 | 0 22 11 | 29 | 1 11 28 | 49 | 02 00 44 |
| 10 | 0 24 38 | 30 | 1 13 55 | 50 | 02 01 12 |
| 11 | 0 27 06 | 31 | 1 16 24 | 51 | 02 01 40 |
| 12 | 0 29 34 | 32 | 1 18 52 | 52 | 02 08 08 |
| 13 | 0 32 02 | 33 | 1 21 19 | 53 | 02 10 36 |
| 14 | 0 34 30 | 34 | 1 23 46 | 54 | 02 13 04 |
| 15 | 0 36 58 | 35 | 1 26 14 | 55 | 02 15 32 |
| 16 | 0 39 26 | 36 | 1 28 42 | 56 | 02 18 00 |
| 17 | 0 41 53 | 37 | 1 31 10 | 57 | 02 20 28 |
| 18 | 0 44 21 | 38 | 1 33 38 | 58 | 02 22 56 |
| 19 | 0 46 39 | 39 | 1 36 06 | 59 | 02 25 24 |
| 20 | 0 49 17 | 40 | 1 38 34 | 60 | 02 28 00 |

If you would know the equal Motion of ☉ Longitude in Hours, you must have recourse to the top of the Table. But for Minutes, you must mind the bottom of the Table. Example, The Motion for 20 Hours is 49': 17". But for 20', 'tis 49": 17". &c.

A TABLE of the Equal Motion of Sol Apogeon: The Radix at the be- ginning of the Julian Period.

| S. 0 | | | Y | | |
|-------------|-----|-------|---------|-----------|------|
| II 17 39 54 | | | | | |
| Years. | De. | Min. | Years. | De. | Min. |
| 1 | 0 | 1 2 | 40 | 0 41 05 | |
| 2 | 0 | 2 4 | 60 | 1 01 38 | |
| 3 | 0 | 3 5 | 80 | 1 22 10 | |
| 4 | 0 | 4 7 | 100 | 1 42 43 | |
| 5 | 0 | 5 8 | 200 | 3 25 25 | |
| 6 | 0 | 6 10 | 300 | 5 08 08 | |
| 7 | 0 | 7 12 | 400 | 6 50 51 | |
| 8 | 0 | 8 13 | 500 | 8 33 34 | |
| 9 | 0 | 9 15 | 600 | 10 16 18 | |
| 10 | 0 | 10 16 | 700 | 11 58 59 | |
| 11 | 0 | 11 18 | 800 | 13 41 42 | |
| 12 | 0 | 12 20 | 900 | 15 24 25 | |
| 13 | 0 | 13 21 | 1000 | 17 07 07 | |
| 14 | 0 | 14 23 | 2000 | 34 14 15 | |
| 15 | 0 | 15 25 | 3000 | 51 21 22 | |
| 16 | 0 | 16 26 | 4000 | 68 28 29 | |
| 17 | 0 | 17 28 | 5000 | 85 35 37 | |
| 18 | 0 | 18 29 | 6000 | 102 42 45 | |
| 19 | 0 | 19 31 | 7000 | 119 49 50 | |
| 20 | 0 | 20 33 | 8000 | 137 27 00 | |
| Months. | | | Months. | | |
| Janua. | 0 | 01 5 | July | 0 0 36 | |
| Febru. | 0 | 0 10 | Aug. | 0 0 41 | |
| Marc. | 0 | 0 15 | Sept. | 0 0 46 | |
| April | 0 | 0 20 | Octo. | 0 0 51 | |
| May | 0 | 0 25 | Nov. | 0 0 56 | |
| June | 0 | 0 30 | Dece. | 0 1 02 | |

A TABLE of Lutations. The Radix of the Julian Period, from Midnight Current.

Days 19. Hours 1. Minutes 50. Scruples 25.

| Years. | Da. Ho. ' " | Years. | Day. Ho. ' " |
|--------|-------------|--------|--------------|
| 1 | 11 15 11 22 | 40 | 21 21 17 49 |
| 2 | 22 06 22 44 | 60 | 03 07 12 40 |
| 3 | 03 08 50 02 | 80 | 14 05 51 34 |
| 4 | 14 00 01 24 | 100 | 25 04 30 29 |
| 5 | 25 15 12 46 | 200 | 20 20 16 54 |
| 6 | 06 17 40 05 | 300 | 16 12 03 20 |
| 7 | 17 08 51 26 | 400 | 12 03 49 45 |
| 8 | 28 00 02 48 | 500 | 07 19 36 10 |
| 9 | 10 02 30 07 | 600 | 03 11 22 36 |
| 10 | 20 17 41 29 | 700 | 28 15 53 04 |
| 11 | 01 20 08 47 | 800 | 24 07 39 30 |
| 12 | 12 11 20 09 | 900 | 19 23 25 55 |
| 13 | 24 02 31 31 | 1000 | 15 15 12 21 |
| 14 | 05 04 58 50 | 2000 | 01 17 40 38 |
| 15 | 15 20 10 12 | 3000 | 17 08 52 59 |
| 16 | 26 11 21 33 | 4000 | 03 11 1 17 |
| 17 | 08 13 48 52 | 5000 | 19 02 33 38 |
| 18 | 19 05 00 14 | 6000 | 05 05 10 55 |
| 19 | 00 47 27 33 | 7000 | 10 20 14 16 |
| 20 | 10 22 38 54 | 8000 | 06 22 42 34 |

Epacts of Months.

Canons of Conjunctions of ☉ and ☾

| | | | |
|------------|-------------|---------------|--------------|
| January | 01 11 15 57 | I. | 29 12 44 03 |
| Feb. Co. | 29 11 15 57 | II. | 59 01 28 06 |
| Feb. Biff. | 0 28 31 54 | III. | 88 14 12 10 |
| March | 1 09 47 50 | IV. | 118 02 56 13 |
| April | 1 21 03 47 | V. | 147 15 40 16 |
| May | 3 08 19 44 | VI. | 177 04 24 19 |
| June | 3 19 35 41 | VII. | 206 17 08 22 |
| July | 5 06 51 38 | VIII. | 236 05 52 25 |
| August | 6 18 07 35 | IX. | 265 18 36 29 |
| | | X. | 295 07 20 32 |
| September | 7 5 23 31 | XI. | 324 20 04 35 |
| October | 8 16 39 28 | XII. | 354 08 48 38 |
| November | 9 03 55 25 | XIII. | 383 21 32 41 |
| December | 10 15 11 22 | Part of half | 14 18 22 02 |
| | | Part of four. | 7 09 11 1 |

In Bissextile one Day more.

Part

A TABLE of the Anomaly, or Reason of Latitude : The Epocha or Radix at the Mid-night of the beginning of the *Julian* Period.

Signs 4. Deg. 19. Minutes 43. Seconds 38.

| Biffer | Si. | De. | " | Years. | Si. | De. | " |
|---------|-----|-----|-------|---------|-----|-----|-------|
| Bif. 1 | 05 | 11 | 56 32 | 40 | 10 | 20 | 47 58 |
| 2 | 10 | 10 | 39 17 | 60 | 04 | 01 | 11 57 |
| 3 | 03 | 09 | 22 02 | 80 | 09 | 11 | 35 56 |
| 4 | 08 | 08 | 04 48 | 100 | 02 | 21 | 59 58 |
| B. 5 | 08 | 20 | 01 20 | 200 | 05 | 13 | 59 56 |
| 6 | 06 | 18 | 44 06 | 300 | 08 | 05 | 59 54 |
| 7 | 11 | 17 | 26 49 | 400 | 10 | 27 | 59 52 |
| 8 | 04 | 16 | 09 35 | 500 | 01 | 19 | 59 50 |
| B. 9 | 09 | 28 | 06 07 | 600 | 04 | 11 | 59 46 |
| 10 | 02 | 26 | 48 53 | 700 | 07 | 03 | 59 44 |
| 11 | 07 | 25 | 31 39 | 800 | 09 | 25 | 59 49 |
| 12 | 00 | 24 | 14 24 | 900 | 00 | 17 | 59 39 |
| B. 13 | 06 | 06 | 10 56 | 1000 | 03 | 09 | 59 37 |
| 14 | 11 | 04 | 53 42 | 2000 | 06 | 19 | 59 14 |
| 15 | 04 | 03 | 36 28 | 3000 | 09 | 29 | 58 50 |
| 16 | 09 | 02 | 19 12 | 4000 | 01 | 09 | 58 27 |
| B. 17 | 02 | 14 | 15 44 | 5000 | 04 | 19 | 58 13 |
| 18 | 07 | 12 | 58 30 | 6000 | 07 | 29 | 57 40 |
| 12 | 00 | 11 | 41 16 | 7000 | 11 | 09 | 57 17 |
| 20 | 05 | 10 | 23 59 | 8000 | 02 | 19 | 56 53 |
| Months. | | | | Months. | | | |
| Janu. | 01 | 20 | 06 36 | July | 09 | 14 | 37 19 |
| Febr. | 02 | 00 | 31 54 | Augu. | 11 | 04 | 43 55 |
| Marc. | 03 | 20 | 38 29 | Sept. | 00 | 11 | 36 45 |
| April | 04 | 27 | 31 18 | Octo. | 02 | 01 | 43 20 |
| May | 06 | 17 | 37 54 | Nove. | 03 | 08 | 36 10 |
| June | 07 | 24 | 30 45 | Dece. | 04 | 28 | 42 46 |

In the Bissextile Year after the compleat Day of the Month of February, with that which you take out, you must encrease it by Unity.

A Table of the Anomaly of ϵ in Days, Hours and Scruples.

| Days | Sig. | Deg. | Days | Sig. | Deg. | Days | Sig. | Deg. | |
|------------------------|----------|----------|----------|------|----------|------|----------|----------|----------|
| 1 | 00 | 13 13 46 | 11 | 04 | 25 31 22 | 21 | 09 | 07 48 56 | |
| 2 | 00 | 26 27 31 | 12 | 05 | 08 45 08 | 22 | 09 | 21 02 44 | |
| 3 | 1 | 9 41 17 | 13 | 5 | 21 58 53 | 23 | 10 | 04 16 30 | |
| 4 | 1 | 22 55 3 | 14 | 6 | 5 12 35 | 24 | 10 | 17 30 18 | |
| 5 | 2 | 6 8 48 | 15 | 6 | 18 26 25 | 25 | 11 | 0 44 2 | |
| 6 | 2 | 19 22 34 | 16 | 7 | 01 40 15 | 26 | 11 | 12 57 48 | |
| 7 | 3 | 2 36 19 | 17 | 7 | 14 53 55 | 27 | 11 | 27 11 33 | |
| 8 | 3 | 15 50 5 | 18 | 7 | 28 7 41 | 28 | 9 | 10 25 19 | |
| 9 | 3 | 29 3 5 | 19 | 8 | 11 21 27 | 29 | 9 | 23 39 6 | |
| 10 | 4 | 12 17 36 | 20 | 8 | 24 35 15 | 30 | 1 | 06 52 50 | |
| In Hours and Scruples. | | | | | | | | | |
| Hou. | Deg. | Hou. | Deg. | Hou. | Deg. | Hou. | Deg. | Hou. | |
| 1 | 0 33 04 | 21 | 11 34 33 | 41 | 22 36 0 | 61 | 33 37 29 | 81 | 44 38 58 |
| 2 | 1 6 9 | 22 | 12 7 37 | 42 | 23 09 6 | 62 | 34 11 32 | 82 | 45 11 51 |
| 3 | 1 39 13 | 23 | 12 40 42 | 43 | 23 42 10 | 63 | 34 44 36 | 83 | 45 44 55 |
| 4 | 2 12 18 | 24 | 13 13 46 | 44 | 24 15 14 | 64 | 35 17 40 | 84 | 46 17 58 |
| 5 | 2 45 22 | 25 | 13 46 50 | 45 | 24 48 18 | 65 | 35 50 44 | 85 | 46 50 52 |
| 6 | 3 18 27 | 26 | 14 19 55 | 46 | 25 21 22 | 66 | 36 23 48 | 86 | 47 23 56 |
| 7 | 3 51 31 | 27 | 14 52 58 | 47 | 25 54 26 | 67 | 36 56 52 | 87 | 47 56 52 |
| 8 | 4 24 36 | 28 | 15 26 3 | 48 | 26 27 30 | 68 | 37 29 56 | 88 | 48 29 52 |
| 9 | 4 57 40 | 29 | 15 59 7 | 49 | 27 0 36 | 69 | 38 02 50 | 89 | 48 02 48 |
| 10 | 5 30 44 | 30 | 16 32 12 | 50 | 27 33 40 | 70 | 38 35 54 | 90 | 48 35 44 |
| 11 | 6 03 48 | 31 | 17 5 16 | 51 | 28 6 46 | 71 | 39 08 58 | 91 | 49 08 40 |
| 12 | 6 36 53 | 32 | 17 38 20 | 52 | 28 39 50 | 72 | 39 41 52 | 92 | 49 41 36 |
| 13 | 7 9 57 | 33 | 18 11 25 | 53 | 29 12 54 | 73 | 40 14 56 | 93 | 50 14 32 |
| 14 | 7 43 01 | 34 | 18 44 30 | 54 | 29 45 58 | 74 | 40 47 50 | 94 | 50 47 28 |
| 15 | 8 16 6 | 35 | 19 17 34 | 55 | 30 19 0 | 75 | 41 20 54 | 95 | 51 20 24 |
| 16 | 8 49 10 | 36 | 19 50 38 | 56 | 30 52 6 | 76 | 41 53 58 | 96 | 51 53 20 |
| 17 | 9 22 14 | 37 | 20 23 42 | 57 | 31 25 10 | 77 | 42 26 52 | 97 | 52 26 16 |
| 18 | 9 55 19 | 38 | 20 56 48 | 58 | 31 58 14 | 78 | 42 59 56 | 98 | 52 59 12 |
| 19 | 10 28 28 | 39 | 21 9 52 | 59 | 32 31 18 | 79 | 43 32 50 | 99 | 53 32 8 |
| 20 | 11 1 28 | 40 | 22 2 56 | 60 | 32 5 22 | 80 | 44 05 54 | 100 | 54 05 18 |
| Min | " " | Min | " " | Min | " " | Min | " " | Min | |
| Sec. | " " iv | Sec. | " " iv | Sec. | " " iv | Sec. | " " iv | Sec. | |

Con-

A GEOGRAPHICAL TABLE, containing the degrees of the parallels of the Equator turned into Miles.

| Degr. | Miles | Scr. | Degr. | Miles | Scr. | Degr. | Miles | Scruples |
|-------|-------|------|-------|-------|------|-------|-------|----------|
| 1 | 14 | 59 | 31 | 12 | 81 | 61 | 7 | 15 |
| 2 | 14 | 59 | 32 | 12 | 43 | 62 | 7 | 9 |
| 3 | 14 | 58 | 33 | 12 | 35 | 63 | 6 | 48 |
| 4 | 14 | 58 | 34 | 12 | 26 | 64 | 6 | 34 |
| 5 | 14 | 56 | 35 | 12 | 17 | 65 | 6 | 20 |
| 6 | 14 | 55 | 36 | 12 | 8 | 66 | 6 | 6 |
| 7 | 14 | 53 | 37 | 11 | 59 | 67 | 5 | 52 |
| 8 | 14 | 51 | 38 | 11 | 49 | 68 | 5 | 37 |
| 9 | 14 | 48 | 39 | 11 | 39 | 69 | 5 | 23 |
| 10 | 14 | 46 | 40 | 11 | 29 | 70 | 5 | 8 |
| 11 | 14 | 43 | 41 | 11 | 19 | 71 | 4 | 53 |
| 12 | 14 | 40 | 42 | 11 | 9 | 72 | 4 | 38 |
| 13 | 14 | 37 | 43 | 10 | 58 | 73 | 4 | 23 |
| 14 | 14 | 33 | 44 | 10 | 47 | 74 | 4 | 8 |
| 15 | 14 | 29 | 45 | 10 | 36 | 75 | 3 | 53 |
| 16 | 14 | 25 | 46 | 10 | 25 | 76 | 3 | 38 |
| 17 | 14 | 21 | 47 | 10 | 14 | 77 | 3 | 22 |
| 18 | 14 | 16 | 48 | 10 | 2 | 78 | 3 | 07 |
| 19 | 14 | 11 | 49 | 9 | 50 | 79 | 2 | 52 |
| 20 | 14 | 6 | 50 | 9 | 38 | 80 | 2 | 36 |
| 21 | 14 | 0 | 51 | 9 | 26 | 81 | 2 | 21 |
| 22 | 13 | 54 | 52 | 9 | 14 | 82 | 2 | 5 |
| 23 | 13 | 48 | 53 | 9 | 2 | 83 | 1 | 50 |
| 24 | 13 | 42 | 54 | 8 | 49 | 84 | 1 | 34 |
| 25 | 13 | 36 | 55 | 8 | 36 | 85 | 1 | 18 |
| 26 | 13 | 29 | 56 | 8 | 23 | 86 | 1 | 3 |
| 27 | 13 | 22 | 57 | 8 | 10 | 87 | 0 | 47 |
| 28 | 13 | 15 | 58 | 7 | 57 | 88 | 0 | 31 |
| 29 | 13 | 7 | 59 | 7 | 43 | 89 | 0 | 16 |
| 30 | 12 | 59 | 60 | 7 | 20 | 90 | 0 | 0 |

Note, This Table consists of German Miles and Scruples: but if you would know how many English Miles and Scruples any parallel contains, you must multiply the German Miles and Scruples by 4. Example, the 15th parallel contains 14 : 29, which multiplied by 4, produces 57 : 56.

A Catalogue of some of the chiefeſt Cities in the World, ſhewing the temporary Differences of their Meridians from *London*; with the Height of the Pole in each Place.

| <i>Names of the Cities.</i> | Differen. of Meridians.
Ho. | | | Height of Pole.
De. | |
|---|--------------------------------|----|---|------------------------|----|
| <i>Aberdeen in Scotland</i> | 0 | 9 | S | 58 | 40 |
| <i>Belgrade in Hungary</i> | 1 | 24 | A | 45 | 16 |
| <i>Alexandria in Egypt</i> | 2 | 36 | A | 30 | 58 |
| <i>Amberg</i> | 0 | 48 | A | 49 | 32 |
| <i>Amſterdam in Holland</i> | 0 | 21 | A | 52 | 24 |
| <i>Antwerp in Brabant</i> | 0 | 18 | A | 51 | 12 |
| <i>Aquiſgrane</i> | 0 | 26 | A | 50 | 48 |
| <i>Arim of the Arabian Aſtrologer call'd the middle of the World.</i> | 5 | 37 | A | 00 | 00 |
| <i>Athens in Greece</i> | 1 | 51 | A | 37 | 42 |
| <i>Anſpurg in Swethland</i> | 0 | 46 | A | 48 | 22 |
| <i>Orleans in France</i> | 0 | 0 | | 48 | 08 |
| <i>Bamberge</i> | 0 | 43 | A | 49 | 57 |
| <i>Babylon in Caldea</i> | 3 | 40 | A | 35 | 00 |
| <i>Barcimo in Catalonia</i> | 0 | 10 | A | 40 | 45 |
| <i>Bethlehem</i> | 2 | 46 | A | 31 | 50 |
| <i>Breme in Saxony</i> | 0 | 34 | A | 53 | 08 |
| <i>Bruxels in Brabant</i> | 0 | 16 | A | 50 | 48 |
| <i>Brunſwick in Saxony</i> | 0 | 41 | A | 52 | 16 |
| <i>Burdeaux in France</i> | 0 | 12 | S | 45 | 47 |
| <i>Calice in France</i> | 0 | 00 | | 50 | 52 |
| <i>Calicute in Eaſt India</i> | 5 | 50 | A | 11 | 30 |
| <i>Carthage in Affricka</i> | 0 | 40 | A | 34 | 50 |

Names

| <i>Names of the Cities.</i> | Differen. of
Meridians.
Ho. | | | Height
of Pole.
De. | |
|-----------------------------|-----------------------------------|----|---|---------------------------|----|
| Cassel in Hefia | 0 | 36 | A | 51 | 19 |
| Compostella in Spain | 0 | 38 | S | 43 | 00 |
| Constantinople | 2 | 18 | A | 43 | 00 |
| Conimbrã in Lusitania | 0 | 37 | S | 40 | 15 |
| Cracovia in Poland | 1 | 21 | A | 49 | 58 |
| Damascus in Syria | 3 | 16 | A | 34 | 00 |
| Dantzick | 1 | 18 | A | 54 | 23 |
| Edenburgh in Scotland | 0 | 11 | S | 57 | 06 |
| Ephesus | 2 | 17 | A | 38 | 18 |
| Ferraria in Italy | 0 | 43 | A | 44 | 45 |
| Fesse in Mauritania | 0 | 16 | S | 33 | 15 |
| Francford upon Odar | 1 | 00 | A | 52 | 20 |
| Frœnburg in Prussia | 1 | 20 | A | 54 | 29 |
| Francford upon Moene | 0 | 32 | A | 50 | 07 |
| Gadiz or Cadiz | 0 | 24 | S | 36 | 10 |
| Geneva | 0 | 26 | A | 45 | 54 |
| Gaunt in Flanders | 0 | 16 | A | 50 | 54 |
| Goa in India | 5 | 48 | A | 16 | 00 |
| Ter-Goose in Zealand | 0 | 16 | A | 51 | 30 |
| Hamburg in Holsatia | 0 | 39 | A | 33 | 43 |
| Heidelburge | 0 | 35 | A | 49 | 26 |
| Hierusalem | 3 | 05 | A | 32 | 10 |
| Ingolstade | 0 | 47 | A | 48 | 42 |
| Lisborn in Portugal | 0 | 36 | S | 38 | 45 |
| Lyons in France | 0 | 20 | A | 45 | 16 |
| Leydon | 0 | 19 | A | 32 | 10 |
| LONDON | 0 | 00 | | 51 | 32 |
| Lubeck in Saxony | 0 | 40 | A | 53 | 58 |
| Magdeburg in Saxony | 0 | 47 | A | 52 | 16 |

Names

| <i>Names of the Cities.</i> | Differen. of
Meridians.
Ho. | | | Height
of Pole.
De. | |
|--------------------------------|-----------------------------------|----|---|---------------------------|----|
| <i>Saint Malo</i> | 0 | 20 | S | 48 | 44 |
| <i>Mexico</i> | 6 | 50 | S | 20 | 06 |
| <i>Macao in Kingd. Pegu</i> | 7 | 09 | A | 19 | 30 |
| <i>Malico in East India</i> | 7 | 19 | A | 02 | 24 |
| <i>Maliapur St. Tho. Sepu.</i> | 6 | 13 | A | 13 | 00 |
| <i>Morocco</i> | 0 | 31 | S | 31 | 15 |
| <i>Marpurg in Hestia</i> | 0 | 33 | A | 50 | 43 |
| <i>Mecha in Arabia</i> | 3 | 23 | A | 23 | 00 |
| <i>Messane in Sicilia</i> | 1 | 06 | A | 37 | 52 |
| <i>Midleburg in Zealand</i> | 0 | 16 | A | 51 | 30 |
| <i>Coninsburge</i> | 1 | 28 | A | 55 | 08 |
| <i>Mentz in Germany</i> | 0 | 45 | A | 00 | 00 |
| <i>Munster in Westphalia</i> | 0 | 28 | A | 51 | 54 |
| <i>Madrid Seat of K. Spain</i> | 0 | 11 | S | 40 | 45 |
| <i>Mount Sinai</i> | 2 | 40 | A | 30 | 00 |
| <i>Narbon in France</i> | 0 | 01 | A | 43 | 26 |
| <i>Nattaw</i> | 0 | 42 | A | 50 | 16 |
| <i>Ninive in Assyria</i> | 3 | 38 | A | 35 | 50 |
| <i>Naples in Italy</i> | 1 | 01 | A | 40 | 42 |
| <i>Norimberg in Germany</i> | 0 | 46 | A | 49 | 26 |
| <i>Presburg in Hungary</i> | 1 | 09 | A | 48 | 25 |
| <i>Paris in France</i> | 0 | 02 | A | 48 | 51 |
| <i>Prague in Bohemia</i> | 0 | 56 | A | 50 | 56 |
| <i>Padua</i> | 0 | 49 | A | 36 | 25 |
| <i>Pergamus</i> | 2 | 10 | A | 41 | 08 |
| <i>Quinzoy</i> | 11 | 28 | A | 37 | 40 |
| <i>Rome in Italy</i> | 0 | 50 | A | 42 | 02 |
| <i>Ravensburg in Bavaria</i> | 0 | 41 | A | 47 | 45 |

Names.

| <i>Names of the Cities.</i> | Differen. of
Meridians.
Ho. | Height
of Pole.
De. |
|------------------------------|-----------------------------------|---------------------------|
| Rotterdam <i>in</i> Holland | 0 14 A | 51 55 |
| Rochel | 0 04 S | 45 49 |
| Roan <i>in</i> Normandy | 0 06 A | 49 30 |
| Salisburge <i>in</i> Bavaria | 0 54 A | 47 42 |
| Spier <i>upon</i> the Rhine | 0 35 A | 49 24 |
| Sretine <i>in</i> Germany | 0 58 A | 53 36 |
| Smyrna <i>in</i> Asia | 2 14 A | 38 50 |
| Stockholm <i>in</i> Swedland | 1 01 A | 58 50 |
| Syracusa <i>in</i> Sicily | 1 05 A | 36 50 |
| Theffalonica | 1 45 A | 41 32 |
| Tripolis <i>in</i> Syria | 3 00 A | 34 50 |
| Tubing Wittenburge | 0 38 A | 48 34 |
| Trier | 0 26 A | 49 50 |
| Torge <i>in</i> Misnai | 0 53 A | 51 33 |
| Copenhagen <i>in</i> Denmar. | 0 50 A | 55 43 |
| Valence <i>in</i> Spain | 0 04 A | 39 30 |
| Venice | 0 48 A | 45 15 |
| Vienna <i>in</i> Austria | 1 05 A | 48 22 |
| Weymar | 0 46 A | 51 08 |
| Ulme <i>in</i> Lower Swedl. | 0 42 A | 48 24 |
| Uraniburg | 0 50 A | 55 54 |
| Uliſſippo <i>in</i> Portugal | 0 35 S | 38 45 |
| Worms | 0 29 A | 50 25 |
| Wittenburg <i>in</i> Saxony | 0 52 A | 51 52 |

F I N I S.

Advertisements.

*The Author doth intend, God willing, very quickly
to Publish the following Treatises.*

THE Art of Dyalling Geometrically and Arithmetically, divided into 4 Parts; each Part being a compleat Treatise of Dyalling for the whole Terrene Globe: With Gnomonick Tables, for making all manner of Dyals to any Declination, and inciribing their Furniture. Together with the Method of making the Tables, both Linearly and Logarithmically for any Latitude.

- I. A Tractate in proportional Numbers; containing Part the 1st. concerning the Ratio of Numbers: Part the 2d. of finding proportional Numbers.
- II. A Tractate in numerical Proportions; containing Part the 1st. concerning Geometrical Proportion continued: Part the 2d. of Arithmetical Proportion: Part the 3d. of Harmonical Proportion.
- III. A Tractate concerning the Proportions of Lines and of Segments.
- IV. A Tractate of proportional Progression; wherein Part the 1st. treats of the Geometrical Progression of Lines: Part the 2d. of the Harmonical Progression of Lines.
- V. A Tractate concerning the Proportionalities of Ratio's.



